Exercise 14.1

Question 1:

Find the common factors of the given terms.

(i) $12x, 36$

(ii) $2y, 22xy$

(iii) $14pq, 28p^2q^2$

(iv) $2x, 3x^2, 4$

(v) $6abc, 24ab^2, 12a^2b$

(vi) $16x^3, -4x^2, 32x$

(vii) $10pq, 20qr, 30rp$

(viii) $3x^2y^3, 10x^3y^2, 6x^2y^2z$

Answer 1:

(i) $12x = 2 \times 2 \times 3 \times x$

$36 = 2 \times 2 \times 3 \times 3$

Hence, the common factors are 2, 2 and 3 = 2 x 2 x 3 = 12

(ii) $2y = 2 \times y$

$22xy = 2 \times 11 \times x \times y$

Hence, the common factors are 2 and $y = 2 \times y = 2y$

(iii) $14pq = 2 \times 7 \times p \times q$

$28p^2q^2 = 2 \times 2 \times 7 \times p \times p \times q \times q$

Hence, the common factors are $2 \times 7 \times p \times q = 14pq$

(iv) $2x = 2 \times x \times 1$

$3x^2 = 3 \times x \times x \times 1$

$4 = 2 \times 2 \times 1$

Hence, the common factor is 1.

(v) $6abc = 2 \times 3 \times a \times b \times c$

$24ab^2 = 2 \times 2 \times 2 \times 3 \times a \times b \times b$

$12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

Hence, the common factors are $2 \times 3 \times a \times b = 6ab$

(vi) $16x^3 = 2 \times 2 \times 2 \times 2 \times x \times x \times x$

$-4x^2 = (-1) \times 2 \times 2 \times x \times x$

$32x = 2 \times 2 \times 2 \times 2 \times 2 \times x$

Hence, the common factors are $2 \times 2 \times x = 4x$

(vii) $10pq = 2\times5\times p\times q$

$20qr = 2\times2\times5\times q\times r$

$30rp = 2\times3\times5\times r\times p$

Hence, the common factors are $2\times5=10$

(viii) $3x^2y^3 = 3\times x\times x\times y\times y\times y$

$10x^3y^2 = 2\times5\times x\times x\times x\times y\times y$

$6x^2y^2z = 2\times3\times x\times x\times y\times y\times z$

Hence, the common factors are $x\times x\times y\times y = x^2y^2$

Question 2:

Factorize the following expressions.

(i) $7x-42$ (ii) $6p-12q$

(iii) $7a^2+14a$ (iv) $-16z+20z^3$

(v) $20l^2m+30alm$ (vi) $5x^2y-15xy^2$

(vii) $10a^2-15b^2+20c^2$ (viii) $-4a^2+4ab-4ca$

(ix) $x^2yz+xy^2z+xyz^2$ (x) ax^2y+bxy^2+cxyz

Answer 2:

(i) $7x-42 = 7\times x-2\times3\times7$

Taking common factors from each term,

$= 7(x-2\times3)$

$= 7(x-6)$

(ii) $6p-12q = 2\times3\times p-2\times2\times3\times q$

Taking common factors from each term,

$= 2\times3(p-2q)$

$= 6(p-2q)$

(iii) $7a^2+14a = 7\times a\times a+2\times7\times a$

Taking common factors from each term,

$= 7\times a(a+2)$

$= 7a(a+2)$

(iv) $-16z+20z^3=(-1)\times 2\times 2\times 2\times 2\times z+2\times 2\times 5\times z\times z\times z$

Taking common factors from each term,

$= 2\times 2\times z(-2\times 2+5\times z\times z)$

$= 4z(-4+5z^2)$

(v) $20l^2m+30alm=2\times 2\times 5\times l\times l\times m+2\times 3\times 5\times a\times l\times m$

Taking common factors from each term,

$= 2\times 5\times l\times m(2\times l+3\times a)$

$= 10lm(2l+3a)$

(vi) $5x^2y-15xy^2=5\times x\times x\times y+3\times 5\times x\times y\times y$

Taking common factors from each term,

$= 5\times x\times y(x-3y)$

$= 5xy(x-3y)$

(vii) $10a^2-15b^2+20c^2=2\times 5\times a\times a-3\times 5\times b\times b+2\times 2\times 5\times c\times c$

Taking common factors from each term,

$= 5(2\times a\times a-3\times b\times b+2\times 2\times c\times c)$

$= 5(2a^2-3b^2+4c^2)$

(viii) $-4a^2+4ab-4ca=(-1)\times 2\times 2\times a\times a+2\times 2\times a\times b-2\times 2\times c\times a$

Taking common factors from each term,

$= 2\times 2\times a(-a+b-c)$

$= 4a(-a+b+c)$

(ix) $x^2yz+xy^2z+xyz^2=x\times x\times y\times z+x\times y\times y\times z+x\times y\times z\times z$

Taking common factors from each term,

$= x\times y\times z(x+y+z)$

$= xyz(x+y+z)$

(x) $ax^2y+bxy^2+cxyz=a\times x\times x\times y+b\times x\times y\times y+c\times x\times y\times z$

Taking common factors from each term,

$= x\times y(a\times x+b\times y+c\times z)$

$= xy(ax+by+cz)$

Question 3:

Factorize:

(i) $x^2+xy+8x+8y$ (ii) $15xy-6x+5y-2$

(iii) $ax+bx-ay-by$ (iv) $15pq+15+9q+25p$

(v) $z-7+7xy-xyz$

Answer 3:

(i) $x^2+xy+8x+8y=x(x+y)+8(x+y)$

$=(x+y)(x+8)$

(ii) $15xy-6x+5y-2=3x(5y-2)+1(5y-2)$

$=(5y-2)(3x+1)$

(iii) $ax+bx-ay-by=(ax+bx)-(ay+by)$

$=x(a+b)-y(a+b)$

$=(a+b)(x-y)$

(iv) $15pq+15+9q+25p=15pq+25p+9q+15$

$=5p(3q+5)+3(3q+5)$

$=(3q+5)(5p+3)$

(v) $z-7+7xy-xyz=7xy-7-xyz+z$

$=7(xy-1)-z(xy-1)$

$=(xy-1)(7-z)=(-1)(1-xy)(-1)(z-7)$

$=(1-xy)(z-7)$

Exercise 14.2

Question 1:

Factorize the following expressions:

(i) $a^2+8a+16$ (ii) $p^2-10p+25$

(iii) $25m^2+30m+9$ (iv) $49y^2+84yz+36z^2$

(v) $4x^2-8x+4$ (vi) $121b^2-88bc+16c^2$

(vii) $(l+m)^2-4lm$ [Hint: Expand $(l+m)^2$ first]

(viii) $a^4+2a^2b^2+b^4$

Answer 1:

(i) $a^2+8a+16=a^2+(4+4)a+4\times4$

Using identity $x^2+(a+b)x+ab=(x+a)(x+b)$,

Here $x=a, a=4$ and $b=4$

$a^2+8a+16 = (a+4)(a+4)=(a+4)^2$

(ii) $p^2-10p+25=p^2+(-5-5)p+(-5)(-5)$

Using identity $x^2+(a+b)x+ab=(x+a)(x+b)$,

Here $x=p, a=-5$ and $b=-5$

$p^2-10p+25=(p-5)(p-5)=(p-5)^2$

(iii) $25m^2+30m+9=(5m)^2+2\times5m\times3+(3)^2$

Using identity $a^2+2ab+b^2=(a+b)^2$, here $a=5m, b=3$

$25m^2+30m+9=(5m+3)^2$

(iv) $49y^2+84yz+36z^2=(7y)^2+2\times7y\times6z+(6z)^2$

Using identity $a^2+2ab+b^2=(a+b)^2$, here $a=7y, b=6z$

$49y^2+84yz+36z^2=(7y+6z)^2$

(v) $4x^2-8x+4=(2x)^2-2\times 2x\times 2+(2)^2$

Using identity $a^2-2ab+b^2=(a-b)^2$, here $a=2x, b=2$

$4x^2-8x+4=(2x-2)^2=(2)^2(x-1)^2=4(x-1)^2$

(vi) $121b^2-88bc+16c^2=(11b)^2-2\times 11b\times 4c+(4c)^2$

Using identity $a^2-2ab+b^2=(a-b)^2$, here $a=11b, b=4c$

$121b^2-88bc+16c^2=(11b-4c)^2$

(vii) $(l+m)^2-4lm = l^2+2\times l\times m+m^2-4lm$ $\quad[\because\ (a+b)^2=a^2+2ab+b^2]$

$= l^2+2lm+m^2-4lm$

$= l^2-2lm+m^2$

$= (l-m)^2$ $\quad[\because\ (a-b)^2=a^2-2ab+b^2]$

(viii) $a^4+2a^2b^2+b^4=(a^2)^2+2\times a^2\times b^2+(b^2)^2$

$= (a^2+b^2)^2$ $\quad[\because\ (a+b)^2=a^2+2ab+b^2]$

Question 2:

Factorize:

(i) $4p^2-9q^2$ (ii) $63a^2-112b^2$

(iii) $49x^2-36$ (iv) $16x^5-144x^2$

(v) $(l+m)^2-(l-m)^2$ (vi) $9x^2y^2-16$

(vii) $(x^2-2xy+y^2)-z^2$ (viii) $25a^2-4b^2+28bc-49c^2$

Answer 2:

(i) $4p^2-9q^2=(2p)^2-(3q)^2$

$= (2p-3q)(2p+3q)$ $\quad[\because\ a^2-b^2=(a-b)(a+b)]$

(ii) $63a^2-112b^2=7(9a^2-16b^2)=7[(3a)^2-(4b)^2]$

$= 7(3a-4b)(3a+4b)$ $\quad[\because\ a^2-b^2=(a-b)(a+b)]$

(iii) $49x^2-36 = (7x)^2-(6)^2$

$= (7x-6)(7x+6)$ $\left[\because\ a^2-b^2=(a-b)(a+b)\right]$

(iv) $16x^5-144x^3 = 16x^3(x^2-9)$

$= 16x^3\left[(x)^2-(3)^2\right]$

$= 16x^3(x-3)(x+3)$ $\left[\because\ a^2-b^2=(a-b)(a+b)\right]$

(v) $(l+m)^2-(l-m)^2 = \left[(l+m)+(l-m)\right]\left[(l+m)-(l-m)\right]$

$\left[\because\ a^2-b^2=(a-b)(a+b)\right]$

$= (l+m+l-m)(l+m-l+m)$

$= (2m)(2l) = 4lm$

(vi) $9x^2y^2-16 = (3xy)^2-(4)^2$

$= (3xy-4)(3xy+4)$ $\left[\because\ a^2-b^2=(a-b)(a+b)\right]$

(vii) $(x^2-2xy+y^2)-z^2 = (x-y)^2-z^2$

$\left[\because\ (a-b)^2=a^2-2ab+b^2\right]$

$= (x-y-z)(x-y+z)$ $\left[\because\ a^2-b^2=(a-b)(a+b)\right]$

(viii) $25a^2-4b^2+28bc-49c^2 = 25a^2-(4b^2-28bc+49c^2)$

$= 25a^2-\left[(2b)^2-2\times 2b\times 7c+(7c)^2\right]$

$= 25a^2-(2b-7c)^2$ $\left[\because\ (a-b)^2=a^2-2ab+b^2\right]$

$= (5a)^2-(2b-7c)^2$

$= \left[5a-(2b-7c)\right]\left[5a+(2b-7c)\right]$ $\left[\because\ a^2-b^2=(a-b)(a+b)\right]$

$= (5a-2b+7c)(5a+2b-7c)$

Question 3:

Factorize the expressions:

(i) ax^2+bx (ii) $7p^2+21q^2$

(iii) $2x^3+2xy^2+2xz^2$ (iv) $am^2+bm^2+bn^2+an^2$

(v) $(lm+l)+m+1$ (vi) $y(y+z)+9(y+z)$

(vii) $5y^2-20y-8z+2yz$ (viii) $10ab+4a+5b+2$

(ix) $6xy-4y+6-9x$

Answer 3:

(i) $ax^2+bx = x(ax+b)$

(ii) $7p^2+21q^2 = 7(p^2+3q^2)$

(iii) $2x^3+2xy^2+2xz^2 = 2x(x^2+y^2+z^2)$

(iv) $am^2+bm^2+bn^2+an^2 = m^2(a+b)+n^2(a+b)$

$= (a+b)(m^2+n^2)$

(v) $(lm+l)+m+1 = l(m+1)+1(m+1) = (m+1)(l+1)$

(vi) $y(y+z)+9(y+z) = (y+z)(y+9)$

(vii) $5y^2-20y-8z+2yz = 5y^2-20y+2yz-8z$

$= 5y(y-4)+2z(y-4)$

$= (y-4)(5y+2z)$

(viii) $10ab+4a+5b+2 = 2a(5b+2)+1(5b+2)$

$= (5b+2)(2a+1)$

(ix) $6xy-4y+6-9x = 6xy-9x-4y+6$

$= 3x(2y-3)-2(2y-3)$

$= (2y-3)(3x-2)$

Question 4:

Factorize:

(i) a^4-b^4 (ii) p^4-81

(iii) $x^4-(y+z)^4$ (iv) $x^4-(x-z)^4$

(v) $a^4-2a^2b^2+b^4$

Answer 4:

(i) $a^4-b^4=\left(a^2\right)^2-\left(b^2\right)^2$

$=\left(a^2-b^2\right)\left(a^2+b^2\right)$ $\left[\because\ a^2-b^2-(a-b)(a+b)\right]$

$=(a-b)(a+b)\left(a^2+b^2\right)$ $\left[\because\ a^2-b^2-(a-b)(a+b)\right]$

(ii) $p^4-81=\left(p^2\right)^2-(9)^2$

$=\left(p^2-9\right)\left(p^2+9\right)$ $\left[\because\ a^2-b^2-(a-b)(a+b)\right]$

$=\left(p^2-3^2\right)\left(p^2+9\right)$

$=(p-3)(p+3)\left(p^2+9\right)$ $\left[\because\ a^2-b^2-(a-b)(a+b)\right]$

(iii) $x^4-(y+z)^4=\left(x^2\right)^2-\left[(y+z)^2\right]^2$

$=\left[x^2-(y+z)^2\right]\left[x^2+(y+z)^2\right]$ $\left[\because\ a^2-b^2-(a-b)(a+b)\right]$

$=\left[x-(y+z)\right]\left[x+(y+z)\right]\left[x^2+(y+z)^2\right]$ $\left[\because\ a^2-b^2-(a-b)(a+b)\right]$

$=(x-y+z)(x+y+z)\left[x^2+(y+z)^2\right]$

(iv) $x^4-(x-z)^4=\left(x^2\right)^2-\left[(x-z)^2\right]^2$

$=\left[x^2-(x-z)^2\right]\left[x^2+(x-z)^2\right]$ $\left[\because\ a^2-b^2-(a-b)(a+b)\right]$

$=\left[x-(x-z)\right]\left[x+(x+z)\right]\left[x^2+(x-z)^2\right]$ $\left[\because\ a^2-b^2-(a-b)(a+b)\right]$

$=(x-x+z)(x+x+z)\left(x^2+x^2-2xz+z^2\right)$ $\left[\because\ (a-b)^2=a^2-2ab+b^2\right]$

$=x(2x+z)\left(2x^2-2xz+z^2\right)$

(v) $a^4-2a^2b^2+b^4 = \left(a^2\right)^2-2a^2b^2+\left(b^2\right)^2$

$= \left(a^2-b^2\right)^2$ $\left[\because \ (a-b)^2 = a^2-2ab+b^2\right]$

$= \left[(a-b)(a+b)\right]^2$ $\left[\because \ a^2-b^2-(a-b)(a+b)\right]$

$= (a-b)^2(a+b)^2$ $\left[\because \ (xy)^m = x^m.y^m\right]$

Question 5:

Factorize the following expressions:

(i) p^2+6p+8 (ii) $q^2-10q+21$

(iii) $p^2+6p-16$

Answer 5:

(i) $p^2+6p+8 = p^2+(4+2)p+4\times 2$

$= p^2+4p+2p+4\times 2$

$= p(p+4)+2(p+4)$

$= (p+4)(p+2)$

(ii) $q^2-10q+21 = q^2-(7+3)q+7\times 3$

$= q^2-7q-3q+7\times 3$

$= q(q-7)-3(q-7)$

$= (q-7)(q-3)$

(iii) $p^2+6p-16 = p^2+(8-2)p-8\times 2$

$= p^2+8p-2p-8\times 2$

$= p(p+8)-2(p+8)$

$= (p+8)(p-2)$

Exercise 14.3

Question 1:

Carry out the following divisions:

(i) $28x^4 \div 56x$ (ii) $-36y^3 \div 9y^2$

(iii) $66pq^2r^3 \div 11qr^2$ (iv) $34x^3y^3z^3 \div 51xy^2z^3$

(v) $12a^8b^8 \div \left(-6a^6b^4\right)$

Answer 1:

(i) $28x^4 \div 56x = \frac{28x^4}{56x} = \frac{28}{56} \times \frac{x^4}{x}$

$= \frac{1}{2}x^3$ $\left[\because\ x^m \div x^n = x^{m-n}\right]$

(ii) $-36y^3 \div 9y^2 = \frac{-36y^3}{9y^2} = \frac{-36}{9} \times \frac{y^3}{y^2}$

$= -4y$ $\left[\because\ x^m \div x^n = x^{m-n}\right]$

(iii) $66pq^2r^3 \div 11qr^2 = \frac{66pq^2r^3}{11qr^2} = \frac{66}{11} \times \frac{pq^2r^3}{qr^2}$

$= 6pqr$ $\left[\because\ x^m \div x^n = x^{m-n}\right]$

(iv) $34x^3y^3z^3 \div 51xy^2z^3 = \frac{34x^3y^3z^3}{51xy^2z^3} = \frac{34}{51} \times \frac{x^3y^3z^3}{xy^2z^3}$

$= \frac{2}{3}x^2y$ $\left[\because\ x^m \div x^n = x^{m-n}\right]$

(v) $12a^8b^8 \div \left(-6a^6b^4\right) = \frac{12a^8b^8}{-6a^6b^4} = \frac{12}{-6} \times \frac{a^8b^8}{a^6b^4}$

$= -2a^2b^4$ $\left[\because\ x^m \div x^n = x^{m-n}\right]$

Question 2:

Divide the given polynomial by the given monomial:

(i) $\left(5x^2-6x\right)\div 3x$ (ii) $\left(3y^8-4y^6+5y^4\right)\div y^4$

(iii) $8\left(x^3y^2z^2+x^2y^3z^2+x^2y^2z^3\right)\div 4x^2y^2z^2$

(iv) $\left(x^3+2x^2+3x\right)\div 2x$ (v) $\left(p^3q^6-p^6q^3\right)\div p^3q^3$

Answer 2:

(i) $\left(5x^2-6x\right)\div 3x=\dfrac{5x^2-6x}{3x}$

$=\dfrac{5x^2}{3x}-\dfrac{6x}{3x}=\dfrac{5}{3}x-2=\dfrac{1}{3}(5x-6)$

(ii) $\left(3y^8-4y^6+5y^4\right)\div y^4=\dfrac{3y^8-4y^6+5y^4}{y^4}$

$=\dfrac{3y^8}{y^4}-\dfrac{4y^6}{y^4}+\dfrac{5y^4}{y^4}=3y^4-4y^2+5$

(iii) $8\left(x^3y^2z^2+x^2y^3z^2+x^2y^2z^3\right)\div 4x^2y^2z^2=\dfrac{8\left(x^3y^2z^2+x^2y^3z^2+x^2y^2z^3\right)}{4x^2y^2z^2}$

$=\dfrac{8x^3y^2z^2}{4x^2y^2z^2}+\dfrac{8x^2y^3z^2}{4x^2y^2z^2}+\dfrac{8x^2y^2z^3}{4x^2y^2z^2}=2x+2y+2z$

$=2\left(x+y+z\right)$

(iv) $\left(x^3+2x^2+3x\right)\div 2x=\dfrac{x^3+2x^2+3x}{2x}$

$=\dfrac{x^3}{2x}+\dfrac{2x^2}{2x}+\dfrac{3x}{2x}=\dfrac{x^2}{2}+\dfrac{2x}{2}+\dfrac{3}{2}=\dfrac{1}{2}\left(x^2+2x+3\right)$

(v) $\left(p^3q^6-p^6q^3\right)\div p^3q^3=\dfrac{p^3q^6-p^6q^3}{p^3q^3}$

$=\dfrac{p^3q^6}{p^3q^3}-\dfrac{p^6q^3}{p^3q^3}=q^3-p^3$

Question 3:

Work out the following divisions:

(i) $(10x-25)\div 5$ (ii) $(10x-25)\div(2x-5)$

(iii) $10y(6y+21)\div 5(2y+7)$ (iv) $9x^2y^2(3z-24)\div 27xy(z-8)$

(v) $96abc(3a-12)(5b-30)\div 144(a-4)(b-6)$

Answer 3:

(i) $(10x-25)\div 5=\dfrac{10x-25}{5}$

$=\dfrac{5(2x-5)}{5}=2x-5$

(ii) $(10x-25)\div(2x-5)=\dfrac{10x-25}{(2x-5)}$

$=\dfrac{5(2x-5)}{(2x-5)}=5$

(iii) $10y(6y+21)\div 5(2y+7)=\dfrac{10y(6y+21)}{5(2y+7)}$

$=\dfrac{2\times 5\times y\times 3(2y+7)}{5(2y+7)}=2\times y\times 3=6y$

(iv) $9x^2y^2(3z-24)\div 27xy(z-8)=\dfrac{9x^2y^2(3z-24)}{27xy(z-8)}$

$=\dfrac{9}{27}\times\dfrac{xy\times xy\times 3(z-8)}{xy(z-8)}=xy$

(v) $96abc(3a-12)(5b-30)\div 144(a-4)(b-6)=\dfrac{96abc(3a-12)(5b-30)}{144(a-4)(b-6)}$

$=\dfrac{12\times 4\times 2\times abc\times 3(a-4)\times 5(b-6)}{12\times 4\times 3(a-4)(b-6)}=10abc$

Question 4:

Divide as directed:

(i) $5(2x+1)(3x+5)\div(2x+1)$

(ii) $26xy(x+5)(y-4)\div 13x(y-4)$

(iii) $52pqr(p+q)(q+r)(r+p)\div 104pq(q+r)(r+p)$

(iv) $20(y+4)(y^2+5y+3)\div 5(y+4)$

(v) $x(x+1)(x+2)(x+3)\div x(x+1)$

Answer 4:

(i) $5(2x+1)(3x+5)\div(2x+1)=\dfrac{5(2x+1)(3x+5)}{(2x+1)}$

$= 5(3x+5)$

(ii) $26xy(x+5)(y-4)\div 13x(y-4)=\dfrac{26xy(x+5)(y-4)}{13x(y-4)}$

$=\dfrac{13\times 2\times xy(x+5)(y-4)}{13x(y-4)} = 2y(x+5)$

(iii)

$52pqr(p+q)(q+r)(r+p)\div 104pq(q+r)(r+p)=\dfrac{52pqr(p+q)(q+r)(r+p)}{104pq(q+r)(r+p)}$

$=\dfrac{52pqr(p+q)(q+r)(r+p)}{52\times 2\times pq(q+r)(r+p)} = \dfrac{1}{2}r(p+q)$

(iv) $20(y+4)(y^2+5y+3)\div 5(y+4)=\dfrac{20(y+4)(y^2+5y+3)}{5(y+4)}$

$= 4(y^2+5y+3)$

(v) $x(x+1)(x+2)(x+3)\div x(x+1)=\dfrac{x(x+1)(x+2)(x+3)}{x(x+1)}$

$= (x+2)(x+3)$

Question 5:

Factorize the expressions and divide them as directed:

(i) $\left(y^2+7y+10\right)\div\left(y+5\right)$

(ii) $\left(m^2-14m-32\right)\div\left(m+2\right)$

(iii) $\left(5p^2-25p+20\right)\div\left(p-1\right)$

(iv) $4yz\left(z^2+6z-16\right)\div 2y\left(z+8\right)$

(v) $5pq\left(p^2-q^2\right)\div 2p\left(p+q\right)$

(vi) $12xy\left(9x^2-16y^2\right)\div 4xy\left(3x+4y\right)$

(vii) $39y^3\left(50y^2-98\right)\div 26y^2\left(5y+7\right)$

Answer 5:

(i) $\left(y^2+7y+10\right)\div\left(y+5\right)=\dfrac{y^2+7y+10}{\left(y+5\right)}$

$=\dfrac{y^2+\left(2+5\right)y+2\times 5}{\left(y+5\right)}=\dfrac{y^2+2y+5y+2\times 5}{\left(y+5\right)}$

$=\dfrac{\left(y+2\right)\left(y+5\right)}{\left(y+5\right)}\qquad\left[\because x^2+\left(a+b\right)x+ab=\left(x+a\right)\left(x+b\right)\right]$

$=y+2$

(ii) $\left(m^2-14m+32\right)\div\left(m+2\right)=\dfrac{m^2-14m+32}{\left(m+2\right)}$

$=\dfrac{m^2+\left(-16+2\right)m+\left(-16\right)\times 2}{\left(m+2\right)}$

$=\dfrac{\left(m-16\right)\left(m+2\right)}{\left(m+2\right)}\qquad\left[\because x^2+\left(a+b\right)x+ab=\left(x+a\right)\left(x+b\right)\right]$

$=\left(m+2\right)$

(iii) $\left(5p^2-25p+20\right)\div\left(p-1\right)=\dfrac{5p^2-25p+20}{\left(p-1\right)}$

$=\dfrac{5p^2-20p-5p+20}{\left(p-1\right)}$

$$= \frac{5p(p-4)-5(p-4)}{(p-1)}$$

$$= \frac{(5p-5)(p-4)}{(p-1)} = \frac{5(p-1)(p-4)}{(p-1)}$$

$$= 5(p-4)$$

(iv) $$4yz\left(z^2+6z-16\right) \div 2y(z+8) = \frac{4yz\left(z^2+6z-16\right)}{2y(z+8)}$$

$$= \frac{4yz\left[z^2+(8-2)z+8\times(-2)\right]}{2y(z+8)}$$

$$= \frac{4yz(z-2)(z+8)}{2y(z+8)} \qquad \left[\because x^2+(a+b)x+ab=(x+a)(x+b)\right]$$

$$= 2z(z-2)$$

(v) $$5pq\left(p^2-q^2\right) \div 2p(p+q) = \frac{5pq\left(p^2-q^2\right)}{2p(p+q)}$$

$$= \frac{5pq(p-q)(p+q)}{2p(p+q)} \qquad \left[\because \ a^2-b^2=(a-b)(a+b)\right]$$

$$= \frac{5}{2}q(p-q)$$

(vi) $$12xy\left(9x^2-16y^2\right) \div 4xy(3x+4y) = \frac{12xy\left(9x^2-16y^2\right)}{4xy(3x+4y)}$$

$$= \frac{12xy\left[(3x)^2-(4y)^2\right]}{4xy(3x+4y)}$$

$$= \frac{12xy(3x-4y)(3x+4y)}{4xy(3x+4y)} \qquad \left[\because \ a^2-b^2=(a-b)(a+b)\right]$$

$$= 3(3x-4y)$$

(vii) $39y^3\left(50y^2-98\right)\div 26y^2\left(5y+7\right)=\dfrac{39y^3\left(50y^2-98\right)}{26y^2\left(5y+7\right)}$

$$= \frac{39y^3\times 2\left(25y^2-49\right)}{26y^2\left(5y+7\right)}$$

$$= \frac{39y^2\times 2\left[\left(5y\right)^2-\left(7\right)^2\right]}{26y^2\left(5y+7\right)}$$

$$= \frac{39y^2\times 2\left(5y-7\right)\left(5y+7\right)}{26y^2\left(5y+7\right)} \qquad \left[\because\ a^2-b^2=\left(a-b\right)\left(a+b\right)\right]$$

$$= 3y\left(5y-7\right)$$

Exercise 14.4

Question 1:

Find and correct the errors in the following mathematical statement: $4(x-5)=4x-5$

Answer 1:

L.H.S. = $4(x-5)=4x-20 \neq$ R.H.S.

Hence, the correct mathematical statements is $4(x-5)=4x-20.$

Question 2:

Find and correct the errors in the following mathematical statement: $x(3x+2)=3x^2+2$

Answer 2:

L.H.S. = $x(3x+2)=3x^2+2x \neq$ R.H.S.

Hence, the correct mathematical statements is $x(3x+2)=3x^2+2x.$

Question 3:

Find and correct the errors in the following mathematical statement: $2x+3y=5xy$

Answer 3:

L.H.S. = $2x+3y \neq$ R.H.S.

Hence, the correct mathematical statements is $2x+3y=2x+3y.$

Question 4:

Find and correct the errors in the following mathematical statement: $x+2x+3x=5x$

Answer 4:

L.H.S. = $x+2x+3x=6x \neq$ R.H.S.

Hence, the correct mathematical statements is $x+2x+3x=6x.$

Question 5:

Find and correct the errors in the following mathematical statement: $5y+2y+y-7y=0$

Answer 5:

L.H.S. = $5y+2y+y-7y=8y-7y=y\neq$ R.H.S.

Hence, the correct mathematical statements is $5y+2y+y-7y=y$.

Question 6:

Find and correct the errors in the following mathematical statement: $3x+2x=5x^2$

Answer 6:

L.H.S. = $3x+2x=5x\neq$ R.H.S.

Hence, the correct mathematical statements is $3x+2x=5x$.

Question 7:

Find and correct the errors in the following mathematical statement:

$(2x)^2+4(2x)+7=2x^2+8x+7$

Answer 7:

L.H.S. = $(2x)^2+4(2x)+7=4x^2+8x+7\neq$ R.H.S.

Hence, the correct mathematical statements is $(2x)^2+4(2x)+7=4x^2+8x+7$.

Question 8:

Find and correct the errors in the following mathematical statement:

$(2x)^2+5x=4x+5x=9x$

Answer 8:

L.H.S. = $(2x)^2+5x=4x^2+5x\neq$ R.H.S.

Hence, the correct mathematical statements is $(2x)^2+5x=4x^2+5x$.

Question 9:

Find and correct the errors in the following mathematical statement:

$(3x+2)^2 = 3x^2 + 6x + 4$

Answer 9:

L.H.S. = $(3x+2)^2 = (3x)^2 + 2\times 3x \times 2 + (2)^2 = 9x^2 + 12x + 4 \neq$ R.H.S.

Hence, the correct mathematical statements is $(3x+2)^2 = 9x^2 + 12x + 4.$

Question 10:

Find and correct the errors in the following mathematical statements:

Substituting $x = -3$ in:

(a) $x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 + 2 + 4 = 15$

(b) $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 - 15 + 4 = -2$

(c) $x^2 + 5x$ gives $(-3)^2 + 5(-3) = -9 - 15 = -24$

Answer 10:

(a) L.H.S. = $x^2 + 5x + 4$

Putting $x = -3$ in given expression,

= $(-3)^2 + 5(-3) + 4 = 9 - 15 + 4 = -2 \neq$ R.H.S.

Hence, $x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 - 15 + 4 = -2.$

(b) L.H.S. = $x^2 - 5x + 4$

Putting $x = -3$ in given expression,

= $(-3)^2 - 5(-3) + 4 = 9 + 15 + 4 = 28 \neq$ R.H.S.

Hence, $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 + 15 + 4 = 28.$

(c) L.H.S. = $x^2 + 5x$

Putting $x = -3$ in given expression,

= $(-3)^2 + 5(-3) = 9 - 15 = -6 \neq$ R.H.S.

Hence, $x^2 + 5x$ gives $(-3)^2 + 5(-3) = 9 - 15 = -6.$

Question 11:

Find and correct the errors in the following mathematical statement: $(y-3)^2 = y^2 - 9$

Answer 11:

L.H.S. = $(y-3)^2 = y^2 - 2\times y\times 3 + (3)^2$ $\left[\because (a-b)^2 = a^2 - 2ab + b^2\right]$

$= y^2 - 6y + 9 \neq$ R.H.S.

Hence, the correct statements is $(y-3)^2 = y^2 - 6y + 9$.

Question 12:

Find and correct the errors in the following mathematical statement: $(z+5)^2 = z^2 + 25$

Answer 12:

L.H.S. = $(z+5)^2 = z^2 + 2\times z\times 5 + (5)^2$ $\left[\because (a+b)^2 = a^2 + 2ab + b^2\right]$

$= z^2 + 10z + 25$

Hence, the correct statement is $(z+5)^2 = z^2 + 10z + 25$.

Question 13:

Find and correct the errors in the following mathematical statement:

$(2a+3b)(a-b) = 2a^2 - 3b^2$

Answer 13:

L.H.S. = $(2a+3b)(a-b) = 2a(a-b) + 3b(a-b)$

$= 2a^2 - 2ab + 3ab - 3b^2 = 2a^2 + ab - 3b^2 \neq$ R.H.S.

Hence, the correct statement is $(2a+3b)(a-b) = 2a^2 + ab - 3b^2$.

Question 14:

Find and correct the errors in the following mathematical statement:

$(a+b)(a+2) = a^2 + 8$

Answer14:

L.H.S. = $(a+4)(a+2) = a(a+2)+4(a+2)$

$= a^2+2a+4a+8 = a^2+6a+8 \neq$ R.H.S.

Hence, the correct statement is $(a+4)(a+2) = a^2+6a+8.$

Question 15:

Find and correct the errors in the following mathematical statement:

$(a-4)(a-2) = a^2-8$

Answer 15:

L.H.S. = $(a-4)(a-2) = a(a-2)-4(a-2)$

$= a^2-2a-4a+8 = a^2-6a+8 \neq$ R.H.S.

Hence, the correct statement is $(a-4)(a-2) = a^2-6a+8.$

Question 16:

Find and correct the errors in the following mathematical statement: $\frac{3x^2}{3x^2} = 0$

Answer 16:

L.H.S. = $\frac{3x^2}{3x^2} = \frac{1}{1} = 1 \neq$ R.H.S.

Hence, the correct statement is $\frac{3x^2}{3x^2} = 1.$

Question 17:

Find and correct the errors in the following mathematical statement: $\frac{3x^2+1}{3x^2} = 1+1 = 2$

Answer 17:

$$\text{L.H.S.} = \frac{3x^2+1}{3x^2} = \frac{3x^2}{3x^2} + \frac{1}{3x^2}$$

$$= 1 + \frac{1}{3x^2} \neq \text{R.H.S.}$$

Hence, the correct statement is $\frac{3x^2+1}{3x^2} = 1 + \frac{1}{3x^2}$.

Question 18:

Find and correct the errors in the following mathematical statement: $\frac{3x}{3x+2} = \frac{1}{2}$

Answer 18:

$$\text{L.H.S.} = \frac{3x}{3x+2} \neq \text{R.H.S.}$$

Hence, the correct statement is $\frac{3x}{3x+2} = \frac{3x}{3x+2}$.

Question 19:

Find and correct the errors in the following mathematical statement: $\frac{3}{4x+3} = \frac{1}{4x}$

Answer 19:

$$\text{L.H.S.} = \frac{3}{4x+3} \neq \text{R.H.S.}$$

Hence, the correct statement is $\frac{3}{4x+3} = \frac{3}{4x+3}$.

Question 20:

Find and correct the errors in the following mathematical statement: $\frac{4x+5}{4x} = 5$

Answer 20:

$$\text{L.H.S.} = \frac{4x+5}{4x} = \frac{4x}{4x} + \frac{5}{4x} = 1 + \frac{5}{4x} \neq \text{R.H.S.}$$

Hence, the correct statement is $\frac{4x+5}{4x} = 1 + \frac{5}{4x}$.

Question 21:

Find and correct the errors in the following mathematical statement: $\frac{7x+5}{5} = 7x$

Answer 21:

$$\text{L.H.S.} = \frac{7x+5}{5} = \frac{7x}{5} + \frac{5}{5} = \frac{7x}{5} + 1 \neq \text{R.H.S.}$$

Hence, the correct statement is $\frac{7x+5}{5} = \frac{7x}{5} + 1$.