Unit 3(Square-Square Root & Cube-Cube Root)

Multiple Choice Questions

Question1 196 is the square of

(a) 11 (b) 12

(c) 14 (d) 16

Solution.

(c) Square of 11 = 11 x 11 = 121

Square of 12 = 12 x 12 = 144

Square of 14 = 14 x 14 = 196

Clearly, 196 is the square of 14

Question 2 Which of the following is a square of an even number?

(a) 144 (b) 169

(c) 441 (d) 625

Solution.

(a) Here, $144 = (12)^2$

Similarly, $169 = (13)^2$

$441 = (21)^2$

$625 = (25)^2$

Thus, 144 is a square of an even number.

Alternate Method

We know that, square of an even number is always an even number. Hence, 169, 441 and 625 are not even numbers. So, only 144 is an even number, which is the square of 12.

Question 3 A number ending in 9 will have the unit's place of its square as

(a) 3 (b) 9

(c) 1 (d) 6

Solution.

(c) We know that, if a number is ending in 1 or 9 in the unit's place, then its square ends in 1.

The number ending in 9, will have the unit's place of its square as 1. $[\because 9 \times 9 = 81]$

Question 4 Which of the following will have 4 at the unit's place?

(a) 14^2 (b) 62^2 (c) 27^2 (d)35^2

Solution.

(b) The unit place of the square of $14 = 4^2 = 16 = 6$

The unit place of the square of $62 = 2^2 = 4$ $[\because 2^2 = 4]$

The unit place of the square of $27 = 7^2 = 49 = 9$

The unit place of the square of $35 = 5^2 = 25 = 5$

Clearly, 62^2 has 4 at the unit's place.

Question 5 How many natural numbers lie between 5^2 and 6^2?

(a) 9 (b) 10 (c)11 (d) 12

Solution. (b) The natural numbers lying between 5^2 and 6^2, i.e. between 25 and 36 are 26, 27, 28, 29, 30, 31,32, 33, 34 and 35.

Hence, 10 natural numbers lie between 5^2 and 6^2.

Question 6 Which of the following cannot be a perfect square?

(a) 841 (b) 529 (c) 198 (d) All of these

Solution.(c) We know that, a number ending with digits 2, 3, 7 or 8 can never be a perfect square. So, 198 cannot be written in the form of a perfect square.

Question 7 The one's digit of the cube of 23 is

(a) 6 (b) 7 (c) 3 (d) 9

Solution. (b) We know that, the cubes of the numbers ending with digits 3 and 7, have 7 and 3 at one's digit, respectively.

So, the one's digit of the cube of 23 is 7.

Question 8 A square board has an area of 144 sq units. How long is each side of the board?

(a) 11 units (b) 12 units (c) 13 units (d) 14 units

Solution.

(b) Given, area of square board = 144 sq units

$\therefore$ $(\text{Side})^2 = 144$ $[\because \text{area of square} = (\text{side})^2]$

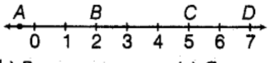
$\Rightarrow$ $(\text{Side})^2 = (12)^2$

$\Rightarrow$ Side = 12 units

Hence, the length of each side of the board is 12 units.

Question 9

Which letter best represents the location of $\sqrt{25}$ on a number line?



(a) *A* (b) *B* (c) *C* (d) *D*

Solution.

(c) We have, $\sqrt{25} = 5$

Therefore, 5 at letter *C* represents the best location of $\sqrt{25}$ on number line.

Question 10 If one member of a Pythagorean triplet is 2m, then the other two members are

(a) $m, m^2 + 1$ (b) $m^2 + 1, m^2 - 1$ (c) $m^2, m^2 - 1$ (d) $m^2, m + 1$

Solution.

$$2m = 4$$
$$\Rightarrow \quad m = 2$$
$$m^2 + 1 = 2^2 + 1 = 4 + 1 = 5$$
and $\quad m^2 - 1 = 2^2 - 1 = 4 - 1 = 3$

Now, $\quad 3^2 + 4^2 = 5^2$

$$\Rightarrow \quad 9 + 16 = 25$$
$$\Rightarrow \quad 25 = 25$$

So, 3, 4 and 5 are pythagorean triplets.

Question 11 The sum of successive odd numbers 1, 3, 5, 7, 9, 11, 13 and 15 is

(a) 61 (b) 64 (c) 49 (d) 36

Solution. (b) We know that, the sum of first n odd natural numbers is n^2.

Given odd numbers are 1,3, 5, 7, 9,11,13 and 15.

So, number of odd numbers, n = 8

The sum of given odd numbers $= n^2 = (8)^2 = 64$

Question 12 The sum of first n odd natural numbers is

(a) 2n +1 (b) n^2 (c) $n^2 - 1$ (d) $n^2 + 1$

Solution.

(b) Sum of first n odd natural numbers $= \Sigma(2n - 1) = 2\,\Sigma n - n$

$$= \frac{2 \times n(n+1)}{2} - n = n(n+1) - n = n^2 + n - n = n^2$$

Question 13 Which of the following numbers is a perfect cube?

(a) 243 (b) 216 (c) 392 (d) 8640

Solution.(b) For option (a) We have, 243

Resolving 243 into prime factors, we have

243= 3 x 3 x 3 x 3 x 3

Grouping the factors in triplets of equal factors, we get

243 = (3 x 3 x 3) x 3 x 3

Clearly, in grouping, the factors in triplets of equal factors, we are left with two factors 3 x 3.

Therefore, 243 is not a perfect cube.

For option (b) We have, 216 Resolving 216 into prime factqrs, we have

216 = 2 x 2 x 2 x 3 x 3 x 3

Grouping the factors in triplets of equal factors, we get 216 = (2 x 2 x 2) x (3 x 3 x 3)

Clearly, in grouping, the factors of triplets of equal factors, no factor is left over.

So, 216 is a perfect cube.

For option (c) We have, 392

Resolving 392 into prime factors, we get

392 = 2 x 2 x 2 x 7 x 7

Grouping the factors in triplets of equal factors, we get

392 = (2 x 2 x 2) x 7 x 7

Clearly, in grouping, the factors in triplets of equal factors, we are left with two factors 7 x 7.

Therefore, 392 is not a perfect cube.

For option (d) We have, 8640

Resolving 8640 into prime factors, we get

8640=2 x 2 x 2 x 2 x 2 x 2 x 3 x 3 x 3 x 5

Grouping the factors in triplets of equal factors, we get

8640 = (2 x 2 x 2) x (2 x 2 x 2) x (3 x 3 x 3) x 5

Clearly, in grouping, the factors in triplets of equal factors, we are left with one factor 5.

Therefore, 8640 in not a perfect cube.

After solving, it is clear that option (b) is correct.

Question 14 The hypotenuse of a right angled triangle with its legs of lengths 3x x 4x is

(a) 5X (b)7x (c) 16x (d) 25x

Solution.

(a) Given, lengths of the legs of right angled triangle are $3x$ and $4x$.

Now, hypotenuse $= \sqrt{(3x)^2 + (4x)^2}$ [using Pythagoras theorem]

$= \sqrt{9x^2 + 16x^2}$

$= \sqrt{25x^2} = 5x$

Question 15 The next two numbers in the number pattern 1, 4, 9,16, 25,... are

(a) 35, 48 (b) 36, 49 (c) 36, 48 (d) 35, 49

Solution. (b) We have, 1,4, 9,16, 25, ...

The number pattern can be written as $(1)^2, (2)^2, (3)^2, (4)^2, (5)^2$

Hence, the next two numbers are $(6)^2$ and $(7)^2$, i.e. 36 and 49.

Question 16 Which among 43^2 , 67^2 , 52^2 , 59^2 would end with digit 1?

(a) 43^2 (b) 67^2 (c) 52^2 (d) 59^2

Solution.

(d) We know that, the unit's digit of the square of a natural number is the unit's digit of the square of the digit at unit's place of the given natural number.

$\therefore$ Unit's digit of $43^2 = 9$ [$\because 3^2 = 9$]

Unit's digit of $67^2 = 9$ [$\because$ unit's digit of 7^2 is 9]

Unit's digit of $52^2 = 4$ [$\because 2^2 = 4$]

Unit's digit of $59^2 = 1$ [$\because$ unit's digit of 9^2 is 1]

Clearly, the square of 59 end with digit 1.

Question 17 A perfect square can never have the following digit in its one's place.

(a) 1 (b) 8 (c) 0 (d) 6

Solution.

(b) We know that, a number ending with digits 2, 3, 7 or 8 can never be a perfect square.

Clearly, a perfect square can never have the digit 8 in its one's place.

Question 18 Which of the following numbers is not a perfect cube?

(a) 216 (b) 567 (c) 125 (d) 343

Solution.

(b) 216=6 x 6 x 6, 567 = 3 x 3 x 3 x 3 x 7

125 = 5 x 5 x 5, 343 = 7 x 7 x 7

Clearly, 567 is not a perfect cube, because in grouping, the factors in triplets of equal factors, we are left with two factors 3 x 7.

Question 19

$\sqrt[3]{1000}$ is equal to

(a) 10 (b) 100 (c) 1 (d) None of these

Solution.

(a) We have, $\sqrt[3]{1000} = \sqrt[3]{10 \times 10 \times 10} = \sqrt[3]{(10)^3} = (10)^{3/3} = 10$

Question 20

If m is the square of a natural number n, then n is

(a) the square of m (b) greater than m

(c) equal to m (d) $\sqrt{m}$

Solution.

(d) Given, m is the square of n, i.e. $m = n^2$

Taking square root both sides, we get

$n = \sqrt{m}$

Question 21 A perfect square number having n digits, where n is even, will have square root with

(a) $(n+1)$ digit (b) $\frac{n}{2}$ digit (c) $\frac{n}{3}$ digit (d) $\left(\frac{n+1}{2}\right)$ digit

Solution. (b) A perfect square number having n digits, where n is even, will have square root with n/2 digit.

Question 22

If m is the cube root of n, then n is

(a) m^3 (b) $\sqrt{m}$ (c) $\frac{m}{3}$ (d) $\sqrt[3]{m}$

solution.

(a) Given, m is the cube root of n, i.e. $m = \sqrt[3]{n}$

$\Rightarrow \quad m = (n)^{1/3}$

$\Rightarrow \quad m^3 = (n)^{3/3}$ [taking cube on both sides]

$\therefore \quad m^3 = n$

Question 23

The value of $\sqrt{248 + \sqrt{52 + \sqrt{144}}}$ is

(a) 14 (b) 12 (c) 16 (d) 13

Solution.

(c) We have, $\sqrt{248 + \sqrt{52 + \sqrt{144}}}$

$= \sqrt{248 + \sqrt{52 + 12}}$ [$\because$ square root of 144 = 12]

$= \sqrt{248 + \sqrt{64}}$

$= \sqrt{248 + 8}$ [$\because$ square root of 64 = 8]

$= \sqrt{256} = 16$ [$\because$ square root of 256 = 16]

Question 24

Given that, $\sqrt{4096} = 64$, the value of $\sqrt{4096} + \sqrt{40.96}$ is

(a) 74 (b) 60.4 (c) 64.4 (d) 70.4

Solution.

(d) Given, $\sqrt{4096} = 64$

So, $\sqrt{4096} + \sqrt{40.96}$

$= 64 + \sqrt{4096 \times 10^{-2}}$

$= 64 + \sqrt{4096}\sqrt{10^{-2}}$

$= 64 + 64 \times 10^{-1}$

$= 64 + 6.4 = 70.4$

Fill in the Blanks

In questions 25 to 48, fill in the blanks to make the statements true.

Question 25 There are________perfect squares between 1 and 100.

Solution. 8

There are 8 perfect squares between 1 and 100, i.e. 4, 9,16, 25, 36, 49, 64 and 81.

Question 26 There are________ perfect cubes between 1 and 1000.

Solution. 8

There are 8 perfect cubes between 1 and 1000, i.e. 8, 27,64,125, 216, 343 and 729.

Question 27 The unit's digit in the square of 1294 is________

Solution. 6

We know that, the unit's digit of the square of a number having digit .at unit's place as 4 or 6 is

6.

Hence, the units digit in the square of 1294 is 6 as 4 x 4 = 16.

Question 28 The square of 500 will have zeroes.

Solution. four

The square of 500 = $(500)^2$

= 500 x 500 = 250000

Hence, the square of 500 will have four zeroes.

Question 29 There are natural numbers between n^2 and $(n + l)^2$

Solution.

2*n*

Natural numbers between n^2 and $(n+1)^2 = [(n+1)^2 - n^2] - 1$

$= [n^2 + 2n + 1 - n^2] - 1$

$= 2n + 1 - 1 = 2n$

Question 30 The square root of 24025 will have________digits.

Solution.

3

Given number is 24025.

Here, number of digits, $n = 5$ (odd)

$\therefore$ Number of digits in the square root of 24025 $= \frac{n+1}{2} = \frac{5+1}{2} = 3$

Question 31 The square of 5.5 is________

Solution. 30.25

Square of 5.5= $(5.5)^2$ = 55 x 5.5= 30.25

Question 32 The square root of 5.3 x 5.3 is________

Solution.

5.3

Square root of $5.3 \times 5.3 = \sqrt{(5.3)^2}$

$= (5.3)^{2/2} = 5.3$

Question 33 The cube of 100 will havfe________zeroes.

Solution. 6

Cubeof100 = 100^3

= 100x100x100 = 1000000

Question 34 $1m^2$ =________ cm^2.

Solution.

10000 cm^2

$1 m^2 = (100 cm)^2$ $[\because 1 m = 100 cm]$

$= 10000 cm^2$

Question 351m^3 =________ cm^3.

Solution.

1000000 cm^3

$1 m^3 = (100 cm)^3$ $[\because 1 m = 100 cm]$

$= (100 \times 100 \times 100) cm^3$

$= 1000000 cm^3$

Question 36 One's digit in the cube of 38 is________

Solution.

2

We know that, the cube of a number having 8 at one's place

∴One's digit in cube of 38 = 2

Question 37 The square of 0.7 is________

Solution.0.49

Square of 0.7 = $(0.7)^2$ = 07 x 07 = 0.49

Question 38 The sum of first six odd natural numbers is________

Solution.

We know that, sum of first n odd natural numbers = n^2

∴ Sum of first six odd natural numbers = $(6)^2 = 36$

Question 39 The digit at the one's place of 572 is________

Solution.

9

We know that, the unit's digit of the square of a number having digit at unit's place as 3 or 7 is 9.

The digit at one's place of $57^2 = 9$ [$\because 7 \times 7 = 49$]

Question 40 The sides of a right angled triangle whose hypotenuse is 17cm, are________and________

Solution.

8, 15

As, hypotenuse of right angled triangle is 17 cm.

So, $17^2 = a^2 + b^2$ [by Pythagoras theorem]

where, a and b are other sides of right angled triangle.

$= 8^2 + 15^2$ [$\because 17^2 = 8^2 + 15^2$]

Hence, $a = 8$ and $b = 15$.

As, hypotenuse of right angled triangle is 17 cm.

Question 41

1.4

We have, $\sqrt{1.96} = \sqrt{\frac{196}{100}} = \sqrt{\frac{14 \times 14}{10 \times 10}} = \frac{14}{10} = 1.4$

Solution.

1.728

We have, $(1.2)^3 = \left(\frac{12}{10}\right)^3 = \frac{12}{10} \times \frac{12}{10} \times \frac{12}{10} = \frac{1728}{1000} = 1.728$

Question 42 $(1.2)^3$=________

Solution.

1.728

We have, $(1.2)^3 = \left(\frac{12}{10}\right)^3 = \frac{12}{10} \times \frac{12}{10} \times \frac{12}{10} = \frac{1728}{1000} = 1.728$

Question 43 The cube of an odd number is always an________number.

Solution.odd

We know that, the cubes of all odd natural numbers are odd.

Question 44 The cube root of a number x is denoted by________

Solution.

$\sqrt[3]{x}$ or $x^{1/3}$

The cube root of a number x is denoted by $\sqrt[3]{x}$ or $x^{1/3}$.

Question 45 The least number by which 125 be multiplied to make it a perfect square, is________

Solution.

5

Resolving 125 into prime factors, we get

$125 = 5 \times 5 \times 5$

5	125
5	25
5	5
	1

Grouping the factors in triplets of equal factors, we get

$125 = (5 \times 5) \times 5$

We observe that 5 is without a pair.

Thus, we must multiply the number by 5, so that the product is a perfect square.

Hence, the least number, which should be multiplied with 125 to make it a perfect square, is 5.

Question 46 The least number by which 72 be multiplied to make it a perfect cube, is________

Solution. 3

Resolving 72 into prime factors, we get

72=2 x 2 x 2 x 3 x 3

Grouping the factors in triplets of equal factors, we get

72 = (2 x 2 x 2) x 3 x 3

We find that 2 occurs as a prime factor of 72 thrice, but 3 occurs as a prime factor only twice.

Thus, if we multiply 72 by 3, 3 will also occurs as a prime factor thrice and the product will be 2 x 2 x 2 x 3 x 3 x 3, which is a perfect cube.

Hence, the least number, which should be multiplied with 72 to get perfect cube, is 3.

Question 47 The least number by which 72 be divided to make it a perfect cube, is________

Solution. 9

Resolving 72 into prime factors, we get

72=2 x 2 x 2 x 3 x 3

Grouping the factors in triplets of equal factors, we get

72 = (2 x 2 x 2) x 3 x 3

Clearly, if we divide 72 by 3 x 3, the quotient would be 2 x 2 x 2, which is a perfect cube.

Hence, the least number by which 72 be divided to make it, a perfect cube, is 9.

Question 48 Cube of a number ending in 7 will end in the digit________

Solution 3

We know that, the cubes of the numbers ending in digits 3 or 7 ends in digits 7 or 3, respectively.

i.e 7 x 7 x 7 = 343

Hence, the cube of a number ending in 7 will end in the digit 3.

True/False

In questions 49 to 86, state whether the statements are True or False.

Question 49 The square of 86 will have 6 at the unit's place.

Solution True

We know that, the unit's digit of the square of a number having digit at unit's place as 4 or 6 is 6.

Question 50 The sum of two perfect squares is a perfect square.

Solution False

e.g. 16 and 25 are the perfect squares, but 16 + 25 = 41 is not a perfect square.

Question 51 The product of twtfperfect squares is a perfect square.

Solution True

e.g. If 4 and 25 are the perfect square, then 4 x 25 = 100 is also a perfect square.

Clearly, the product of two perfect squares is a perfect square.

Question 52 There is no square number between 50 and 60.

Solution True

Numbers between 50 and 60 are 51,52, 53, 54, 55, 56, 57, 58 and 59.

We observed that there is no square number between 50 and 60.

Question 53 The square root of 1521 is 31.

Solution Falsie %

As, the square of 31 = $(31)^2$ = 31 x 31 = 961

Question 54 Each prime factor appears 3 times in its cube.

Solution True

If a3 is the cube and m is one of the prime factors of a. Then, m appears three times in a3.

Question 55 The square of 2.8 is 78.4.

Solution False

The square of 2.8 = $(2.8)^2$ = 2.8×2.8 = 7.84

Question 56 The cube of 0.4 is 0.064.

Solution True

Cube of 0.4 = $(0.4)^2$ = 0.4 x 0.4 x 0.4 = 0.064

Question 57 The square root of 0.9 is 0.3.

Solution False

As, the square of 0.3 = $(0.3)^2$ = 0.3 x 0.3 =0.09

Question 58 The square of every natural number is always greater than the number itself.

Solution False

1 is a natural number and square of 1 is not greater than 1.

Question 59 The cube root of 8000 is 200.

Solution

False

We have, $\sqrt[3]{8000} = \sqrt[3]{(2\times2\times2)\times(2\times2\times2)\times(5\times5\times5)}$

$= 2\times2\times5 = 20$

Question60 There are five perfect cubes between 1 and 100.

Solution False

There are eight perfect cubes between 1 and 100, i.e. 8,27,64,125,216,343,512 and 729.

Question 61 There are 200 natural numbers between 100^2 and 101^2.

Solution.

True

Natural numbers between 100^2 and 101^2

$= 101^2 - 100^2 - 1$ [$\because$ natural numbers between a and $b = b - a - 1$]

$= (101 + 100)(101 - 100) - 1 = 201 - 1 = 200$

Question 62 The sum of first n odd natural numbers is n^2.

Solution.

True

Sum of odd natural numbers $= \Sigma(2n - 1) = \frac{2\times n\times(n+1)}{2} - n$

$= n^2 + n - n = n^2$

Question 63 1000 is a perfect square.

Solution False

1000 = 2 x 2 x 2 x 5 x 5 x 5 = $2^2 \times 5^2 \times 2 \times 5$ Clearly, it is not a perfect square, because it has two unpaired factors 2 and 5.

Question 64 A perfect square can have 8 as its unit's digit.

Solution False

A perfect square can never have 8 as its unit's digit.

Question65

For every natural number m, $(2m - 1, 2m^3 - 2m, 2m^2 - 2m + 1)$ is a pythagorean triplet.

Solution

False

$$\because \quad (2m-1)^2 \neq (2m^3 - 2m)^2 + (2m^2 - 2m + 1)^2$$

$$(2m^3 - 2m)^2 \neq (2m-1)^2 + (2m^2 - 2m + 1)^2$$

and $$(2m^2 - 2m + 1)^2 \neq (2m-1)^2 + (2m^3 - 2m)^2$$

Question 66 All numbers of a Pythagorean triplet are odd.

Solution False

3, 4 and 5 are the numbers of Pythagorean triplet as $5^2 = 4^2 + 3^2$ where, 4 is not an odd number.

Question 67 For an integer a, a^3 is always greater than a^2.

Solution

False

e.g. – 1 is an integer and $(-1)^3$, i.e. – 1 is not greater than $(-1)^2$, i.e. 1.

Qustion68

If x and y are integers such that $x^2 > y^2$, then $x^3 > y^3$.

Solution

False

Suppose, – 1 and – 2 are integers

Then, $(-2)^2 > (-1)^2 = 4 > 1$

and $(-2)^3 < (-1)^3 = -8 < -1$

Question 69 Let x and y be natural numbers. If x divides y, then x^3 divides y^3.

Solution.

True

If x divides y, then $\frac{y}{x}$ is a natural number.

$\Rightarrow \quad \left(\frac{y}{x}\right)^3$ is also a natural number.

$\Rightarrow \quad \frac{y^3}{x^3}$ is a natural number.

$\Rightarrow \quad x^3$ divides y^3.

Question 70 If a^2 ends in 5, then a^3 ends in 25:

Solution.

False

$\because (35)^2 = 1225$ ends in 5 and $(35)^3$, i.e. 42875 does not end in 25.

Question 71 If a^2 ends in 9, then a^3 ends in 7.
Solution

False

$\because (7)^2 = 49$ ends in 9 and $(7)^3$, i.e. 343 does not end in 7.

Question72

The square root of a perfect square of *n* digits will have $\left(\frac{n+1}{2}\right)$ digits, if *n* is odd.

Solution.

True

If the square has 3 digits, then its square root has 2 i.e. $\left(\frac{3+1}{2}\right)$ digits.

Similarly, if the square has 5 digits, then its square root has 3 i.e. $\left(\frac{5+1}{2}\right)$ digits.

Question 73 Square root of a number x is denoted by 4x.
Solution True
Square root of a number x is denoted by 4x.

Question 74 A number having 7 at its one's place will have 3 at the unit's place of its square.
Solution False
Square of 7 = 7 x 7 = 49
Square of 17 = 17 x 17 = 289
Square of 27 = 27 x 27 = 729
and so on.

Question 75 A number having 7 at its one's place will have 3 at the one's place of its cube.
Solution True
Cube of 7 = 7 x 7 x 7 =343
Cube of 17 = 17 x 17 x 17=4913
Cube of 27 = 27 x 27 x 27 = 19683
and so on.

Question 76 The cube of a one-digit number cannot be a two-digit number.
Solution

False

e.g. 3 is a one-digit number and $(3)^3$, i.e. 27 is a two-digit number.

Question 77 Cube of an even number is odd.
Solution. False
We know that, the cube of an even number is always an even number,
e.g. 2 is an even number. Then, 2^3 = 2 x 2 x 2 = 8
Clearly, 8 is also an even number.

Question 78 Cube of an odd number is even.
Solution. False
We know that, the cube of an odd number is always an odd number,
e.g. 3 is an odd number. Then, 33 = 3 x 3 x 3 = 27
Clearly, 27 is not an even number.

Question 79 Cube of an even number is even.
Solution. True
We know that, the cube of an even number is always an even number,

e.g. 4 is an even number. Then, 43 = 4 x 4 x 4 = 64

Clearly, 64 is also an even number.

Question 80 Cube of an odd number is odd.

Solution. True

We know that, the cube of an odd number is always an odd number,

e.g. 9 is an odd number.

Then,93 =9 x 9 x 9 = 729

Clearly, 729 is also an odd number.

Question 81 999 is a perfect cube.

Solution.

False

Resolving 999 into prime factors, we get

$$999 = 3 \times 3 \times 3 \times 37$$

3	999
3	333
3	111
37	37
	1

Grouping the factors in triplets of equal factors, we get

$$999 = (3 \times 3 \times 3) \times 37$$

Clearly, in grouping, the factors in triplets of equal factors, we are left with one factor 37.

Therefore, 999 is not a perfect cube.

Question 82 363 x 81 is a perfect cube.

Solution.

False

$\because$ $363 \times 81 = 3 \times 11 \times 11 \times 3 \times 3 \times 3 \times 3$

$= 3 \times 3 \times 3 \times 3 \times 3 \times 11 \times 11$

Grouping the factors in triplets of equal factors, we have

$$363 \times 81 = (3 \times 3 \times 3) \times 3 \times 3 \times 11 \times 11$$

Clearly, in grouping, the factors in triplets of equal factors, we are left with two factors 3×3 and 11×11.

Hence, 363×81 is not a perfect cube.

Question 83 Cube roots of 8 are + 2 and – 2 .

Sol. False

Cube root of 8 is 2 only and cube root of – 8 is – 2.

Question 84

$$\sqrt[3]{8 + 27} = \sqrt[3]{8} + \sqrt[3]{27}$$

Solution.

False

We have, $\sqrt[3]{8 + 27} \neq \sqrt[3]{8} + \sqrt[3]{27}$

$\because$ $\sqrt[3]{8} + \sqrt[3]{27} = \sqrt[3]{2 \times 2 \times 2} + \sqrt[3]{3 \times 3 \times 3} = 2 + 3 = 5$

But $\sqrt[3]{8 + 27} = \sqrt[3]{35} \neq \sqrt[3]{125} = 5$

Question 85 There is no cube root of a negative integer.

Solution.

False

e.g. Let – 8 be a negative number.

Then, cube root of $-8 = \sqrt[3]{-8} = \sqrt[3]{(-2) \times (-2)(-2)} = -2$

Question 86 Square of a number is positive, so the cube of positive.

Solution.

False

e.g. $(-2)^2$, i.e. 4 is a positive number and $(-2)^3$. i.e. -8 is a negative number.

Question 87 Write the first five square numbers.

Solution. First five square numbers are $1^2, 2^2, 3^2, 4^2$ and 5^2, i.e. 1, 4, 9,16and 25.

Question 88 Write cubes of first three multiples of 3.

Solution. Since, the first three multiples of 3 are 3, 6 and 9.

Hence, the cubes of first three multiples of 3 are $(3)^3$, $(6)^3$ and $(9)^3$, i.e. 27, 216 and 729.

Question 89 Show that 500 is not a perfect square.

Solution.

Resolving 500 into prime factors, we have

2	500
2	250
5	125
5	25
5	5
	1

$$500 = 2 \times 2 \times 5 \times 5 \times 5$$

Grouping the factors into pairs of equal factors, we get

$$500 = (2 \times 2) \times (5 \times 5 \times 5)$$

Clearly, by grouping into pairs of equal factors, we are left with one factor 5, which cannot be paired.

Hence, 500 is not a perfect square.

Question 90 Express 81 as the sum of first nine consecutive odd numbers.

Solution. $81 = (9)^2 = 1+3+5+7+9+11+13+15+17$ = Sum of first nine consecutive odd numbers

Question 91 Using prime factorisation, find which of the following are perfect squares.

(a) 484 **(b) 11250**

(c) 841 **(d) 729 .**

Solution.

(a) Prime factors of $484 = (2 \times 2) \times (11 \times 11)$

As grouping, there is no unpaired factor left over.

2	484
2	242
11	121
11	11
	1

So, 484 is a perfect square.

So, 484 is a perfect square.

(b) Prime factors of 11250 = $2 \times (3 \times 3) \times (5 \times 5) \times (5 \times 5)$

As grouping, 2 has no pair.

2	11250
3	5625
3	1875
5	625
5	125
5	25
5	5
5	1

So, 11250 is not a perfect square,

(c) Prime factors of 841 = (29×29)

29	841
29	29
	1

As grouping, there is no unpaired factor left over. So, 841 is a perfect square.

(d) Prime factors of 729 = (3 x 3) x (3 x 3) x (3 x 3)

As grouping, there is no unpaired factor left over.

3	729
3	243
3	81
3	27
3	9
3	3
3	1

So, 729 is a perfect square.

Question 92 Using prime factorisatioji, find which of the following are perfect cubes,

(a) 128 (b) 343 (c) 729 (d) 1331

Solution.(a) We have, 128 = 2 x 2 x 2 x 2 x 2 x 2 x 2

Since, 2 remains after grouping in triplets.

So, 128 is not a perfect cube.

(b) We have, 343 = 7 x 7 x 7

Since, the prime factors appear in triplets.

So, 343 is a perfect cube.

(c) We have, 729 = 3 x 3 x 3 x 3 x 3 x3

Since, the prime factors appear in triplets.

So, 729 is a perfect cube.

(d) We have, 1331 =11x11x11

Since, the prime factors appear in triplets.

So, 1331 is a perfect cube.

Question 93 Using distributive law, find the squares of (a) 101 (b) 72

Solution. (a) We have, 101^2 = 101 x 101

= 101(100+ 1)= 10100+ 101 = 10201

(b) We have, 72^2 = 72 x 72 = 72 x (70 + 2)

= 5040+ 144= 5184

Question 94 Can a right angled triangle with sides 6cm, 10cm and 8cm be formed? Give reason.

Solution.

We know that, the sum of the squares of two smaller sides is equal to the square of longer side.

$\because 10^2 = 100 = 64 + 36 = 8^2 + 6^2$

Hence, 6 cm, 8 cm and 10 cm are the sides of a right angled triangle.

Question 95 Write the Pythagorean triplet whose one of the numbers is 4.

Solution.

We know that, for any natural number greater than 1, $(2m, m^2 - 1, m^2 + 1)$ is a pythagorean triplet.

So, one number is $2m$, then other two numbers are m^2+1 and $m^2 - 1$.

Hence, one number is 4, then pythagorean triplet,

$2m = 4 \Rightarrow m = 2$

$\therefore \quad m^2 + 1 = 2^2 + 1 = 4 + 1 = 5$

and $\quad m^2 - 1 = 2^2 - 1 = 4 - 1 = 3$

Now, $\quad 3^2 + 4^2 = 5^2$

$\Rightarrow \quad 9 + 16 = 25 \Rightarrow 25 = 25$

So, 3, 4 and 5 are pythagorean triplets.

Question 96 Using prime factorisation, find the square roots of (a) 11025 (b) 4761

Solution.

(a) We have, 11025

3	11025
3	3675
5	1225
5	245
7	49
7	7
	1

Now, $11025 = \underline{3 \times 3} \times \underline{5 \times 5} \times \underline{7 \times 7}$

$\therefore$ Square root of $11025 = \sqrt{11025} = 3 \times 5 \times 7 = 105$

(b) We have, 4761

3	4761
3	1587
23	529
23	23
	1

Now, $4761 = \underline{3 \times 3} \times \underline{23 \times 23}$

$\therefore$ Square root of $4761 = \sqrt{4761} = 3 \times 23 = 69$

Question 97 Using prime factorisation, find the cube roots of

(a) 512

(b) 2197

Solution.

(a) We have, 512

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Now, $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\therefore$ $\sqrt[3]{512} = 2 \times 2 \times 2$

$= 8$

(b) We have, 2197

13	2197
13	169
13	13
	1

Now, $2197 = 13 \times 13 \times 13$

$\therefore$ $\sqrt[3]{2197} = 13$

Question 98 Is 176 a perfect square? If not;- find the smallest number by which it should be multiplied to get a perfect square.

Solution.

Prime factors of $176 = \underline{2 \times 2} \times \underline{2 \times 2} \times 11$

Here, 11 has no pair.

So, it is not a perfect square and 11 is the smallest number by which 176 should be multiplied to get a perfect square.

Then, $176 \times 11 = 2^2 \times 2^2 \times 11 \times 11$

$= 2^2 \times 2^2 \times 11^2$

$= 1936 = (44)^2$

which is a perfect square of 44.

Hence, the smallest number is 11 by which it should be multiplied to get a perfect square.

Question 99 Is 9720 a perfect cube? If not, find the smallest number by which it should be divided to get a perfect cube.

If we divide the number by $3 \times 3 \times 5$, then the prime factorisation of the quotient will not contain $3 \times 3 \times 5 = 45$.

2	9720
2	4860
2	2430
3	1215
3	405
3	135
3	45
3	15
5	5
	1

$\therefore \quad 9720 \div 45 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$= 216$

$= (6)^3$

Hence, the smallest number by which 9720 should be divided to get a perfect cube, is 45.

Solution. Prime factors of 9720 = 2 x 2 x 2 x 3 x 3 x 3 x 3 x 3 x5

The prime factors 3 and 5 do not appear in group of triplets.

So, 9720 is not a perfect cube.

If we divide the number by 3 x 3 x 5, then the prime factorisation of the quotient will not contain 3 x 3 x 5 = 45.

Question 100 Write two Pythagorean triplets, each having one of the numbers as 5.

Solution.

As, $5^2 = 3^2 + 4^2$

and $13^2 = 12^2 + 5^2$

Hence, 3, 4, 5 and 12, 5, 13 are the two pythagorean triplets.

Question 101 By what smallest number should 216 be divided, so that the quotient is' a perfect square? Also, find the square root of the quotient.

Solution.

Prime factors of $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Grouping the factors into pairs of equal factors, we get

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

We find that there is no prime factor to form a pair with 2 and 3.

Therefore, we must divide the number by 6, so that the quotient becomes a perfect square.

If we divide the given number by 2×3 i.e.6, then

New number $= \frac{216}{6} = 36$

Taking one factor from each, we get square root of new number (quotient)

$= 2 \times 3 = 6$

Question 102 By what smallest number should 3600 be multiplied, so that the quotient is a perfect cube. Also, find the cube root of the quotient.

Solution. Prime factors of 3600 = 2x2x2x2x3x3x5x5

Grouping the factors into triplets of equal factors, we get

$$3600 = \underline{2\times2\times2}\times2\times3\times3\times5\times5$$

2	3600
2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

We know that, if a number is to be a perfect cube, then each of its prime factors must occur thrice.

We find that 2 occurs once 3 and 5 occurs twice only.

Hence, the smallest number, by which the given number must be multiplied in order that the product is a perfect cube $= 2\times2\times3\times5 = 60$

Also, product $= 3600\times60 = 216000$

Now, arranging into triplets of equal prime factors, we have

$$216000 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{3\times3\times3}\times\underline{5\times5\times5}$$

Taking one factor from each triplets, we get

$$\sqrt[3]{216000} = 2\times2\times3\times5 = 60$$

Question 103 Find the square root of the following by long division method.

(a) 1369 (b) 5625

Solution.

(a) We have, 1369

	37
3	$\overline{13}\,\overline{69}$
	9
67	469
	469
	0

$\therefore \quad \sqrt{1369} = 37$

(b) We have, 5625

	75
7	$\overline{56}\,\overline{25}$
	49
145	725
	725
	0

$\therefore \quad \sqrt{5625} = 75$

Question 104 Find the square root of the following by long division method. : (a) 27.04 (b) 1.44

Solution.

(a) We have, 27.04

	5.2
5	$\overline{27}.\overline{04}$
	25
102	204
	204
	0

$\therefore \quad \sqrt{27.04} = 5.2$

Question 105 What is the least number, that should be subtracted from 1385 to get a perfect square? Also, find the square root of the perfect square.

Solution.

(b) We have, 1.44

	1.2
1	$1.\overline{44}$
	1
22	44
	44
	0

$\therefore \quad \sqrt{1.44} = 1.2$

Question 106 What is the least number that should be added to 6200 to make it a perfect square?

Solution.

First, find the square root of 1385 by long division method.

	37
3	$\overline{13}\,\overline{85}$
	9
67	485
	469
	16

Hence, the least number is 16, which should be subtracted from 1385 to get a perfect square and the required perfect square number = 1385 − 16 = 1369

$\therefore$ Square root of 1369 = $\sqrt{37 \times 37} = 37$

Question 107 Find the least number of four digits that is a perfect square.

Solution.

Since, the least number of four digits is 1000.

Now, find the square root of 1000 by long division method.

	32
3	$\overline{10}\,\overline{00}$
	9
62	100
	124
	− 24

Hence, the least number of four digits, i.e. a perfect square = 1000 + 24 = 1024

Question 108 Find the greatest^umber of three digits that is a perfect square.

Solution.

Since, the greatest number of three digits is 1000.

Now, find the square root of 1000 by long division method.

	31
3	$\overline{10}\,\overline{00}$
	9
61	100
	61
	39

Hence, the greatest number of three digits, i.e. a perfect square = 1000 − 39 = 961

Question 109 Find the least square, number, which is exactly divisible by 3, 4, 5, 6 and 8.

Solution.

The least square number divisible by each of 3, 4, 5, 6 and 8, is equal to the LCM of 3, 4, 5, 6 and 8.

2	3, 4, 5, 6, 8
2	3, 2, 5, 3, 4
2	3, 1, 5, 3, 2
3	3, 1, 5, 3, 1
5	1, 1, 5, 1, 1
	1, 1, 1, 1, 1

$\therefore$ LCM of 3, 4, 5, 6 and 8 $= 2\times2\times2\times3\times5 = 120$

The prime factorisation of $120 = (2\times2)\times2\times3\times5$

Here, prime factors 2, 3 and 5 are unpaired. Clearly, to make it a perfect square, it must be multiplied by $2\times3\times5$, i.e. 30.

Therefore, required number $= 120\times30 = 3600$

Hence, the least square number is 3600.

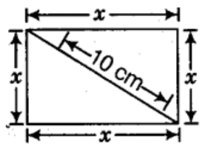
Question 110 Find the length of the side of a square, if the length of its diagonal is 10 cm.

Solution.

Given, length of diagonal = 10 cm

Suppose, the length of side of a square is x cm.

By using Pythagoras theorem,



$$(10)^2 = x^2 + x^2$$
$\Rightarrow$ $$100 = 2x^2$$
$\Rightarrow$ $$x^2 = 50$$
$\Rightarrow$ $$x = \sqrt{50}$$ [taking square root on both sides]
$\therefore$ $$x = 5\sqrt{2}\text{ cm}$$

Hence, the length of the side of square is $\sqrt{50}$ or $5\sqrt{2}$ cm.

Question 111 A decimal number is multiplied by itself. If the product is 51.84, then find the number.

Solution.

Let the number be x. Then, product $= x\times x = x^2$

But product = 51.84 [given]

$\therefore$ $$x^2 = 51.84$$
$\Rightarrow$ $$x = \sqrt{51.84}$$

Now, place the bar over the numbers, then square root is given below.

$\therefore$ $$x = \sqrt{51.84} = 7.2$$

	7.2
7	$\overline{51}.\overline{84}$
	49
142	284
	284
	0

Hence, the required number is 7.2.

Question 112 Find the decimal fraction, which when multiplied by itself, gives 84.64.

Solution.

Let the decimal fraction be x.

When a decimal fraction is multiplied by itself, then product $= x \times x = x^2$

But product = 84.64 [given]

$\therefore \quad x^2 = 84.64$

$\Rightarrow \quad x = \sqrt{84.64}$

Now, place the bar over the numbers, then square root is given below.

	9.2
9	$\overline{84}.\overline{64}$
	81
182	364
	364
	0

$\therefore \quad x = \sqrt{84.64} = 9.2$

Hence, the required decimal is 9.2.

Question 113 A farmer wants to plough his square field of side 150m. How much area will he have to plough?

Solution.

Given, side of square field = 150 m

$\therefore$ Area of square field = Side $\times$ Side $= 150 \times 150 = 22500\ m^2$

Question 114 What will be the number of unit squares on each side of a square graph paper, if the total number of unit squares is 256?

Solution.

Let the number be x. Then, number of unit squares $= x \times x = x^2$

But total number of unit squares = 256

$\therefore \quad x^2 = 256$

$\Rightarrow \quad x = \sqrt{256} = \sqrt{16 \times 16} = 16$

Hence, the required number of squares is 16.

Question 115 If one side of a cube is 15m in Length, then find its volume.

Solution.

Given, one side of a cube = 15 m

$\therefore$ Volume of cube $= (\text{Side})^3 = (15)^3 = 3375\ m^3$

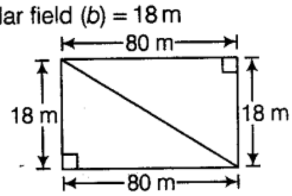
Hence, the volume of cube is $3375\ m^3$.

Question 116 The dimensions of a rectangular field are 80m and 18m. Find the length of its diagonal.

Solution.

Here, length of a rectangular field $(l) = 80$ m

and breadth of a rectangular field $(b) = 18$ m



$\therefore$ Length of diagonal $= \sqrt{l^2 + b^2}$

$= \sqrt{(80)^2 + (18)^2}$

$= \sqrt{6400 + 324}$

$= \sqrt{6724} = 82$ m

Question 117 Find the area of a square field, if its perimeter is 96 m.

Solution.

Given, perimeter of square field = 96 m

$\because$ Perimeter of square = $4 \times$ Side

$\Rightarrow 4 \times (\text{Side}) = 96$

$\Rightarrow \quad \text{Side} = 24\text{ m}$

$\therefore$ Area of square field = $(\text{Side})^2 = (24)^2 = 576\text{ m}^2$

Question 118 Find the length of each side of a cube, if its volume is 512 cm3.

Solution.

Given, volume of a cube = 512 cm^3

$\Rightarrow (\text{Side of a cube})^3 = 8 \times 8 \times 8 = (8)^3$

$\therefore \quad \text{Side of a cube} = 8\text{ cm}$

Hence, the length of each side of the cube is 8 cm.

Question 119 Three numbers are in the ratio 1:2:3 and the sum of their cubes is 4500. Find the numbers.

Solution.

Let the three numbers be x, $2x$ and $3x$.

According to the question,

$$(x)^3 + (2x)^3 + (3x)^3 = 4500$$

$$\Rightarrow \quad x^3 + 8x^3 + 27x^3 = 4500$$

$$\Rightarrow \quad 36x^3 = 4500$$

$$\Rightarrow \quad x^3 = \frac{4500}{36}$$

Question 120 How many square metres of carpet will be required for a square room of side 6.5m to be carpeted?

Solution.

$$\Rightarrow \quad x^3 = 125 \quad \text{[taking cube root on both sides]}$$

$$\Rightarrow \quad x = \sqrt[3]{125}$$

$$\Rightarrow \quad x = \sqrt[3]{5 \times 5 \times 5}$$

$$\therefore \quad x = 5$$

Hence, the numbers are 5, 2×5 and 3×5, i.e. 5, 10 and 15.

Question 121 Find the side of a square, whose area is equal to the area of a rectangle with sides 6.4m and 2.5m.

Solution.

Given dimensions of a rectangle are 6.4 m and 2.5 m.

Area of a rectangle = Length $\times$ Breadth = $6.4 \times 2.5 = 16\text{ m}^2$

Let the side of square be x m.

According to the question,

Area of square = Area of rectangle

$$\Rightarrow \quad x^2 = 16$$

$$\Rightarrow \quad x = \sqrt{16}$$

$$\Rightarrow \quad x = \sqrt{4 \times 4}$$

$$\therefore \quad x = 4\text{ m}$$

Hence, the side of a square is 4 m.

Question 122 Difference of two perfect cubes is" 189. If the cube root of the smaller of the two numbers is 3, then find the cube root of the larger number.

Solution.

Given different of two perfect cubes = 189

and cube root of the smaller number = 3

∴ Cube of smaller number = $(3)^3 = 27$

Let cube root of the larger number be x.

Then, cube of larger number = x^3

According to the question,

$$x^3 - 27 = 189$$

⇒ $$x^3 = 189 + 27$$

⇒ $$x^3 = 216$$

⇒ $$x = \sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6}$$

∴ $$x = 6$$

Hence, the cube root of the larger number is 6.

Question 123 Find the number of plants in each row, if 1024 plants are arranged, so that number of plants in a row is the same as the number of rows.

Solution.

Let the number of plants in each row be x.

Then, number of rows = Number of plants in each row = x

∴ Total plants = $x \times x = x^2$

According to the question,

$$x^2 = 1024$$

⇒ $$x = \sqrt{1024} = \sqrt{32 \times 32}$$

∴ $$x = 32$$

Hence, there are 32 plants in each row.

	32
3	$\overline{10}\,\overline{24}$
	9
62	124
	124
	0

Question 124 A hall has a capacity of 2704 seats. If the number of rows is equal to the number of seats in each row, then find the number of seats in each row.

Solution.

Let the number of seats in each row be x.

Then, number of rows = Number of seats in each row = x

∴ Total seats = $x \times x = x^2$

According to the question,

$$x^2 = 2704$$

⇒ $$x = \sqrt{2704} = \sqrt{52 \times 52}$$

⇒ $$x = 52$$

Hence, there are 52 seats in each row.

	52
5	$\overline{27}\,\overline{04}$
	25
102	204
	204
	0

Question 125 A General wishes to draw up his 7500 soldiers in the form of a square. After arranging, he found out that some of them are left out. How many soldiers were left out?

Solution.

Given, total number of soldiers = 7500

	86
8	$\overline{75}\,\overline{00}$
	64
166	1100
	996
	104

Hence, 104 soldiers were left out.

Question 126 8649 students were sitting in a lecture room in such a manner that there were as many students in the row as there were rows in the lecture room. How many students were there in each row of the lecture room?

Solution.

Let number of students in each row of the lecture room be x.

Then, number of rows = x

$\therefore$ Total students = $x \times x = x^2$

According to the question,

$$x^2 = 8649$$

$$\Rightarrow \quad x = \sqrt{8649}$$

$$\therefore \quad x = 93$$

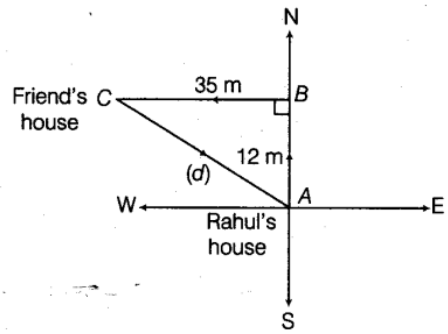
Hence, there are 93 students in each row of the lecture room.

	93
9	$\overline{86}\,\overline{49}$
	81
183	549
	549
	0

Question 127 Rahul walks 12m North from his house and turns West to walk 35m to reach his friend's house. While returning, he walks diagonally from his friend's house to reach back to his house. What distance did he walk, while returning?

Solution.

Let Rahul walked x m, while returning home.



In ΔABC, by using Pythagoras theorem, we get

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad AC^2 = (12)^2 + (35)^2$$

$$\Rightarrow \quad AC^2 = 144 + 1225 = 1369$$

$$\Rightarrow \quad AC = \sqrt{1369}$$

$$\Rightarrow \quad AC = 37 \text{ m}$$

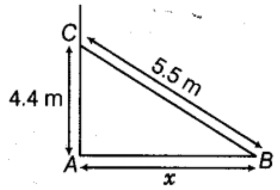
Hence, Rahul walked 37m distance for returning to his house diagonally.

	37
3	$\overline{13}\,\overline{69}$
	9
67	469
	469
	0

Question 128 A 5.5m long ladder is leaned against a wall. The ladder reaches the wall to a height of 4.4m. Find^the distance between the wall and the foot of the ladder.

Solution.

Let the distance between the wall and the foot of the ladder be x m



In right angled ΔABC, by using Pythagoras theorem, we get

$$BC^2 = AB^2 + AC^2$$

$$\Rightarrow \quad (5.5)^2 = x^2 + (4.4)^2$$

$$\Rightarrow \quad x^2 = (5.5)^2 - (4.4)^2$$

$$\Rightarrow \quad x^2 = 30.25 - 19.36$$

$$\Rightarrow \quad x^2 = 10.89$$

$$\Rightarrow \quad x = \sqrt{10.89}$$

$$\therefore \quad x = 3.3 \text{ m}$$

Hence, the distance between the wall and the foot of the ladder is 3.3 m.

	3.3
3	$\overline{10}\,.\,\overline{89}$
	9
63	189
	189
	0

Question 129 A king wanted to reward his advisor, a wiseman of the kingdom. So, he asked the wiseman to name his own reward. The wiseman thanked the king, but said that he would ask only for some gold coins each day for a month. The coins were to be counted out in a pattern of one coin for the first day, 3 coins for the second day, 5 coins for the third day and

so on for 30 days. Without making calculations, find how many coins will the advisor get in that month?

Solution.

Let the advisor get x coins in that month.

According to the question, advisor get coins upto 30 days as pattern

$$1+3+5+\ldots$$

Here, $n = 30$

We know that, sum of first n odd natural numbers $= n^2 = (30)^2 = 30 \times 30 = 900$

Hence, the advisor get 900 coins in that month.

Question 130 Find three numbers in the ratio 2 : 3 : 5, the sum of whose squares is 608.

Solution.

Let the three numbers be $2x$, $3x$ and $5x$, respectively.

According to the question,

$$(2x)^2 + (3x)^2 + (5x)^2 = 608 \quad \text{[given]}$$

$$\Rightarrow \quad 4x^2 + 9x^2 + 25x^2 = 608$$

$$\Rightarrow \quad 38x^2 = 608$$

$$\Rightarrow \quad x^2 = \frac{608}{38}$$

$$\Rightarrow \quad x^2 = 16 = (4)^2$$

$$\therefore \quad x = 4$$

Hence, the numbers are 2×4, 3×4, 5×4, i.e. 8, 12 and 20.

Question 131 Find the smallest square number divisible by each of the numbers 8, 9 and 10.

Solution.

The least number divisible by each of the numbers 8, 9 and 10 is equal to the LCM of 8, 9 and 10.

2	8, 9, 10
2	4, 9, 5
2	2, 9, 5
3	1, 9, 5
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

$\therefore$ LCM of 8, 9 and 10 $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

Prime factors of 360 $= (2 \times 2) \times 2 \times (3 \times 3) \times 5$

Here, prime factors 2 and 5 are unpaired. Clearly, to make it a perfect square, it must be multiplied by 2×5, i.e. 10.

Therefore, required number $= 360 \times 10 = 3600$

Hence, the smallest square number divisible by each of the numbers 8, 9 and 10 is 3600.

Question 132

The area of a square plot is $101\frac{1}{400}$ m². Find the length of one side of the plot.

Solution.

Let length of the square plot be a. Then, the area of square $= a^2$

According to the question,

$$\text{Area} = 101\frac{1}{400}\ \text{m}^2 \quad \text{[given]}$$

$$\therefore \quad a^2 = 101\frac{1}{400} \Rightarrow a^2 = \frac{40401}{400}$$

$$\Rightarrow \quad a = \sqrt{\frac{40401}{400}} \Rightarrow a = \sqrt{\frac{201 \times 201}{20 \times 20}}$$

$$\therefore \quad a = \frac{201}{20} = 10\frac{1}{20}\ \text{m} \quad \text{[taking square root on both sides]}$$

Hence, the length of one side of the plot is $10\frac{1}{20}$ m.

Question 133 Find the square root of 324 by the method of repeated subtraction.

Solution.

Given number is 324.

Now, we subtract successive odd numbers starting from 1 as follows:

Here, $324 - 1 = 323, \; 323 - 3 = 320$

$320 - 5 = 315, \; 315 - 7 = 308$

$308 - 9 = 299, \; 299 - 11 = 288$

$288 - 13 = 275, \; 275 - 15 = 260$

$260 - 17 = 243, \; 243 - 19 = 224$

$224 - 21 = 203, \; 203 - 23 = 180$

$180 - 25 = 155, \; 155 - 27 = 128$

$128 - 29 = 99, \; 99 - 31 = 68$

$68 - 33 = 35, \; 35 - 35 = 0$

We observe that the number 324 reduced to zero (0) after subracting 18 odd numbers.

So, 324 is a perfect square of 18.

$\therefore \; \sqrt{324} = 18$

Hence, the square root of 324 is 18.

Question 134 Three numbers are in the ratio 2 : 3 : 4. The sum of their cubes is 0.334125. Find the numbers.

Solution.

Let the numbers be $2x$, $3x$ and $4x$, respectively.

$\because$ Sum of their cubes = 0.334125 [given]

According to the question,

$$(2x)^3 + (3x)^3 + (4x)^3 = 0.334125$$

$$\Rightarrow 8x^3 + 27x^3 + 64x^3 = 0.334125$$

$$\Rightarrow 99x^3 = 0.334125$$

$$\Rightarrow x^3 = \frac{0.334125}{99}$$

$$\Rightarrow x^3 = 0.003375$$

$$\Rightarrow x^3 = \frac{3375}{1000000}$$

$$\Rightarrow x = \sqrt[3]{\frac{15\times15\times15}{10\times10\times10\times10\times10\times10}}$$ [taking cube root on both sides]

$$\Rightarrow x = \frac{15}{10\times10\times10}$$

$$\therefore x = 0.015$$

Hence, the required numbers are 2×0.015, 3×0.015 and 4×0.015, i.e. 0.03, 0.045 and 0.06.

Question 135

Evaluate: $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$

Solution.

We have, $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$

$$= (27)^{1/3} + (0.008)^{1/3} + (0.064)^{1/3}$$

$$= (3\times3\times3)^{\frac{1}{3}} + (0.2\times0.2\times0.2)^{\frac{1}{3}} + (0.4\times0.4\times0.4)^{\frac{1}{3}}$$

$$= (3)^{3\times\frac{1}{3}} + (0.2)^{3\times\frac{1}{3}} + (0.4)^{3\times\frac{1}{3}}$$

$$= 3 + 0.2 + 0.4 = 3.6$$

Question 136

Evaluate: $\{5^2 + (12^2)^{1/2}\}^3$

Solution.

We have, $\{5^2 + (12^2)^{1/2}\}^3 = \{25 + 12\}^3$

$= (37)^3 = 37 \times 37 \times 37 = 50653$

Question137

Evaluate: $\{6^2 + (8^2)^{1/2}\}^3$

Solution.

We have, $\{6^2 + (8^2)^{1/2}\}^3 = \{36 + 8\}^3 = (44)^3 = 44 \times 44 \times 44 = 85184$

Question 138

A perfect square number has four digits, none of which is zero. The- digits from left to right have valfles, that are even, even, odd, even. Find the number.

Solution.

Suppose abed is a perfect square.

where, a = even number

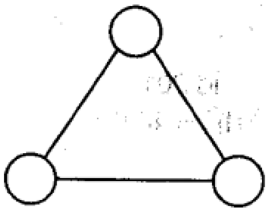
b = even number

c = odd number

d = even number

Hence, 8836 is one of the number which satisfies the given condition.

Question 139 Put three different numbers in the circles, so that when you add the numbers at the end of each line, you always get a perfect square.

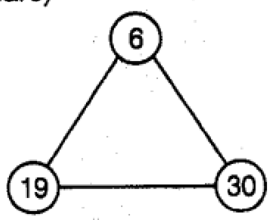


Solution.

$\because$ $6 + 19 = 25$ (perfect square)

$19 + 30 = 49$ (perfect square)

and $30 + 6 = 36$ (perfect square)



Question 140 The perimeters of two squares are 40m and 96m, respectively. Find the perimeter of another square equal in area to the sum of the first two squares.

Solution.

Let the side of two squares be a_1 and a_2.
Then, perimeter of first square $= 4a_1$
Given, perimeter of first square = 40
According to the question,

$$4a_1 = 40 \Rightarrow a_1 = 10$$

Similarly, perimeter of second square = $4a_2$
Given, perimeter of second square = 96
According to the question,

$$4a_2 = 96 \Rightarrow a_2 = 24$$

Let a be the side of another square.
Then, area of another square = a^2
∴ Area of another square = Sum of the areas of first and second squares
= Area of first square + Area of second square
$= a_1^2 + a_2^2$

$$\Rightarrow a^2 = (10)^2 + (24)^2 = 100 + 576 = 676 \Rightarrow a = \sqrt{676} = 26 \text{ m}$$

[taking square root on both sides]

Perimeter of another square $= 4a = 4 \times 26 = 104$ m
So, the perimeter of another square is 104 m.

Question 141 A three-digit perfect square is such that, if it is viewed upside down, the number seen is also a perfect square. What is the number?
[Hint The digits 1, 0 and 8 stay the same when viewed upside down, whereas 9 becomes 6 and 6 becomes 9]
Solution. Three-digit perfect squares are 196 and 961, which looks same when viewed upside down.

Question 142 13 and 31 is a strange pair of numbers, such that their squares 169 and 961 are also mirror of each other. Can yqu find two other such pairs?
Solution.
(i) Let a pair of numbers be 12 and 21.
Also, $(12)^2 = 144$ and $(21)^2 = 441$
(ii) Let a pair of numbers be 102 and 201.
Also, $(102)^2 = 10404$ and $(201)^2 = 40401$