Unit 7(Algebraic Expression, Identities & Factorisation)

Multiple Choice Questions

Question. 1 The product of a monomial and a binomial is a
(a) monomial (b) binomial
(c) trinomial (d) None of these

Solution. (b) Monomial consists of only single term and binomial contains two terms. So, the multiplication of a binomial by a monomial will always produce a binomial, whose first term is the product of monomial and the binomial's first term and second term is the product of monomial and the binomial's second term.

Question. 2 In a polynomial, the exponents of the variables are always (a)'integers (b) positive integers (c) non-negative integers (d) non-positive integers

Solution. (c) In a polynomial, the exponents of the variables are either positive integers or 0. Constant term C can be written as C x°. We do not consider the expressions as a polynomial which consist of the variables having negative/fractional exponent.

Question. 3 Which of the following is correct?
(a) $(a-b)^2=a^2+2ab-b^2$ **(b)** $(a-b)^2=a^2-2ab+b^2$
(c) $(a-b)^2=a^2-b^2$ **(d)** $(a+b)^2=a^2+2ab-b^2$

Solution.

(b) We have,

$(a-b)^2 = (a-b)(a-b)$

$= a(a-b) - b(a-b)$

$= a \cdot a - a \cdot b - b \cdot a + b \cdot b$

$= a^2 - ab - ab + b^2$ $[\because a \cdot b = b \cdot a]$

$= a^2 - 2ab + b^2$

and $(a+b)^2 = (a+b)(a+b)$

$= a \cdot a + a \cdot b + b \cdot a + b \cdot b$

$= a^2 + 2ab + b^2$

Question. 4 The sum of -7pq and 2pq is

(a) -9pq (b) 9pq

(c) 5pq (d) -5pq

Solution.

(d) Given, monomials are $-7pq$ and $2pq$.

$\therefore$ Their sum $= -7pq + 2pq = (-7+2)\,pq$

[both monomials consist of like terms, so adding their numerical coefficient]

$= -5pq$

Question. 5 If we subtract $-3x^2y^2$ from x^2y^2, then we get

(a) $-4x^2y^2$ (b) $-2x^2y^2$

(c) $2x^2y^2$ (d) $4x^2y^2$

Solution.

(d) Given, monomials are $-3x^2y^2$ and x^2y^2. Now, we have to subtract the first one from the second one.

i.e. $x^2y^2 - (-3x^2y^2) = x^2y^2 - (-3)x^2y^2$

$= x^2y^2 + 3x^2y^2$

$= (1+3)\,x^2y^2$

$= 4x^2y^2$

Question. 6 Like term as $4m^3n^2$ is

(a) $4m^2n^2$ (b) $-6m^3n^2$

(c) $6pm^3n^2$ (d) $4m^3n$

Solution. (b) We know that, the like terms contain the same literal factor. So, the like term as $4m^3n^2$, is $-6m^3n^2$, as it contains the same literal factor m^3n^2.

Question. 7 Which of the following is a binomial?

(a) $7 \times a + a$ (b) $6a^2 + 7b + 2c$

(c) $4a \times 3b \times 2c$ (d) $6(a^2 + b)$

Solution.

(d) Binomials are algebraic expressions consisting of two unlike terms.

(a) $7 \times a + a = 7a + a = 8a$ [monomial]

(b) $6a^2 + 7b + 2c$ [trinomial]

(c) $4a \times 3b \times 2c = 24abc$ [monomial]

(d) $6(a^2 + b) = 6a^2 + 6b$ [binomial]

Question. 8 Sum of a – b + ab, b + c – bc and c – a – ac is

(a) $2c + ab - ac - bc$ (b) $2c - ab - ac - bc$

(c) $2c + ab + ac + bc$ (d) $2c - ab + ac + bc$

Solution.

(a) Required sum = $(a - b + ab) + (b + c - bc) + (c - a - ac)$

$= a - b + ab + b + c - bc + c - a - ac$

$= 2c + ab - ac - bc$ [adding the like terms and retaining others]

Question. 9 Product of the monomials 4p, $-7q^3$, -7pq is

(a) $196\ p^2q^4$
(b) $196\ pq^4$
(c) $-196\ p^2q^4$
(d) $196\ p^2q^3$

Solution.

(a) Required product = $4p \times (-7q^3) \times (-7\ pq)$

$= 4 \times (-7) \times (-7)\ p \times q^3 \times pq$ [multiplying the numerical coefficients]

$= 196\ p^2q^4$ [multiplying the literal factors having same variables]

Question. 10 Area of a rectangle with length 4ab and breadth $6b^2$ is

(a) $24a^2b^2$
(b) $24\ ab^3$
(c) $24\ ab^2$
(d) $24ab$

Solution.

(b) We know that, area of a rectangle = Length × Breadth = $4ab \times 6b^2$

This is the product of two monomials.

∴ Area of rectangle = $(4 \times 6)\ ab \times b^2$

$= 24\ ab^3$

Question. 11 Volume of a rectangular box (cuboid) with length = 2ab, breadth = 3ac and height = 2ac is

(a) $12\ a^3bc^2$
(b) $12\ a^3bc$
(c) $12\ a^2bc$
(d) $2\ ab + 3\ ac + 2ac$

Solution.

(a) We know that, volume of a cuboid = Length × Breadth × Height = $2ab \times 3ac \times 2ac$

$= (2 \times 3 \times 2)\ ab \times ac \times ac = 12\ a \times a \times a \times b \times c \times c = 12a^3bc^2$

Question. 12 Product of $6a^2$ -7b + 5ab and 2ab is

(a) $12a^3b - 14ab^2 + 10ab$
(b) $12a^3b - 14ab^2 + 10a^2b^2$
(c) $6a^2 - 7b + 7ab$
(d) $12a^2b - 7ab^2 + 10ab$

Solution.

(b) Required product $= 2ab \times (6a^2 - 7b + 5ab)$

This is the product of a trinomial by a monomial, so we multiply monomial with each term of the trinomial.

∴ $2ab \times (6a^2 - 7b + 5ab) = 2ab \times 6a^2 + 2ab\ (-7b) + 2ab \times 5ab$

$= 12a^3b - 14ab^2 + 10a^2b^2$

Question. 13 Square of 3x – 4y is

(a) $9x^2 - 16y^2$
(b) $6x^2 - 8y^2$
(c) $9x^2 + 16y^2 + 24xy$
(d) $9x^2 + 16y^2 - 24xy$

Solution.

(d) Square of $(3x - 4y)$ will be $(3x - 4y)^2$.

Comparing $(3x - 4y)^2$ with $(a - b)^2$, we get $a = 3x$ and $b = 4y$.

Now, using the identity $(a - b)^2 = a^2 - 2ab + b^2$

$$(3x - 4y)^2 = (3x)^2 - 2 \cdot 3x \cdot 4y + (4y)^2$$
$$= 9x^2 - 24xy + 16y^2$$

Question. 14 Which of the following are like terms?

(a) $5xyz^2, -3xy^2z$ (b) $-5xyz^2, 7xyz^2$
(c) $5xyz^2, 5x^2yz$ (d) $5xyz^2, x^2y^2z^2$

Solution.

(b) We know that, the terms having same algebraic (literal) factors are called like terms.

(a) $5\,xyz^2, -3\,xy^2z$ [unlike terms]

(b) $-5\,xyz^2, 7\,xyz^2$ [like terms]

(c) $5\,xyz^2, 5\,x^2yz$ [unlike terms]

(d) $5\,xyz^2, x^2y^2z^2$ [unlike terms]

Question. 15 Coefficient of y in the term of $-y^3$ is

(a)-1 (b)-3 (c)-1^3 (d)1^3

Solution.

(c) We can write $\frac{-y}{3}$ as $-\frac{1}{3} \times y$.

So, the coefficient of y is $-\frac{1}{3}$.

Question. 16 $a^2 - b^2$ is equal to

(a) $(a - b)^2$ (b) $(a - b)(a - b)$ (c) $(a + b)(a - b)$ (d) $(a + b)(a + b)$

Solution.

(c) (a) $(a - b)^2 = a^2 - 2ab + b^2$

(b) $(a - b)(a - b) = (a - b)^2 = a^2 - 2ab + b^2$

(c) $(a + b)(a - b) = a(a - b) + b(a - b) = a^2 - ab + ba - b^2 = a^2 - b^2$ [$\because ab = ba$]

(d) $(a + b)(a + b) = (a + b)^2 = a^2 + 2ab + b^2$

Question. 17 Common factor Of 17abc, 34ab^2, 51a^2b is

(a)17abc (b)17ab (c)17ac (d)17a^2b^2c

Solution.

(b) Given, $17abc = 17 \times a \times b \times c$

$$34ab^2 = 2 \times 17 \times a \times b \times b$$
$$51a^2b = 3 \times 17 \times a \times a \times b$$

Now, collecting the common factors, we get $17 \times a \times b = 17ab$

Question. 18 Square of 9x – 7xy is

(a) $81x^2 + 49x^2y^2$ (b) $81x^2 - 49x^2y^2$
(c) $81x^2 + 49x^2y^2 - 126x^2y$ (d) $81x^2 + 49x^2y^2 - 63x^2y$

Solution.

(c) Square of $(9x - 7xy) = (9x - 7xy)^2$

Comparing with $(a - b)^2$, we get $a = 9x$ and $b = 7xy$

$(9x - 7xy)^2 = (9x)^2 - 2 \cdot 9x \cdot 7xy + (7xy)^2$ [using the identity, $(a - b)^2 = a^2 - 2ab + b^2$]

$$= 81x^2 - 126x^2y + 49x^2y^2$$
$$= 81x^2 + 49x^2y^2 - 126x^2y$$

Question. 19 Factorised form of 23xy – 46x + 54y -108 is

(a) $(23x + 54)(y - 2)$ (b) $(23x + 54y)(y - 2)$
(c) $(23xy + 54y)(-46x - 108)$ (d) $(23x + 54)(y + 2)$

Solution.

(a) We have, $23xy - 46x + 54y - 108 = 23xy - 2 \times 23x + 54y - 2 \times 54$

$= 23x(y-2) + 54(y-2)$

[taking common out in I and II expressions]

$= (y-2)(23x + 54)$ [taking $(y-2)$ common]

$= (23x + 54)(y-2)$

Question. 20 Factorised form of r^2-10r + 21 is

(a)(r-1)(r-4) (b)(r-7)(r-3) (c)(r-7)(r+3) (d)(r+7)(r+3)

Solution.

(b) We have, $r^2 - 10r + 21$

$= r^2 - 7r - 3r + 21 = r(r-7) - 3(r-7)$

[by splitting the middle term, so that the product of their numerical coefficients is equal constant term]

$= (r-7)(r-3)$ [$\because x^2 + (a+b)x + ab = (x+a)(x+b)$]

Question. 21 Factorised form of p^2 – 17p – 38 is

(a) (p -19)(p + 2) (b) (p -19) (p – 2) (c) (p +19) (p + 2) (d) (p + 19) (p – 2)

Solution.

(a) We have, $p^2 - 17p - 38 = p^2 - 19p + 2p - 38$

[by splitting the middle term, so that the product of their numerical coefficients is equal constant term]

$= p(p-19) + 2(p-19) = (p-19)(p+2)$ [$\because x^2 + (a+b)x + ab = (x+a)(x+b)$]

Question. 22 On dividing 57 p^2 qr by 114pq, we get

(a) $\frac{1}{4}pr$ (b) $\frac{3}{4}pr$ (c) $\frac{1}{2}pr$ (d) $2pr$

Solution.

(c) Required value $= \frac{57p^2qr}{114pq} = \frac{57 \times p \times p \times q \times r}{114 \times p \times q} = \frac{57}{114}pr = \frac{1}{2}pr$

Question. 23 On dividing p(4p^2 – 16) by 4p (p – 2), we get

(a) 2p + 4 (b) 2p – 4 (c) p + 2 (d) p – 2

Solution.

(c) We have,

$$\frac{p(4p^2 - 16)}{4p(p-2)} = \frac{p[(2p)^2 - 4^2]}{4p(p-2)}$$

$$= \frac{(2p-4)(2p+4)}{4(p-2)} \quad [\because a^2 - b^2 = (a+b)(a-b)]$$

$$= \frac{2(p-2)\cdot 2(p+2)}{4(p-2)} = \frac{4(p-2)(p+2)}{4(p-2)} = p + 2$$

Question. 24 The common factor of 3ab and 2cd is

(a) 1 (b) -1 (c) a (d) c

Solution. (a) We have, monomials 3ab and 2cd Now, 3ab = 3xaxb 2cd =2 x c x d
Observing the monomials, we see that, there is no common factor (neither numerical nor literal) between them except 1.

Question. 25 An irreducible factor of24x^2y^2 is

(a)a^2 (b)y^2 (c)x (d)24x

Solution. (c) A factor is said to be irreducible, if it cannot be factorised further.
We have, $24x^2y^2$ =2 x 2 x 2 x 3 x x x x x y x y Hence, an irreducible factor of 24 x^2y^2 is x.

Question. 26 Number of factors of $(a+b)^2$ is
(a) 4 (b) 3 (c) 2 (d) 1
Solution. (c) We can write $(a+b)^2$ as, (a + b) (a + b) and this cannot be factorised further.
Hence, number of factors of $(a+b)^2$ is 2.

Question. 27 The factorised form of 3x – 24 is
(a) 3x x 24 (b)3 (x – 8) (c)24(x – 3) (d)3(x-12)
Solution. (b) We have,
3x – 24 = 3 x x – 3 x 8= 3 (x – 8) [taking 3 as common]

Question. 28 The factors of x^2 – 4 are
(a) (x – 2), (x – 2) (b) (x + 2), (x – 2)
(c) (x + 2), (x + 2) (d) (x – 4), (x – 4)
Solution.
(b) We have,
$x^2-4=x^2-2^2=(x+2)(x-2)$ $[\because a^2-b^2=(a+b)(a-b)]$
Hence, $(x+2)$, $(x-2)$ are factors of x^2-4.

Question. 29 The value of $(-27x^2y)\div(-9xy)$ is
(a)3xy (b)-3xy (c)-3x (d)3x
Solution.
(d) We have,
$$(-27x^2y)\div(-9xy)=\frac{-27x^2y}{-9xy}=\frac{27\times x\times x\times y}{9\times x\times y}=\frac{27}{9}x=3x$$

Question. 30 The value of $(2x^2+4)\div(2)$ is
(a) $2x^2+2$ (b) x^2+2 (c) x^2+4 (d) $2x^2+4$

Solution.
(b) We have,
$$(2x^2+4)\div 2=\frac{2x^2+4}{2}=\frac{2(x^2+2)}{2}$$ [taking 2 as common]
$$=x^2+2$$

Question. 31 The value of $(3x^3+9x^2+27x)\div 3x$ is
(a) $x^2+9+27x$ (b) $3x^2+3x^2+27x$
(c) $3x^3+9x^2+9$ (d) x^2+3x+9

Solution.
(d) We have,
$$(3x^3+9x^2+27x)\div 3x=\frac{3x^3+9x^2+27x}{3x}=\frac{3x^3}{3x}+\frac{9x^2}{3x}+\frac{27x}{3x}=x^2+3x+9$$

Question. 32 The value of $(a+b)^2+(a-b)^2$ is
(a) $2a+2b$ (b) $2a-2b$
(c) $2a^2+2b^2$ (d) $2a^2-2b^2$

Solution.

(c) We have,

$(a+b)^2 + (a-b)^2 = (a^2 + b^2 + 2ab) + (a^2 + b^2 - 2ab)$

$[\because (a+b)^2 = a^2 + b^2 + 2ab \text{ and } (a-b)^2 = a^2 + b^2 - 2ab]$

$= (a^2 + a^2) + (b^2 + b^2) + (2ab - 2ab)$ [combining the like terms]

$= 2a^2 + 2b^2$

Question. 33 The value of $(a+b)^2 - (a-b)^2$ is

(a) $4ab$ (b) $-4ab$ (c) $2a^2 + 2b^2$ (d) $2a^2 - 2b^2$

Solution.

(a) We have,

$(a+b)^2 - (a-b)^2 = a^2 + b^2 + 2ab - (a^2 + b^2 - 2ab)$

$[\because (a+b)^2 = a^2 + b^2 + 2ab \text{ and } (a-b)^2 = a^2 + b^2 - 2ab]$

$= a^2 + b^2 + 2ab - a^2 - b^2 + 2ab = a^2 - a^2 + b^2 - b^2 + 2ab + 2ab = 2ab + 2ab = 4ab$

[combining the like terms]

Fill in the Blanks

In questions 34 to 58, fill in the blanks to make the statements true.

Question. 34 The product of two terms with like signs is a term.

Solution. Positive

If both the like terms are either positive or negative, then the resultant term will always be positive.

Question. 35 The product of two terms with unlike signs is a term.

Solution. Negative

As the product of a positive term and a negative term is always negative.

Question. 36 a (b + c) = a x ——– + a x ——-

Solution. b,c

we have , a(b+c)=a x b + a x c [using left distributive law]

Question. 37 (a-b) ———- = $a^2 - 2ab + b^2$

Solution.

(a – b)

We know that, $(a-b)(a-b) = (a-b)^2$

$= a^2 - 2ab + b^2$ $[\because (a-b)^2 = a^2 - 2ab + b^2]$

Question. 38 $a^2 - b^2$=(a+b)————-

Solution.

(a – b)

We have, $a^2 - b^2 = (a+b)(a-b)$ $[\because a^2 - b^2 = (a+b)(a-b)]$

Alternative Method

Let $(a^2 - b^2) = (a+b)x$

$\Rightarrow$ $x = \dfrac{a^2 - b^2}{a+b} = \dfrac{(a+b)(a-b)}{a+b} = a - b$

Question. 39 $(a-b)^2$+————-=$a^2 - b^2$

Solution.

$2ab - 2b^2$

Let $(a-b)^2 + x = a^2 - b^2$

$\Rightarrow a^2 + b^2 - 2ab + x = a^2 - b^2$ $[\because (a-b)^2 = a^2 + b^2 - 2ab]$

$\Rightarrow$ $x = a^2 - b^2 - (a^2 + b^2 - 2ab) = a^2 - b^2 - a^2 - b^2 + 2ab = 2ab - 2b^2$

Question. 40 $(a+b)^2$-2ab=————- + ————.

Solution.

$a^2 + b^2$

We have,

$$(a+b)^2 - 2ab = a^2 + b^2 + 2ab - 2ab \quad [\because (a+b)^2 = a^2 + b^2 + 2ab]$$
$$= a^2 + b^2$$

Question. 41 (x+a)(x+b)=x^2 + (a+b) x + ————.

Solution.

ab

We have,

$$(x+a)(x+b) = x^2 + bx + ax + ab$$
$$= x^2 + (a+b)x + ab$$

Question. 42 The product of two polynomials is a —————.

Solution. Polynomial

As the product of two polynomials is again a polynomial.

Question. 43 Common factor of ax2 + bx is——————.

Solution.

x

We have, $ax^2 + bx = x(ax+b)$ [taking x as common]

Question. 44 Factorised form of 18mn + 10mnp is ——————.

Solution.

$2mn(9+5p)$

We have, $18mn + 10mnp = 2 \times 9 \times m \times n + 2 \times 5 \times m \times n \times p$

$= 2mn(9+5p)$ [taking $2mn$ as common]

Question. 45 Factorised form of $4y^2$ – 12y + 9 is———— .

Solution.

$(2y-3)(2y-3)$

Let

$$4y^2 - 12y + 9 = (2y)^2 - 2 \times 2y \times 3 + 3^2$$
$$= (2y-3)^2 \quad [\because (a-b)^2 = a^2 - 2ab + b^2]$$
$$= (2y-3)(2y-3)$$

Question. 46 $38x^2y^2z \div 19xy^2$ is equal to————.

Solution.

$2x^2z$

We have, $38x^3y^2z \div 19xy^2$

i.e. $$\frac{38x^3y^2z}{19xy^2} = \frac{38 \times x \times x \times x \times y \times y \times z}{19 \times x \times y \times y} = \frac{38}{19}x^2z = 2x^2z$$

Question. 47 Volume of a rectangular box with length 2x, breadth 3y and height 4z is ——.

Solution. 24 xyz

We know that, the volume of a rectangular box,

V = Length x Breadth x Height = 2x x 3y x 4z = (2 x 3 x 4) xyz = 24 xyz

Question. 48 $67^2 - 37^2$ =(67 -37) x ————=————.

Solution.

67 + 37, 3120

We have, $67^2 - 37^2 = (67 - 37)(67 + 37)$ $[\because a^2 - b^2 = (a - b)(a + b)]$

$= 30 \times 104 = 3120$

Question. 49 $103^2 - 102^2$=————- x (103-102)=—————.

Solution.

(103 + 102), 205

We have,

$103^2 - 102^2 = (103 + 102)(103 - 102)$ $[\because a^2 - b^2 = (a + b)(a - b)]$

$= 205 \times 1 = 205$

Question. 50 Area of a rectangular plot with sides $4y^2$ and $3y^2$ is—————.

Solution.

$\mathbf{12x^2y^2}$

We know that, area of rectangle = Length × Breadth

$\therefore$ Area of a rectangular plot = $4x^2 \times 3y^2 = (4 \times 3)x^2y^2 = 12x^2y^2$

Question. 51 Volume of a rectangular box with l = b = h = 2x is ———-.

Solution.

$\mathbf{8x^3}$

Volume of a rectangular box = $l \times b \times h = 2x \times 2x \times 2x$

$= (2 \times 2 \times 2)x^3$

$= 8x^3$

Question. 52 The numerical coefficient in -37abc is—————.

Solution. -37

The constant term (with their sign) involved in term of an algebraic expression is called the numerical coefficient of that term.

Question. 53 Number of terms in the expression a^2 and + bc x d is –.

Solution.

We have, $a^2 + bc \times d = a^2 + bcd$

$\therefore$ The number of terms in this expression is **2** as *bcd* is treated as a single term.

Question. 54 The sum of areas of two squares with sides 4o and 4b is————-.

Solution.

$\mathbf{16(a^2 + b^2)}$

$\because$ Area of a square = $(\text{Side})^2$

$\therefore$ Area of the square whose one side is $4a = (4a)^2 = 16a^2$

Area of the square with side $4b = (4b)^2 = 16b^2$

$\therefore$ Sum of areas = $16a^2 + 16b^2 = 16(a^2 + b^2)$

Question. 55 The common factor method of factorisation for a polynomial is based on ————-property.

Solution. Distributive

In this method, we regroup the terms in such a way, so that each term in the group contains a common literal or number or both.

Question. 56 The side of the square of area 9 y^2 is————.

Solution.

3y

Given, area of a square = $9y^2$

We know that, the area of a square with side a = a^2

$\therefore \quad a^2 = 9y^2$

$\Rightarrow \quad a^2 = (3y)^2$

$\Rightarrow \quad a = 3y$ [taking square root both sides]

Question. 57 On simplification, $\frac{3x+3}{3}$ =————.

Solution.

x + 1

We have, $\frac{3x+3}{3} = \frac{3x}{3} + \frac{3}{3} = x + 1$

Question. 58 The factorisation of 2x + 4y is————.

Solution. 2 (x + 2y)

We have, 2x + 4y = 2x + 2 x 2y = 2 (x + 2y)

True/False

In questions 59 to 80, state whether the statements are True or False

Question. 59 $(a+b)^2 = a^2 + b^2$.

Solution.

False

We have, $(a+b)^2 = a^2 + b^2 + 2ab$ [an algebraic identity]

Question. 60 $(a-b)^2 = a^2 - b^2$.

Solution.

False

We have, $(a-b)^2 = a^2 + b^2 - 2ab$ [an algebraic identity]

Question. 61 (a+b) (a-b)=$a^2 - b^2$

Solution.

True

We know that, $(a+b)(a-b) = a \times a - a \times b + b \times a - b \times b$

$= a^2 - b^2$

$= a^2 - ab + ba - b^2$

Question. 62 The product of two negative terms is a negative term.

Solution. False

Since, the product of two negative terms is always a positive term, i.e. (-) x (-) = (+).

Question. 63 The product of one negative and one positive term is a negative term.

Solution. True

When we multiply a negative term by a positive term, the resultant will be a negative term, i-e. (-) x (+) = (-).

Question. 64 The numerical coefficient of the term -6x^2y^2 is -6.

Solution. True

Since, the constant term (i.e. a number) present in the expression -6x^2y^2 is -6.

Question. 65 p^2q+q^2r+r^2q is a binomial.

Solution. False

Since, the given expression contains three unlike terms, so it is a trinomial.

Question. 66 The factors of a^2 – 2ab + $b^2$are (a + b) and (a + b).

Solution.

False

We have, $a^2 - 2ab + b^2 = (a - b)^2 = (a - b)(a - b)$ [an algebraic identity]

Question. 67 h is a factor of $2\pi(h+r)$.

Solution.

False

h is not a factor of $2\pi(h+r)$.

This expression has only two factors 2π and $(h+r)$.

Question. 68 Some of the factors of $\frac{n^2}{2}+\frac{n}{2}$ are $\frac{1}{2}n$ and (n+1).

Solution.

True

We have, $\frac{n^2}{2}+\frac{n}{2}=\frac{n^2+n}{2}=\frac{1}{2}n(n+1)$

$\therefore$ The factors are $\frac{1}{2}n$ and $(n+1)$.

Question. 69 An equation is true for all values of its variables.

Solution. False

As equation is true only for some values of its variables, e.g. 2x – 4= 0 is true, only for x =2.

Question. 70 x^2 + (a+b)x +ab =(a+b)(x +ab)

Solution.

False

As we know that,

$$x^2 + (a+b)x + ab = (x+a)(x+b)$$

Question. 71 Common factors of $11pq^2, 121p^2q^3, 1331p^2q$ is $11p^2q^2$

Solution.

False

We have,

$$11pq^2 = 11 \times p \times q \times q$$
$$121p^2q^3 = 11 \times 11 \times p \times p \times q \times q \times q$$
$$1331p^2q = 11 \times 11 \times 11 \times p \times p \times q$$

$\therefore$ Common factor $= 11 \times p \times q = 11pq$

Question. 72 Common factors of 12 $11a^2b^2$ +4ab^2 -32 is 4.

Solution.

True

As we have,

$$12a^2b^2 + 4ab^2 - 32 = 2 \times 2 \times 3 \times a \times a \times b \times b + 2 \times 2 \times a \times b \times b - 2^2 \times 2^3$$
$$= 4(3a^2b^2 + ab^2 - 8)$$

Thus, the common factor is 4.

Question. 73 Factorisation of -3a^2+3ab+3ac is 3a (-a-b-c).

Solution.

False

We have,

$$-3a^2 + 3ab + 3ac = 3a(-a + b + c)$$

Question. 74 Factorised form of p^2+30p+216 is (p+18) (p-12).

Solution.

False

We have,

$$p^2 + 30p + 216 = p^2 + (12 + 18)p + 216$$
$$= p^2 + 12p + 18p + 216 \quad \text{[by splitting the middle term]}$$
$$= p(p + 12) + 18(p + 12)$$
$$= (p + 18)(p + 12)$$

Question. 75 The difference of the squares of two consecutive numbers is their sum.

Solution.

True

Let n and $n + 1$ be any two consecutive numbers, then their sum $= n + n + 1 = 2n + 1$

Now, the difference of their squares,

$$(n + 1)^2 - n^2 = n^2 + 1 + 2n - n^2$$
$$= 2n + 1 \quad [\because (a + b)^2 = a^2 + 2ab + b^2]$$

Question. 76 abc + bca + cab is a monomial.

Solution. True

The given expression seems to be a trinomial but it is not as it contains three like terms which can be added to form a monomial, i.e. abc + abc + abc = 3abc

Question. 77 On dividing $\frac{p}{3}$ by $\frac{3}{p}$,the quotient is 9

Solution.

False

We have, $\frac{p}{3} \div \frac{3}{p} = \frac{p}{3} \times \frac{p}{3} = \frac{1}{9}p^2$ $\quad \left[\because \text{reciprocal of } \frac{3}{p} \text{ is } \frac{p}{3}\right]$

Hence, the quotient is $\frac{1}{9}p^2$.

Question. 78 The value of p for 51^2-49^2=100 p is 2.

Solution.

True

We have, $51^2 - 49^2 = 100p$

$\Rightarrow \quad (51 + 49)(51 - 49) = 100p \quad [\because a^2 - b^2 = (a + b)(a - b)]$

$\Rightarrow \quad 100 \times 2 = 100p$

$\Rightarrow \quad p = 2$

Question. 79 $(9x-51) \div 9$ is x-51.

Solution.

False

We have, $(9x - 51) \div 9 = \frac{9x - 51}{9} = \frac{9x}{9} - \frac{51}{9} = x - \frac{51}{9}$

Question. 80 The value of (a+1) (a-1)(a^2 +1) is a^4-1.

Solution.

True

We have, $(a + 1)(a - 1)(a^2 + 1) = (a^2 - 1)(a^2 + 1)$

[using the identity, $(a + b)(a - b) = a^2 - b^2$ in first two factors]

$= (a^2)^2 - 1^2$ [again using the same identity]

$= a^4 - 1$

Question. 81 Add:

(i) $7a^2bc, -3abc^2, 3a^2bc, 2abc^2$

(ii) $9ax + 3by - cz, -5by + ax + 3cz$

(iii) $xy^2z^2 + 3x^2y^2z - 4x^2yz^2, -9x^2y^2z + 3xy^2z^2 + x^2yz^2$

(iv) $5x^2 - 3xy + 4y^2 - 9, 7y^2 + 5xy - 2x^2 + 13$

(v) $2p^4 - 3p^3 + p^2 - 5p + 7, -3p^4 - 7p^3 - 3p^2 - p - 12$

(vi) $3a(a - b + c), 2b(a - b + c)$

(vii) $3a(2b + 5c), 3c(2a + 2b)$

Solution.

(i) We have,

$7a^2bc + (-3abc^2) + 3a^2bc + 2abc^2 = 7a^2bc - 3abc^2 + 3a^2bc + 2abc^2$

$= (7a^2bc + 3a^2bc) + (-3abc^2 + 2abc^2)$ [grouping like terms]

$= 10a^2bc + (-abc^2)$

$= 10a^2bc - abc^2$

(ii) We have,

$(9ax + 3by - cz) + (-5by + ax + 3cz)$

$= 9ax + 3by - cz - 5by + ax + 3cz$

$= (9ax + ax) + (3by - 5by) + (-cz + 3cz)$ [grouping like terms]

$= 10ax - 2by + 2cz$

(iii) We have,

$(xy^2z^2 + 3x^2y^2z - 4x^2yz^2) + (-9x^2y^2z + 3xy^2z^2 + x^2yz^2)$

$= xy^2z^2 + 3x^2y^2z - 4x^2yz^2 - 9x^2y^2z + 3xy^2z^2 + x^2yz^2$

$= (xy^2z^2 + 3xy^2z^2) + (3x^2y^2z - 9x^2y^2z) + (-4x^2yz^2 + x^2yz^2)$

[grouping like terms]

$= 4xy^2z^2 - 6x^2y^2z - 3x^2yz^2$

(iv) We have,

$(5x^2 - 3xy + 4y^2 - 9) + (7y^2 + 5xy - 2x^2 + 13)$

$= 5x^2 - 3xy + 4y^2 - 9 + 7y^2 + 5xy - 2x^2 + 13$

$= (5x^2 - 2x^2) + (-3xy + 5xy) + (4y^2 + 7y^2) + (-9 + 13)$

[grouping like terms]

$= 3x^2 + 2xy + 11y^2 + 4$

(v) We have,

$(2p^4 - 3p^3 + p^2 - 5p + 7) + (-3p^4 - 7p^3 - 3p^2 - p - 12)$

$= 2p^4 - 3p^3 + p^2 - 5p + 7 - 3p^4 - 7p^3 - 3p^2 - p - 12$

$= (2p^4 - 3p^4) + (-3p^3 - 7p^3) + (p^2 - 3p^2) + (-5p - p) + (7 - 12)$

[grouping like terms]

$= -p^4 - 10p^3 - 2p^2 - 6p - 5$

(vi) We have,

$3a(a - b + c) + 2b(a - b + c)$

$= (3a^2 - 3ab + 3ac) + (2ab - 2b^2 + 2bc)$

$= 3a^2 - 3ab + 2ab + 3ac + 2bc - 2b^2 = 3a^2 - ab + 3ac + 2bc - 2b^2$

[grouping like terms]

(vii) We have,

$3a(2b + 5c) + 3c(2a + 2b)$

$= (6ab + 15ac) + (6ac + 6bc) = 6ab + 15ac + 6ac + 6bc$

[grouping like terms]

$= 6ab + 21ac + 6bc$

Question. 82 Subtract

(i) $5a^2b^2c^2$ from $-7a^2b^2c^2$

(ii) $6x^2 - 4xy + 5y^2$ from $8y^2 + 6xy - 3x^2$

(iii) $2ab^2c^2 + 4a^2b^2c - 5a^2bc^2$ from $-10a^2b^2c + 4ab^2c^2 + 2a^2bc^2$

(iv) $3t^4 - 4t^3 + 2t^2 - 6t + 6$ from $-4t^4 + 8t^3 - 4t^2 - 2t + 11$

(v) $2ab + 5bc - 7ac$ from $5ab - 2bc - 2ac + 10abc$

(vi) $7p(3q + 7p)$ from $8p(2p - 7q)$

(vii) $-3p^2 + 3pq + 3px$ from $3p(-p - a - r)$

Solution.

(i) We have, $5a^2b^2c^2$ and $-7a^2b^2c^2$

The required difference is given by $-7a^2b^2c^2 - 5a^2b^2c^2$

$= (-7 - 5)a^2b^2c^2 = -12a^2b^2c^2$

(ii) We have, $6x^2 - 4xy + 5y^2$ and $8y^2 + 6xy - 3x^2$

The required difference is given by $(8y^2 + 6xy - 3x^2) - (6x^2 - 4xy + 5y^2)$

$= 8y^2 + 6xy - 3x^2 - 6x^2 + 4xy - 5y^2$

$= (8y^2 - 5y^2) + (6xy + 4xy) - (3x^2 + 6x^2) = 3y^2 + 10xy - 9x^2$

(iii) We have,

$2ab^2c^2 + 4a^2b^2c - 5a^2bc^2$ and $-10a^2b^2c + 4ab^2c^2 + 2a^2bc^2$

The required difference is given by

$(-10a^2b^2c + 4ab^2c^2 + 2a^2bc^2) - (2ab^2c^2 + 4a^2b^2c - 5a^2bc^2)$

$= -10a^2b^2c + 4ab^2c^2 + 2a^2bc^2 - 2ab^2c^2 - 4a^2b^2c + 5a^2bc^2$

$= (-10a^2b^2c - 4a^2b^2c) + (4ab^2c^2 - 2ab^2c^2) + (2a^2bc^2 + 5a^2bc^2)$

[grouping like terms]

$= -14a^2b^2c + 2ab^2c^2 + 7a^2bc^2$

(iv) We have, $3t^4 - 4t^3 + 2t^2 - 6t + 6$ and $-4t^4 + 8t^3 - 4t^2 - 2t + 11$

The required difference is given by

$(-4t^4 + 8t^3 - 4t^2 - 2t + 11) - (3t^4 - 4t^3 + 2t^2 - 6t + 6)$

$= -4t^4 + 8t^3 - 4t^2 - 2t + 11 - 3t^4 + 4t^3 - 2t^2 + 6t - 6$

$= (-4t^4 - 3t^4) + (8t^3 + 4t^3) + (-4t^2 - 2t^2) + (-2t + 6t) + (11 - 6)$

[grouping like terms]

$= -7t^4 + 12t^3 - 6t^2 + 4t + 5$

(v) We have, $2ab + 5bc - 7ac$ and $5ab - 2bc - 2ac + 10abc$

The required difference is given by $(5ab - 2bc - 2ac + 10abc) - (2ab + 5bc - 7ac)$

$= 5ab - 2bc - 2ac + 10abc - 2ab - 5bc + 7ac$

$= (5ab - 2ab) + (-2bc - 5bc) + (-2ac + 7ac) + 10abc$

[groupting like terms]

$= 3ab - 7bc + 5ac + 10abc$

(vi) We have, $7p(3q + 7p)$ and $8p(2p - 7q)$

The required difference is given by $8p(2p - 7q) - 7p(3q + 7p)$

$= 16p^2 - 56pq - 21pq - 49p^2 = (16p^2 - 49p^2) + (-56pq - 21pq)$

[grouping like terms]

$= -33p^2 - 77pq$

(vii) We have, $-3p^2 + 3pq + 3px$ and $3p(-p - a - r)$

The required difference is given by

$3p(-p - a - r) - (-3p^2 + 3pq + 3px) = -3p^2 - 3ap - 3pr + 3p^2 - 3pq - 3px$

$= (-3p^2 + 3p^2) - 3ap - 3pr - 3pq - 3px = -3ap - 3pr - 3pq - 3px$

[grouping like terms]

Question. 83 Multiply the following:

(i) $-7pq^2r^3, -13p^3q^2r$ (ii) $3x^2y^2z^2, 17xyz$

(iii) $15xy^2, 17yz^2$ (iv) $-5a^2bc, 11ab, 13abc^2$

(v) $-3x^2y, (5y - xy)$ (vi) $abc, (bc + ca)$

(vii) $7pqr, (p-q+r)$ (viii) $x^2y^2z^2, (xy-yz+zx)$

(xi) $(p+6), (q-7)$ (x) $6mn, 0mn$

(xi) a, a^5, a^6 (xii) $-7st, -1, -13st^2$

(xiii) $b^3, 3b^2, 7ab^5$ (xiv) $-\frac{100}{9}rs; \frac{3}{4}r^3s^2$

(xv) $(a^2-b^2), (a^2+b^2)$ (xvi) $(ab+c), (ab+c)$

(xvii) $(pq-2r), (pq-2r)$ (xviii) $\left(\frac{3}{4}x-\frac{4}{3}y\right), \left(\frac{2}{3}x+\frac{3}{2}y\right)$

(xix) $\frac{3}{2}p^2+\frac{2}{3}q^2, (2p^2-3q^2)$ (xx) $(x^2-5x+6), (2x+7)$

(xxi) $(3x^2+4x-8), (2x^2-4x+3)$

(xxii) $(2x-2y-3), (x+y+5)$

Solution.

(i) We have, $-7pq^2r^3$ and $-13p^3q^2r$

$\therefore (-7pq^2r^3)\times(-13p^3q^2r)=(-7)\times(-13)\,p^4q^4r^4=91p^4q^4r^4$

(ii) We have, $3x^2y^2z^2$ and $17xyz$

$\therefore 3x^2y^2z^2\times 17\,xyz=(3\times 17)x^2y^2z^2\times xyz=51\,x^3y^3z^3$

(iii) We have, $15xy^2$ and $17yz^2$

$15xy^2\times 17yz^2=(15\times 17)xy^2\times yz^2=255xy^3z^2$

(iv) We have, $-5a^2bc$, $11ab$ and $13abc^2$

$\therefore -5a^2bc\times 11\,ab\times 13abc^2=(-5\times 11\times 13)\,a^2bc\times ab\times abc^2=-715a^4b^3c^3$

(v) We have, $-3x^2y$ and $(5y-xy)$

$\therefore -3x^2y\times(5y-xy)=-3x^2y\times 5y+3x^2y\times xy=-15x^2y^2+3x^3y^2$

(vi) We have, abc and $(bc+ca)$

$\therefore abc\times(bc+ca)=abc\times bc+abc\times ca=ab^2c^2+a^2bc^2$

(vii) We have, $7pqr$ and $(p-q+r)$

$\therefore 7pqr\times(p-q+r)=7pqr\times p-7pqr\times q+7pqr\times r=7p^2qr-7pq^2r+7pqr^2$

(viii) We have, $x^2y^2z^2$ and $(xy-yz-zx)$

$\therefore x^2y^2z^2\times(xy-yz+zx)=x^2y^2z^2\times xy-x^2y^2z^2\times yz+x^2y^2z^2\times zx$

$=x^3y^3z^2-x^2y^3z^3+x^3y^2z^3$

(ix) We have, $(p+6)$ and $(q-7)$

$\therefore (p+6)\times(q-7)=p(q-7)+6(q-7)=pq-7p+6q-42$

(x) We have, 6mn and 0mn

$\therefore 6mn\times 0mn=(6\times 0)\,mn\times mn=0\times m^2n^2=0$

(xi) We have, a, a^5 and a^6

$\therefore a\times a^5\times a^6=a^{1+5+6}=a^{12}$

(xii) We have, $-7st$, -1 and $-13st^2$

$\therefore -7st\times(-1)\times(-13st^2)=[-7\times(-1)\times(-13)]st\times(st^2)=-91s^2t^3$

(xiii) We have, b^3, $3b^2$ and $7ab^5$

$\therefore b^3 \times 3b^2 \times 7ab^5 = (1 \times 3 \times 7)b^3 \times b^2 \times ab^5 = 21ab^{10}$

(xiv) We have, $\frac{-100}{9}rs$ and $\frac{3}{4}r^3s^2$

$\therefore \frac{-100}{9}rs \times \frac{3}{4}r^3s^2 = \left(\frac{-100}{9} \times \frac{3}{4}\right) rs \times r^3s^2 = \frac{-25}{3} \times r^4s^3$

(xv) We have, $(a^2 - b^2)$ and $(a^2 + b^2)$

$\therefore (a^2 - b^2)(a^2 + b^2) = a^2(a^2 + b^2) - b^2(a^2 + b^2) = a^4 + a^2b^2 - b^2a^2 - b^4 = a^4 - b^4$

(xvi) We have, $(ab + c)$ and $(ab + c)$

$\therefore (ab + c)(ab + c) = ab(ab + c) + c(ab + c)$

$= a^2b^2 + abc + cab + c^2 = a^2b^2 + 2abc + c^2$

(xvii) We have, $(pq - 2r)$ and $(pq - 2r)$

$\therefore (pq - 2r)(pq - 2r) = pq(pq - 2r) - 2r(pq - 2r)$

$= p^2q^2 - 2pqr - 2rpq + 4r^2 = p^2q^2 - 4pqr + 4r^2$

(xviii) We have, $\left(\frac{3}{4}x - \frac{4}{3}y\right)$ and $\left(\frac{2}{3}x + \frac{3}{2}y\right)$

$$\therefore \left(\frac{3}{4}x - \frac{4}{3}y\right)\left(\frac{2}{3}x + \frac{3}{2}y\right) = \frac{3}{4}x\left(\frac{2}{3}x + \frac{3}{2}y\right) - \frac{4}{3}y\left(\frac{2}{3}x + \frac{3}{2}y\right)$$

$$= \frac{3}{4} \times \frac{2}{3}x^2 + \frac{3}{4} \times \frac{3}{2}xy - \frac{4}{3} \times \frac{2}{3}yx - \frac{4}{3} \times \frac{3}{2}y^2$$

$$= \frac{1}{2}x^2 + \frac{9}{8}xy - \frac{8}{9}xy - 2y^2$$

$$= \frac{1}{2}x^2 + \left(\frac{9}{8} - \frac{8}{9}\right)xy - 2y^2$$

$$= \frac{1}{2}x^2 + \left(\frac{81 - 64}{72}\right)xy - 2y^2$$

$$= \frac{1}{2}x^2 + \frac{17}{72}xy - 2y^2$$

(xix) We have, $\left(\frac{3}{2}p^2 + \frac{2}{3}q^2\right)$ and $(2p^2 - 3q^2)$

$$\therefore \left(\frac{3}{2}p^2 + \frac{2}{3}q^2\right)(2p^2 - 3q^2) = \frac{3}{2}p^2(2p^2 - 3q^2) + \frac{2}{3}q^2(2p^2 - 3q^2)$$

$$= \frac{3}{2}p^2 \times 2p^2 - \frac{9}{2}p^2q^2 + \frac{4}{3}q^2p^2 - 2q^4$$

$$= 3p^4 + \left(\frac{4}{3} - \frac{9}{2}\right)p^2q^2 - 2q^4$$

$$= 3p^4 + \left(\frac{8 - 27}{6}\right)p^2q^2 - 2q^4$$

$$= 3p^4 - \frac{19}{6}p^2q^2 - 2q^4$$

(xx) We have, $(x^2 - 5x + 6)$ and $(2x + 7)$

$\therefore (x^2 - 5x + 6)(2x + 7) = x^2(2x + 7) - 5x(2x + 7) + 6(2x + 7)$

$= 2x^3 + 7x^2 - 10x^2 - 35x + 12x + 42$

$= 2x^3 - 3x^2 - 23x + 42$

(xxi) We have, $(3x^2 + 4x - 8)$ and $(2x^2 - 4x + 3)$

$\therefore (3x^2 + 4x - 8)(2x^2 - 4x + 3)$

$= 3x^2(2x^2 - 4x + 3) + 4x(2x^2 - 4x + 3) - 8(2x^2 - 4x + 3)$

$= 6x^4 - 12x^3 + 9x^2 + 8x^3 - 16x^2 + 12x - 16x^2 + 32x - 24$

$= 6x^4 - 12x^3 + 8x^3 + 9x^2 - 16x^2 - 16x^2 + 12x + 32x - 24$

[grouping like terms]

$= 6x^4 - 4x^3 - 23x^2 + 44x - 24$

(xxii) We have, $(2x - 2y - 3)$ and $(x + y + 5)$

$\therefore (2x - 2y - 3)(x + y + 5)$

$= 2x(x + y + 5) - 2y(x + y + 5) - 3(x + y + 5)$

$= 2x^2 + 2xy + 10x - 2yx - 2y^2 - 10y - 3x - 3y - 15$

$= 2x^2 + 2xy - 2yx + 10x - 3x - 2y^2 - 10y - 3y - 15$

[grouping like terms]

$= 2x^2 + 7x - 13y - 2y^2 - 15$

Question. 84 Simplify

(i) $(3x + 2y)^2 + (3x - 2y)^2$

(ii) $(3x + 2y)^2 - (3x - 2y)^2$

(iii) $\left(\frac{7}{9}a + \frac{9}{7}b\right)^2 - ab$

(iv) $\left(\frac{3}{4}x - \frac{4}{3}y\right)^2 + 2xy$

(v) $(1.5p + 1.2q)^2 - (1.5p - 1.2q)^2$

(vi) $(2.5m + 1.5q)^2 + (2.5m - 1.5q)^2$

(vii) $(x^2 - 4) + (x^2 + 4) + 16$

(viii) $(ab - c)^2 + 2abc$

(ix) $(a - b)(a^2 + b^2 + ab) - (a + b)(a^2 + b^2 - ab)$

(x) $(b^2 - 49)(b + 7) + 343$

(xi) $(4.5a + 1.5b)^2 + (4.5b + 1.5a)^2$

(xii) $(pq - qr)^2 + 4pq^2r$

(xiii) $(s^2t + tq^2)^2 - (2stq)^2$

Solution.

(i) We have,

$(3x + 2y)^2 + (3x - 2y)^2 = (3x)^2 + (2y)^2 + 2 \times 3x \times 2y + (3x)^2 + (2y)^2 - 2 \times 3x \times 2y$

[using the identities, $(a + b)^2 = a^2 + b^2 + 2ab$

and $(a - b)^2 = a^2 + b^2 - 2ab$]

$$= 9x^2 + 4y^2 + 12xy + 9x^2 + 4y^2 - 12xy$$
$$= (9x^2 + 9x^2) + (4y^2 + 4y^2) + 12xy - 12xy$$
$$= 18x^2 + 8y^2$$

(ii) We have,

$(3x + 2y)^2 - (3x - 2y)^2$

$$= [(3x + 2y) + (3x - 2y)][(3x + 2y) - (3x - 2y)]$$

[using the identity, $a^2 - b^2 = (a + b)(a - b)$]

$$= (3x + 2y + 3x - 2y)(3x + 2y - 3x + 2y) = 6x \times 4y = (6 \times 4) \times xy = 24xy$$

(iii) We have, $\left(\frac{7}{9}a + \frac{9}{7}b\right)^2 - ab$

$$= \left(\frac{7}{9}a\right)^2 + \left(\frac{9}{7}b\right)^2 + 2 \times \frac{7}{9}a \times \frac{9}{7}b - ab$$

[using the identity, $(a + b)^2 = a^2 + b^2 + 2ab$]

$$= \frac{49}{81}a^2 + \frac{81}{49}b^2 + 2ab - ab$$
$$= \frac{49}{81}a^2 + ab + \frac{81}{49}b^2$$

(iv) We have,

$\left(\frac{3}{4}x - \frac{4}{3}y\right)^2 + 2xy$

$$= \left(\frac{3}{4}x\right)^2 + \left(\frac{4}{3}y\right)^2 - 2 \times \frac{3}{4}x \times \frac{4}{3}y + 2xy$$

[using the identity, $(a - b)^2 = a^2 + b^2 - 2ab$]

$$= \frac{9}{16}x^2 + \frac{16}{9}y^2 - 2xy + 2xy = \frac{9}{16}x^2 + \frac{16}{9}y^2$$

(v) We have,

$(1.5p + 1.2q)^2 - (1.5p - 1.2q)^2$

$$= [(1.5p + 1.2q) + (1.5p - 1.2q)][(1.5p + 1.2q) - (1.5p - 1.2q)]$$

[using the identity, $a^2 - b^2 = (a + b)(a - b)$]

$$= [(1.5p + 1.5p) + (1.2q - 1.2q)][(1.5p - 1.5p) + (1.2q + 1.2q)]$$
$$= 3p \times 2.4q = 7.2pq$$

(vi) We have,

$(2.5m + 1.5q)^2 + (2.5m - 1.5q)^2$

$$= (2.5m)^2 + (1.5q)^2 + 2 \times 2.5m \times 1.5q + (2.5m)^2 + (1.5q)^2 - 2 \times (2.5m) \times (1.5q)$$

[using the identities, $(a + b)^2 = a^2 + b^2 + 2ab$ and $(a - b)^2 = a^2 + b^2 - 2ab$]

$$= 6.25m^2 + 2.25q^2 + 6.25m^2 + 2.25q^2$$
$$= (6.25 + 6.25)m^2 + (2.25 + 2.25)q^2$$
$$= 12.5m^2 + 4.5q^2$$

(vii) We have,

$(x^2 - 4) + (x^2 + 4) + 16$

$$= x^2 - 4 + x^2 + 4 + 16 = 2x^2 + 16$$

(viii) $(ab-c)^2+2abc$

$=(ab)^2+c^2-2abc+2abc$ [using the identity, $(a-b)^2=a^2+b^2-2ab$]

$=a^2b^2+c^2$

(ix) $(a-b)(a^2+b^2+ab)-(a+b)(a^2+b^2-ab)$

$=a(a^2+b^2+ab)-b(a^2+b^2+ab)-a(a^2+b^2-ab)-b(a^2+b^2-ab)$

$=a^3+ab^2+a^2b-ba^2-b^3-ab^2-a^3-ab^2+a^2b-ba^2-b^3+ab^2$

$=(a^3-a^3)+(-b^3-b^3)+(ab^2-ab^2)+(a^2b-a^2b+a^2b-a^2b)$

$=0-2b^3+0+0+0$

$=-2b^3$

(x) We have,

$(b^2-49)(b+7)+343$

$=b^2(b+7)-49(b+7)+343$

$=b^3+7b^2-49b-343+343$

$=b^3-49b+7b^2$

(xi) We have, $(4.5a+1.5b)^2+(4.5b+1.5a)^2$

$=(4.5a)^2+(1.5b)^2+2\times4.5a\times1.5b+(4.5b)^2+(1.5a)^2+2\times4.5b\times1.5a$

[using the identity, $(a+b)^2=a^2+b^2+2ab$]

$=20.25a^2+2.25b^2+13.5ab+20.25b^2+2.25a^2+13.5ab$

$=40.5a^2+4.5b^2+27ab$

(xii) We have, $(pq-qr)^2+4pq^2r$

$=p^2q^2+q^2r^2-2pq^2r+4pq^2r$

[using the identity, $(a-b)^2=a^2+b^2-2ab$]

$=p^2q^2+q^2r^2+2pq^2r$

(xiii) We have, $(s^2t+tq^2)^2-(2stq)^2$

$=(s^2t)^2+(tq^2)^2+2\times s^2t\times tq^2-4s^2t^2q^2$

[using the identity, $(a+b)^2=a^2+b^2+2ab$]

$=s^4t^2+t^2q^4+2s^2t^2q^2-4s^2t^2q^2$

$=s^4t^2+t^2q^4-2s^2t^2q^2$

Question. 85 Expand the following, using suitable identities.

(i) $(xy+yz)^2$

(ii) $(x^2y-xy^2)^2$

(iii) $\left(\frac{4}{5}a+\frac{5}{4}b\right)^2$

(iv) $\left(\frac{2}{3}x-\frac{3}{2}y\right)^2$

(v) $\left(\frac{4}{5}p+\frac{5}{3}q\right)^2$

(vi) $(x+3)(x+7)$

(vii) $(2x+9)(2x-7)$

(viii) $\left(\frac{4x}{5}+\frac{y}{4}\right)\left(\frac{4x}{5}+\frac{3y}{4}\right)$

(ix) $\left(\frac{2x}{3}-\frac{2}{3}\right)\left(\frac{2x}{3}+\frac{2a}{3}\right)$

(x) $(2x-5y)(2x-5y)$

(xi) $\left(\frac{2a}{3}+\frac{b}{3}\right)\left(\frac{2a}{3}-\frac{b}{3}\right)$

(xii) $(x^2+y^2)(x^2-y^2)$

(xiii) $(a^2+b^2)^2$

(xiv) $(7x+5)^2$

(xv) $(0.9p-0.5q)^2$

Solution.

(i) We have,

$$(xy + yz)^2 = (xy)^2 + (yz)^2 + 2 \times xy \times yz$$

[using the identity, $(a+b)^2 = a^2 + b^2 + 2ab$]

$$= x^2y^2 + y^2z^2 + 2xy^2z$$

(ii) We have,

$$(x^2y - xy^2)^2 = (x^2y)^2 + (xy^2)^2 - 2x^2y \times xy^2$$

$$= x^4y^2 + x^2y^4 - 2x^3y^3$$

[using the identity, $(a-b)^2 = a^2 + b^2 - 2ab$]

(iii) We have,

$$\left(\frac{4}{5}a + \frac{5}{4}b\right)^2 = \left(\frac{4}{5}a\right)^2 + \left(\frac{5}{4}b\right)^2 + 2 \times \frac{4}{5}a \times \frac{5}{4}b$$

$$= \frac{16}{25}a^2 + \frac{25}{16}b^2 + 2ab$$ [using the identity, $(a+b)^2 = a^2 + b^2 + 2ab$]

(iv) We have,

$$\left(\frac{2}{4}x - \frac{3}{2}y\right)^2 = \left(\frac{2}{3}x\right)^2 + \left(\frac{3}{2}y\right)^2 - 2 \times \frac{2}{3}x \times \frac{3}{2}y$$

[using the identity, $(a-b)^2 = a^2 + b^2 - 2ab$]

$$= \frac{4}{9}x^2 + \frac{9}{4}y^2 - 2xy$$

(v) We have,

$$\left(\frac{4}{5}p + \frac{5}{3}q\right)^2 = \left(\frac{4}{5}p\right)^2 + \left(\frac{5}{3}q\right)^2 + 2 \times \frac{4}{5}p \times \frac{5}{3}q$$

$$= \frac{16}{25}p^2 + \frac{25}{9}q^2 + \frac{8}{3}pq$$

[using the identity, $(a+b)^2 = a^2 + b^2 + 2ab$]

(vi) We have,

$$(x + 3)(x + 7) = x^2 + (3 + 7)x + 3 \times 7$$

[using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$$= x^2 + 10x + 21$$

(vii) We have,

$$(2x + 9)(2x - 7) = (2x + 9)[2x + (-7)]$$

$$= (2x)^2 + [9 + (-7)]2x + 9 \times (-7)$$

[using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$$= 4x^2 + 4x - 63$$

(viii) We have,

$$\left(\frac{4x}{5}+\frac{y}{4}\right)\left(\frac{4x}{5}+\frac{3y}{4}\right)=\left(\frac{4x}{5}\right)^2+\left(\frac{y}{4}+\frac{3y}{4}\right)\frac{4x}{5}+\frac{y}{4}\times\frac{3y}{4}$$

[using the identity, $(x+a)(x+b)=x^2+(a+b)x+ab$]

$$=\frac{16}{25}x^2+\frac{4xy}{5}+\frac{3y^2}{16}$$

(ix) We have,

$$\left(\frac{2x}{3}-\frac{2}{3}\right)\left(\frac{2x}{3}+\frac{2a}{3}\right)=\left(\frac{2x}{3}\right)^2+\left(\frac{-2}{3}+\frac{2a}{3}\right)\frac{2x}{3}+\left(\frac{-2}{3}\times\frac{2a}{3}\right)$$

[using the identity, $(x+a)(x+b)=x^2+(a+b)x+ab$]

$$=\frac{4x^2}{9}+\frac{2a-2}{3}\times\frac{2}{3}x-\frac{4}{9}a=\frac{4x^2}{9}+\frac{4}{9}(a-1)x-\frac{4}{9}a$$

(x) We have,

$$(2x-5y)(2x-5y)=(2x-5y)^2$$

$$=(4x)^2+(5y)^2-2\times 2x\times 5y$$

[using the identity, $(a-b)^2=a^2+b^2-2ab$]

$$=16x^2+25y^2-20xy$$

(xi) We have,

$$\left(\frac{2a}{3}+\frac{b}{3}\right)\left(\frac{2a}{3}-\frac{b}{3}\right)=\left(\frac{2a}{3}\right)^2-\left(\frac{b}{3}\right)^2$$

[using the identity, $(a+b)(a-b)=a^2-b^2$]

$$=\frac{4}{9}a^2-\frac{1}{9}b^2$$

(xii) We have,

$$(x^2+y^2)(x^2-y^2)=(x^2)^2-(y^2)^2$$

[using the identity, $(a+b)(a-b)=a^2-b^2$]

$$=x^4-y^4$$

(xiii) We have,

$$(a^2+b^2)^2=(a^2)^2+(b^2)^2+2a^2\times b^2$$

$$=a^4+b^4+2a^2b^2$$

[using the identity, $(a+b)^2=a^2+b^2+2ab$]

(xiv) We have,

$$(7x+5)^2=(7x)^2+5^2+2\times 7x\times 5$$

$$=49x^2+25+70x$$

[using the identity, $(a+b)^2=a^2+b^2+2ab$]

(xv) We have,

$$(0.9p-0.5q)^2=(0.9p)^2+(0.5q)^2-2\times 0.9p\times 0.5q$$

[using the identity, $(a-b)^2=a^2+b^2-2ab$]

$$=0.81p^2+0.25q^2-0.9pq$$

Question. 86 Using suitable identities, evaluate the following:

(i) $(52)^2$ (ii) $(49)^2$

(iii) $(103)^2$ (iv) $(98)^2$

(v) $(1005)^2$ (vi) $(995)^2$

(vii) 47×53 (viii) 52×53

(ix) 105×95 (x) 104×97

(xi) 101×103 (xii) 98×103

(xiii) $(9.9)^2$ (xiv) 9.8×10.2

(xv) 10.1×10.2 (xvi) $(35.4)^2 - (14.6)^2$

(xvii) $(69.3)^2 - (30.7)^2$ (xviii) $(9.7)^2 - (0.3)^2$

(xix) $(132)^2 - (68)^2$ (xx) $(339)^2 - (161)^2$

(xxi) $(729)^2 - (271)^2$

Solution.

(i) We have,

$$(52)^2 = (50+2)^2$$
$$= (50)^2 + (2)^2 + 2 \times 50 \times 2 \quad \text{[using the identity, } (a+b)^2 = a^2 + b^2 + 2ab]$$
$$= 2500 + 4 + 200$$
$$= 2704$$

(ii) We have,

$$(49)^2 = (50-1)^2$$
$$= (50)^2 + 1^2 - 2 \times 50 \times 1 \quad \text{[using the identity, } (a-b)^2 = a^2 + b^2 - 2ab]$$
$$= 2500 + 1 - 100,$$
$$= 2401$$

(iii) We have,

$$(103)^2 = (100+3)^2$$
$$= (100)^2 + 3^2 + 2 \times 100 \times 3$$
$$= 10000 + 9 + 600$$
$$= 10609 \quad \text{[using the identity, } (a+b)^2 = a^2 + b^2 + 2ab]$$

(iv) We have,

$$(98)^2 = (100-2)^2$$
$$= (100)^2 + (2)^2 - 2 \times 100 \times 2$$
$$= 10000 + 4 - 400$$
$$= 9604 \quad \text{[using the identity, } (a-b)^2 = a^2 + b^2 - 2ab]$$

(v) We have,

$$(1005)^2 = (1000+5)^2$$
$$= (1000)^2 + 5^2 + 2 \times 1000 \times 5$$
$$= 1000000 + 25 + 10000$$
$$= 1010025 \quad \text{[using the identity, } (a+b)^2 = a^2 + b^2 + 2ab]$$

(vi) We have,

$$(995)^2 = (1000 - 5)^2$$
$$= (1000)^2 + (5)^2 - 2 \times 1000 \times 5$$
$$= 1000000 + 25 - 10000$$
$$= 990025$$

[using the identity, $(a - b)^2 = a^2 + b^2 - 2ab$]

(vii) We have,

$$47 \times 53 = (50 - 3)(50 + 3)$$
$$= (50)^2 - (3)^2$$

[using the identity, $(a - b)(a + b) = a^2 - b^2$]

$$= 2500 - 9$$
$$= 2491$$

(viii) We have,

$$52 \times 53 = (50 + 2)(50 + 3)$$
$$= (50)^2 + (2 + 3) 50 + 2 \times 3$$

[using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$]

$$= 2500 + 250 + 6 = 2756$$

(ix) We have,

$$105 \times 95 = (100 + 5)(100 - 5)$$
$$= (100)^2 - (5)^2$$

[using the identity, $(a + b)(a - b) = a^2 - b^2$]

$$= 10000 - 25$$
$$= 9975$$

(x) We have,

$$104 \times 97 = (100 + 4)(100 - 3)$$
$$= (100)^2 + (4 - 3)100 + 4 \times (-3)$$
$$= 10000 + 100 - 12$$
$$= 10088$$

[using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$]

(xi) We have,

$$101 \times 103 = (100 + 1)(100 + 3)$$
$$= (100)^2 + (1 + 3) 100 + 3 \times 1$$
$$= 10000 + 400 + 3$$
$$= 10403$$

[using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$]

(xii) We have,

$$98 \times 103 = (100 - 2)(100 + 3)$$
$$= (100)^2 + (-2 + 3) 100 + (-2) \times 3$$
$$= 10000 + 100 - 6$$
$$= 10094$$

[using the identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$]

(xiii) We have,

$$(9.9)^2 = (10 - 0.1)^2$$
$$= 10^2 + (0.1)^2 - 2 \times 10 \times 0.1$$
$$= 100 + 0.01 - 2 = 98.01$$

[using the identity, $(a - b)^2 = a^2 + b^2 - 2ab$]

(xiv) We have,

$9.8 \times 10.2 = (10 - 0.2)(10 + 0.2)$
$= 10^2 - (0.2)^2$
$= 100 - 0.04$
$= 100 - 0.04$
$= 99.96$ [using the identity, $(a+b)(a-b) = a^2 - b^2$]

(xv) We have,

$10.1 \times 10.2 = (10 + 0.1)(10 + 0.2)$
$= (10)^2 + (0.1 + 0.2)10 + (0.1)(0.2)$
$= 100 + 0.3 \times 10 + 0.02$
$= 103.02$ [using the identity, $(x+a)(x+b) = x^2 + (a+b)x + ab$]

(xvi) We have,

$(35.4)^2 - (14.6)^2 = (35.4 + 14.6)(35.4 - 14.6)$
$= 50 \times 20.8$
$= 1040$ [using the identity, $(a+b)(a-b) = a^2 - b^2$]

(xvii) We have,

$(69.3)^2 - (30.7)^2 = (69.3 + 30.7)(69.3 - 30.7)$
$= 100 \times 38.6$
$= 3860$ [using the identity, $(a+b)(a-b) = a^2 - b^2$]

(xviii) We have,

$(9.7)^2 - (0.3)^2 = (9.7 + 0.3)(9.7 - 0.3)$
$= 10 \times 9.4$
$= 94$ [using the identity, $a^2 - b^2 = (a+b)(a-b)$]

(xix) We have,

$(132)^2 - (68)^2 = (132 + 68)(132 - 68)$
$= 200 \times 64$
$= 12800$ [using the identity, $a^2 - b^2 = (a+b)(a-b)$]

(xx) We have,

$(339)^2 - (161)^2 = (339 + 161)(339 - 161)$
$= 500 \times 178$
$= 89000$ [using the identity, $a^2 - b^2 = (a+b)(a-b)$]

(xxi) We have,

$(729)^2 - (271)^2 = (729 + 271)(729 - 271)$
$= 1000 \times 458$
$= 458000$ [using the identity, $a^2 - b^2 = (a+b)(a-b)$]

Question. 87 Write the greatest common factor in each of the following terms.

(i) $-18a^2, 108a$ (ii) $3x^2y, 18xy^2, -6xy$
(iii) $2xy, -y^2, 2x^2y$ (iv) $l^2m^2n, lm^2n^2, l^2mn^2$
(v) $21pqr, -7p^2q^2r^2, 49p^2qr$
(vi) $qrxy, pryz, rxyz$
(vii) $3x^3y^2z, -6xy^3z^2, 12x^2yz^3$
(viii) $63p^2a^2r^2s, -9pq^2r^2s^2, 15p^2qr^2s^2, -60p^2a^2rs^2$
(ix) $13x^2y, 169xy$
(x) $11x^2, 12y^2$

Solution.

(i) We have,

$$-18a^2 = -18 \times a \times a$$
$$108a = 18 \times 10 \times a$$

$\therefore$ The greatest common factor i.e. GCF is $18a$.

(ii) We have,

$$3x^2y = 3 \times x \times x \times y$$
$$18xy^2 = 3 \times 6 \times x \times y \times y$$
$$-6xy = -1 \times 3 \times 2 \times x \times y$$

$\therefore$ GCF $= 3xy$

(iii) We have,

$$2xy = 2 \times x \times y$$
$$-y^2 = -y \times y$$
$$2x^2y = 2 \times x \times x \times y$$

$\therefore$ GCF $= y$

(iv) We have,

$$l^2m^2n = l \times l \times m \times m \times n$$
$$lm^2n^2 = l \times m \times m \times n \times n$$
$$l^2mn^2 = l \times l \times m \times n \times n$$

$\therefore$ GCF $= lmn$

(v) We have,

$$21pqr = 7 \times 3 \times p \times q \times r$$
$$-7p^2q^2r^2 = -7 \times p \times p \times q \times q \times r \times r$$
$$49p^2qr = 7 \times 7 \times p \times p \times q \times r$$

$\therefore$ GCF $= 7pqr$

(vi) We have,

$$qrxy = q \times r \times x \times y$$
$$pryz = p \times r \times y \times z$$
$$rxyz = r \times x \times y \times z$$

$\therefore$ GCF $= ry$

(vii) We have,

$$3x^3y^2z = 3 \times x \times x \times x \times y \times y \times z$$
$$-6xy^3z^2 = -3 \times 2 \times x \times y \times y \times y \times z \times z$$
$$12x^2yz^3 = 3 \times 4 \times x \times x \times y \times z \times z \times z$$

$\therefore$ GCF $= 3xyz$

(viii) We have,

$$63p^2a^2r^2s = 3 \times 3 \times 7 \times p \times p \times a \times a \times r \times r \times s$$
$$-9pq^2r^2s^2 = -3 \times 3 \times p \times q \times q \times r \times r \times s \times s$$
$$15p^2qr^2s^2 = 3 \times 5 \times p \times p \times q \times r \times r \times s \times s$$
$$-60p^2a^2rs^2 = -2 \times 2 \times 3 \times 5 \times p \times p \times a \times a \times r \times s \times s$$

$\therefore$ GCF $= 3prs$

(ix) We have,

$$13x^2y = 13 \times x \times x \times y$$
$$169xy = 13 \times 13 \times x \times y$$

$\therefore$ GCF $= 13xy$

(x) We have, $11x^2, 12y^2$

The GCF of 11, 12 is 1.

Also, there is no common factor between x^2 and y^2.

Hence, the GCF of $11x^2$ and $12y^2$ is 1.

Question. 88 Factorise the following expressions.

(i) $6ab + 12bc$

(ii) $-xy - ay$

(iii) $ax^3 - bx^2 + cx$

(iv) $l^2m^2n - lm^2n^2 - l^2mn^2$

(v) $3pqr - 6p^2q^2r^2 - 15r^2$

(vi) $x^3y^2 + x^2y^3 - xy^4 + xy$

(vii) $4xy^2 - 10x^2y + 16x^2y^2 + 2xy$

(viii) $2a^3 - 3a^2b + 5ab^2 - ab$

(ix) $63p^2q^2r^2s - 9pq^2r^2s^2 + 15p^2qr^2s^2 - 60p^2q^2rs^2$

(x) $24x^2yz^3 - 6xy^3z^2 + 15x^2y^2z - 5xyz$

(xi) $a^3 + a^2 + a + 1$

(xii) $lx + my + mx + ly$

(xiii) $a^3x - x^4 + a^2x^2 - ax^3$

(xiv) $2x^2 - 2y + 4xy - x$

(xv) $y^2 + 8zx - 2xy - 4yz$

(xvi) $ax^2y - bxyz - ax^2z + bxy^2$

(xvii) $a^2b + a^2c + ab + ac + b^2c + c^2b$

(xviii) $2ax^2 + 4axy + 3bx^2 + 2ay^2 + 6bxy + 3by^2$

Solution.

(i) We have,

$$6ab + 12bc = 6ab + 6 \times 2 \times bc = 6b\,(a + 2c)$$

(ii) We have,

$$-xy - ay = -y(x + a)$$

(iii) We have,

$$ax^3 - bx^2 + cx = x(ax^2 - bx + c)$$

(iv) We have,

$$l^2m^2n - lm^2n^2 - l^2mn^2 = lmn\,(lm - mn - ln)$$

(v) We have, $3pqr - 6p^2q^2r^2 - 15r^2$

$$= 3pqr - 3 \times 2p^2q^2r^2 - 3 \times 5r^2 = 3r\,(pq - 2p^2q^2r - 5r)$$

(vi) We have, $x^3y^2 + x^2y^3 - xy^4 + xy$

$$= xy\,(x^2y + xy^2 - y^3 + 1)$$

(vii) We have, $4xy^2 - 10x^2y + 16x^2y^2 + 2xy$

$$= 2 \times 2xy^2 - 2 \times 5 \times x^2y + 2 \times 8 \times x^2y^2 + 2xy$$

$$= 2xy\,(2y - 5x + 8xy + 1)$$

(viii) We have, $2a^3 - 3a^2b + 5ab^2 - ab$

$$= a(2a^2 - 3ab + 5b^2 - b)$$

(ix) We have, $63p^2q^2r^2s - 9pq^2r^2s^2 + 15p^2qr^2s^2 - 60p^2q^2rs^2$

$$= 3 \times 21p^2q^2r^2s - 3 \times 3pq^2r^2s^2 + 3 \times 5p^2qr^2s^2 - 3 \times 20p^2q^2rs^2$$

$$= 3pqrs\,(21pqr - 3qrs + 5prs - 20pqs)$$

(x) We have, $24x^2yz^3 - 6xy^3z^2 + 15x^2y^2z - 5xyz$

$$= xyz\,(24xz^2 - 6y^2z + 15xy - 5)$$

(x) We have, $24x^2yz^3 - 6xy^3z^2 + 15x^2y^2z - 5xyz$

$= xyz(24xz^2 - 6y^2z + 15xy - 5)$

(xi) We have, $a^3 + a^2 + a + 1$

$= a^2(a+1) + 1(a+1) = (a+1)(a^2+1)$

(xii) We have, $lx + my + mx + ly$

$= lx + mx + my + ly = x(l+m) + y(m+l) = (l+m)(x+y)$

(xiii) We have, $a^3x - x^4 + a^2x^2 - ax^3$

$= x(a^3 - x^3 + a^2x - ax^2) = x(a^3 + a^2x - x^3 - ax^2)$

$= x[a^2(a+x) - x^2(x+a)]$

$= x[(x+a)(a^2 - x^2)] = x(a^2 - x^2)(a+x)$

(xiv) We have, $2x^2 - 2y + 4xy - x$

$= 2x^2 - x - 2y + 4xy = x(2x-1) - 2y(1-2x)$

$= x(2x-1) + 2y(2x-1) = (2x-1)(x+2y)$

(xv) We have, $y^2 + 8zx - 2xy - 4yz$

$= y^2 - 2xy + 8zx - 4yz = y(y-2x) - 4z(y-2x)$

$= (y-2x)(y-4z)$

(xvi) We have, $ax^2y - bxyz - ax^2z + bxy^2$

$= x(axy - byz - axz + by^2)$

$= x(axy - axz - byz + by^2)$

$= x[ax(y-z) + by(-z+y)]$

$= x[(ax+by)(y-z)]$

(xvii) We have, $a^2b + a^2c + ab + ac + b^2c + c^2b$

$= (a^2b + ab + b^2c) + (a^2c + ac + c^2b)$

$= b(a^2 + a + bc) + c(a^2 + a + bc)$

$= (a^2 + a + bc)(b+c)$

(xviii) We have, $2ax^2 + 4axy + 3bx^2 + 2ay^2 + 6bxy + 3by^2$

$= (2ax^2 + 2ay^2 + 4axy) + (3bx^2 + 3by^2 + 6bxy)$

$= 2a(x^2 + y^2 + 2xy) + 3b(x^2 + y^2 + 2xy)$

$= (2a+3b)(x+y)^2$

Question. 89Factorise the following, using the identity, $a^2 + 2ab + b^2 = (a+b)^2$

(i) $x^2 + 6x + 9$
(ii) $x^2 + 12x + 36$
(iii) $x^2 + 14x + 49$
(iv) $x^2 + 2x + 1$
(v) $4x^2 + 4x + 1$
(vi) $a^2x^2 + 2ax + 1$
(vii) $a^2x^2 + 2abx + b^2$
(viii) $a^2x^2 + 2abxy + b^2y^2$
(ix) $4x^2 + 12x + 9$
(x) $16x^2 + 40x + 25$
(xi) $9x^2 + 24x + 16$
(xii) $9x^2 + 30x + 25$
(xiii) $2x^3 + 24x^2 + 72x$
(xiv) $a^2x^3 + 2abx^2 + b^2x$
(xv) $4x^4 + 12x^3 + 9x^2$
(xvi) $\frac{x^2}{4} + 2x + 4$
(xvii) $9x^2 + 2xy + \frac{y^2}{9}$

Solution.

(i) We have,

$$x^2 + 6x + 9 = x^2 + 2 \cdot 3 \cdot x + 3^2$$
$$= (x+3)^2 \qquad [\because a^2 + 2ab + b^2 = (a+b)^2]$$
$$= (x+3)(x+3)$$

(ii) We have, $x^2 + 12x + 36$

$$= x^2 + 2 \cdot 6 \cdot x + 6^2$$
$$= (x+6)^2 \qquad [\because a^2 + 2ab + b^2 = (a+b)^2]$$
$$= (x+6)(x+6)$$

(iii) We have, $x^2 + 14x + 49$

$$= x^2 + 2 \cdot 7 \cdot x + 7^2 = (x+7)^2 = (x+7)(x+7)$$

(iv) We have, $x^2 + 2x + 1$

$$= x^2 + 2 \cdot 1 \cdot x + 1^2 = (x+1)^2 = (x+1)(x+1)$$

(v) We have, $4x^2 + 4x + 1$

$$= (2x)^2 + 2 \cdot 2x \cdot 1 + 1^2 = (2x+1)^2 = (2x+1)(2x+1)$$

(vi) We have, $a^2x^2 + 2ax + 1$

$$= (ax)^2 + 2 \cdot ax \cdot 1 + (1)^2 = (ax+1)^2 = (ax+1)(ax+1)$$

(vii) We have, $a^2x^2 + 2abx + b^2$

$$= (ax)^2 + 2 \cdot ax \cdot b + b^2 = (ax+b)^2 = (ax+b)(ax+b)$$

(viii) We have, $a^2x^2 + 2abxy + b^2y^2$

$$= (ax)^2 + 2 \cdot ax \cdot by + (by)^2 = (ax+by)^2 = (ax+by)(ax+by)$$

(ix) We have, $4x^2 + 12x + 9$

$$= (2x)^2 + 2 \cdot 2x \cdot 3 + 3^2 = (2x+3)^2 = (2x+3)(2x+3)$$

(x) We have, $16x^2 + 40x + 25$

$$= (4x)^2 + 2 \cdot 4x \cdot 5 + 5^2 = (4x+5)^2 = (4x+5)(4x+5)$$

(xi) We have, $9x^2 + 24x + 16$

$$= (3x)^2 + 2 \cdot 3x \cdot 4 + 4^2 = (3x+4)^2 = (3x+4)(3x+4)$$

(xii) We have, $9x^2 + 30x + 25$

$$= (3x)^2 + 2 \cdot 3x \cdot 5 + 5^2 = (3x+5)^2 = (3x+5)(3x+5)$$

(xiii) We have, $2x^3 + 24x^2 + 72x$

$$= 2x(x^2 + 12x + 36) = 2x(x^2 + 2 \cdot 6 \cdot x + 6^2)$$
$$= 2x(x+6)^2 = 2x(x+6)(x+6)$$

(xiv) We have, $a^2x^3 + 2abx^2 + b^2x$

$$= x(a^2x^2 + 2abx + b^2) = x[(ax)^2 + 2 \cdot ax \cdot b + b^2]$$
$$= x(ax+b)^2 = x(ax+b)(ax+b)$$

(xv) We have, $4x^4 + 12x^3 + 9x^2$

$$= x^2(4x^2 + 12x + 9) = x^2[(2x)^2 + 2 \cdot 2x \cdot 3 + 3^2]$$
$$= x^2(2x+3)^2 = x^2(2x+3)(2x+3)$$

(xvi) We have, $\frac{x^2}{4} + 2x + 4$

$$= \left(\frac{x}{2}\right)^2 + 2 \cdot \frac{x}{2} \cdot 2 + 2^2 = \left(\frac{x}{2} + 2\right)^2 = \left(\frac{x}{2} + 2\right)\left(\frac{x}{2} + 2\right)$$

(xvii) We have, $9x^2 + 2xy + \frac{y^2}{9}$

$$= (3x)^2 + 2 \cdot 3x \cdot \frac{y}{3} + \left(\frac{y}{3}\right)^2 = \left(3x + \frac{y}{3}\right)^2 = \left(3x + \frac{y}{3}\right)\left(3x + \frac{y}{3}\right)$$

Question. 90 Factorise the following, using the identity, $a^2 - 2ab + b^2 = (a-b)^2$

(i) $x^2 - 8x + 16$ (ii) $x^2 - 10x + 25$

(iii) $y^2 - 14y + 49$ (iv) $p^2 - 2p + 1$

(v) $4a^2 - 4ab + b^2$ (vi) $p^2y^2 - 2py + 1$

(vii) $a^2y^2 - 2aby + b^2$ (viii) $9x^2 - 12x + 4$

(ix) $4y^2 - 12y + 9$ (x) $\frac{x^2}{4} - 2x + 4$

(xi) $a^2y^3 - 2aby^2 + b^2y$ (xii) $9y^2 - 4xy + \frac{4x^2}{9}$

Solution.

(i) We have,

$$x^2 - 8x + 16 = x^2 - 2 \cdot x \cdot 4 + 4^2$$
$$= (x-4)^2 \qquad [\because a^2 - 2ab + b^2 = (a-b)^2]$$
$$= (x-4)(x-4)$$

(ii) We have,

$$x^2 - 10x + 25 = x^2 - 2 \cdot x \cdot 5 + 5^2$$
$$= (x-5)^2 = (x-5)(x-5)$$

(iii) We have,

$$y^2 - 14y + 49 = y^2 - 2 \cdot y \cdot 7 + 7^2$$
$$= (y-7)^2 = (y-7)(y-7)$$

(iv) We have,

$$p^2 - 2p + 1 = p^2 - 2 \cdot p \cdot 1 + 1^2$$
$$= (p-1)^2 = (p-1)(p-1)$$

(v) We have,

$$4a^2 - 4ab + b^2 = (2a)^2 - 2 \cdot 2a \cdot b + b^2$$
$$= (2a-b)^2 = (2a-b)(2a-b)$$

(vi) We have,

$$p^2y^2 - 2py + 1 = (py)^2 - 2 \cdot py \cdot 1 + 1^2$$
$$= (py-1)^2 = (py-1)(py-1)$$

(vii) We have,

$$a^2y^2 - 2aby + b^2 = (ay)^2 - 2 \cdot ay \cdot b + b^2$$
$$= (ay-b)^2 = (ay-b)(ay-b)$$

(viii) We have,

$$9x^2 - 12x + 4 = (3x)^2 - 2 \cdot 3x \cdot 2 + 2^2$$
$$= (3x-2)^2 = (3x-2)(3x-2)$$

(ix) We have,

$$4y^2 - 12y + 9 = (2y)^2 - 2 \cdot 2y \cdot 3 + 3^2$$
$$= (2y-3)^2 = (2y-3)(2y-3)$$

(x) We have,

$$\frac{x^2}{4} - 2x + 4 = \left(\frac{x}{2}\right)^2 - 2 \cdot \frac{x}{2} \cdot 2 + 2^2$$
$$= \left(\frac{x}{2} - 2\right)^2 = \left(\frac{x}{2} - 2\right)\left(\frac{x}{2} - 2\right)$$

(xi) We have,

$$a^2y^3 - 2aby^2 + b^2y = y(a^2y^2 - 2aby + b^2) = y[(ay)^2 - 2 \cdot ay \cdot b + b^2]$$
$$= y(ay-b)^2 = y(ay-b)(ay-b)$$

(xii) We have,

$$9y^2 - 4xy + \frac{4x^2}{9} = (3y)^2 - 2 \cdot 3y \cdot \frac{2}{3}x + \left(\frac{2}{3}x\right)^2$$
$$= \left(3y - \frac{2}{3}x\right)^2 = \left(3y - \frac{2x}{3}\right)\left(3y - \frac{2x}{3}\right)$$

Question. 91 Factorise the following

(i) $x^2+15x+26$

(ii) $x^2+9x+20$

(iii) $y^2+18y+65$

(iv) $p^2+14p+13$

(v) $y^2+4y-21$

(vi) $y^2-2y-15$

(vii) $18+11x+x^2$

(viii) $x^2-10x+21$

(ix) $x^2-17x+60$

(x) $x^2+4x-77$

(xi) $y^2+7y+12$

(xii) $p^2-13p-30$

(xiii) $p^2-16p-80$

Solution.

(i) We have, $x^2+15x+26$

$= x^2+2x+13x+2\times 13 = x(x+2)+13(x+2) = (x+2)(x+13)$

(ii) We have, $x^2+9x+20$

$= x^2+5x+4x+5\times 4 = x(x+5)+4(x+5) = (x+5)(x+4)$

(iii) We have, $y^2+18y+65$

$= y^2+13y+5y+5\times 13 = y(y+13)+5(y+13) = (y+13)(y+5)$

(iv) We have, $p^2+14p+13$

$= p^2+13p+p+13\times 1 = p(p+13)+1(p+13) = (p+13)(p+1)$

(v) We have, $y^2+4y-21$

$= y^2+(7-3)y-21 = y^2+7y-3y-21 = y(y+7)-3(y+7) = (y+7)(y-3)$

(vi) We have, $y^2-2y-15$

$= y^2+(3-5)y-15 = y^2+3y-5y-15 = y(y+3)-5(y+3) = (y+3)(y-5)$

(vii) We have, $18+11x+x^2$

$= x^2+11x+18 = x^2+(9+2)x+18 = x^2+9x+2x+18$

$= x(x+9)+2(x+9) = (x+9)(x+2)$

(viii) We have, $x^2-10x+21$

$= x^2-(7+3)x+21 = x^2-7x-3x+21 = x(x-7)-3(x-7)$

$= (x-7)(x-3)$

(ix) We have, $x^2-17x+60$

$= x^2-(12+5)x+60 = x^2-12x-5x+60 = x(x-12)-5(x-12)$

$= (x-12)(x-5)$

(x) We have, $x^2+4x-77$

$x^2+(11-7)x-77 = x^2+11x-7x-77 = x(x+11)-7(x+11)$

$= (x+11)(x-7)$

(xi) We have, $y^2+7y+12$

$y^2+(4+3)y+12 = y^2+4y+3y+12 = y(y+4)+3(y+4) = (y+4)(y+3)$

(xii) We have, $p^2-13p-30$

$= p^2-(15-2)p-30 = p^2-15p+2p-30 = p(p-15)+2(p-15)$

$= (p-15)(p+2)$

(xiii) We have, $p^2-16p-80$

$= p^2-(20-4)p-80 = p^2-20p+4p-80 = p(p-20)+4(p-20)$

$= (p-20)(p+4)$

Question. 92 Factorise the following using the identity, a^2-b^2=(a+b)(a-b).

(i) x^2-9

(ii) $4x^2-25y^2$

(iii) $4x^2-49y^2$

(iv) $3a^2b^3-27a^4b$

(v) $28\,ay^2-175\,ax^2$

(vi) $9x^2-1$

(vii) $25ax^2-25a$

(viii) $\frac{x^2}{9}-\frac{y^2}{25}$

(ix) $\frac{2p^2}{25}-32q^2$

(x) $49x^2-36y^2$

(xi) $y^3-\frac{y}{9}$

(xii) $\frac{x^2}{25}-625$

(xiii) $\frac{x^2}{8}-\frac{y^2}{18}$

(xiv) $\frac{4x^2}{9}-\frac{9y^2}{16}$

(xv) $\frac{x^3y}{9}-\frac{xy^3}{16}$

(xvi) $1331x^3y-11y^3x$

(xvii) $\frac{1}{36}a^2b^2-\frac{16}{49}b^2c^2$

(xviii) $a^4-(a-b)^4$

(xix) x^4-1

(xx) y^4-625

(xxi) p^5-16p

(xxii) $16x^4-81$

(xxiii) x^4-y^4

(xxiv) y^4-81

(xxv) $16x^4-625y^4$

(xxvi) $(a-b)^2-(b-c)^2$

(xxvii) $(x+y)^4-(x-y)^4$

(xxviii) $x^4-y^4+x^2-y^2$

(xxix) $8a^3-2a$

(xxx) $x^2-\frac{y^2}{100}$

(xxxi) $9x^2-(3y+z)^2$

Solution.

(i) We have,

$$x^2-9=x^2-3^2=(x-3)(x+3) \qquad [\because a^2-b^2=(a-b)(a+b)]$$

(ii) We have,

$$4x^2-25y^2=(2x)^2-(5y)^2=(2x-5y)(2x+5y)$$

(iii) We have,

$$4x^2-49y^2=(2x)^2-(7y)^2=(2x-7y)(2x+7y)$$

(iv) We have,

$$3a^2b^3-27a^4b=3a^2b\,(b^2-9a^2)=3a^2b\,[b^2-(3a)^2]$$
$$=3a^2b\,(b+3a)\,(b-3a)$$

(v) We have,

$$28ay^2 - 175ax^2 = 7a(4y^2 - 25x^2)$$
$$= 7a[(2y)^2 - (5x)^2] = 7a(2y - 5x)(2y + 5x)$$

(vi) We have,

$$9x^2 - 1 = (3x)^2 - 1^2 = (3x - 1)(3x + 1)$$

(vii) We have,

$$25ax^2 - 25a = 25a(x^2 - 1^2) = 25a(x - 1)(x + 1)$$

(viii) We have,

$$\frac{x^2}{9} - \frac{y^2}{25} = \left(\frac{x}{3}\right)^2 - \left(\frac{y}{5}\right)^2 = \left(\frac{x}{3} - \frac{y}{5}\right)\left(\frac{x}{3} + \frac{y}{5}\right)$$

(ix) We have,

$$\frac{2p^2}{25} - 32q^2 = 2\left(\frac{p^2}{25} - 16q^2\right) = 2\left[\left(\frac{p}{5}\right)^2 - (4q)^2\right] = 2\left(\frac{p}{5} + 4q\right)\left(\frac{p}{5} - 4q\right)$$

(x) We have,

$$49x^2 - 36y^2 = (7x)^2 - (6y)^2 = (7x - 6y)(7x + 6y)$$

(xi) We have,

$$y^3 - \frac{y}{9} = y\left(y^2 - \frac{1}{9}\right) = y\left[y^2 - \left(\frac{1}{3}\right)^2\right] = y\left(y + \frac{1}{3}\right)\left(y - \frac{1}{3}\right)$$

(xii) We have,

$$\frac{x^2}{25} - 625 = \left(\frac{x}{5}\right)^2 - (25)^2 = \left(\frac{x}{5} - 25\right)\left(\frac{x}{5} + 25\right)$$

(xiii) We have,

$$\frac{x^2}{8} - \frac{y^2}{18} = \frac{1}{2}\left(\frac{x^2}{4} - \frac{y^2}{9}\right) = \frac{1}{2}\left[\left(\frac{x}{2}\right)^2 - \left(\frac{y}{3}\right)^2\right]$$
$$= \frac{1}{2}\left(\frac{x}{2} + \frac{y}{3}\right)\left(\frac{x}{2} - \frac{y}{3}\right)$$

(xiv) We have,

$$\frac{4x^2}{9} - \frac{9y^2}{16} = \left(\frac{2x}{3}\right)^2 - \left(\frac{3y}{4}\right)^2 = \left(\frac{2x}{3} + \frac{3y}{4}\right)\left(\frac{2x}{3} - \frac{3y}{4}\right)$$

(xv) We have,

$$\frac{x^3y}{9} - \frac{xy^3}{16} = xy\left(\frac{x^2}{9} - \frac{y^2}{16}\right) = xy\left[\left(\frac{x}{3}\right)^2 - \left(\frac{y}{4}\right)^2\right] = xy\left(\frac{x}{3} + \frac{y}{4}\right)\left(\frac{x}{3} - \frac{y}{4}\right)$$

(xvi) We have,

$$1331x^3y - 11y^3x = (11)^3x^3y - 11y^3x = 11xy(11^2x^2 - y^2)$$
$$= 11xy[(11x)^2 - y^2] = 11xy(11x + y)(11x - y)$$

(xvii) We have,

$$\frac{1}{36}a^2b^2 - \frac{16}{49}b^2c^2 = \left(\frac{ab}{6}\right)^2 - \left(\frac{4bc}{7}\right)^2 = \left(\frac{ab}{6} + \frac{4bc}{7}\right)\left(\frac{ab}{6} - \frac{4bc}{7}\right)$$
$$= b^2\left(\frac{a}{6} + \frac{4c}{7}\right)\left(\frac{a}{6} - \frac{4c}{7}\right)$$

(xviii) We have,

$$a^4 - (a-b)^4 = (a^2)^2 - [(a-b)^2]^2 = [a^2 + (a-b)^2][a^2 - (a-b)^2]$$
$$= [a^2 + a^2 + b^2 - 2ab][a^2 - (a^2 + b^2 - 2ab)]$$
$$= [2a^2 + b^2 - 2ab][-b^2 + 2ab]$$
$$= (2a^2 + b^2 - 2ab)(2ab - b^2)$$

(xix) We have,

$$x^4 - 1 = (x^2)^2 - 1 = (x^2+1)(x^2-1)$$
$$= (x^2+1)(x+1)(x-1)$$

(xx) We have,

$$y^4 - 625 = (y^2)^2 - (25)^2$$
$$= (y^2+25)(y^2-25)$$
$$= (y^2+25)(y^2-5^2)$$
$$= (y^2+25)(y+5)(y-5)$$

(xxi) We have,

$$p^5 - 16p = p(p^4 - 16) = p[(p^2)^2 - 4^2]$$
$$= p(p^2+4)(p^2-4)$$
$$= p(p^2+4)(p^2-2^2)$$
$$= p(p^2+4)(p+2)(p-2)$$

(xxii) We have,

$$16x^4 - 81 = (4x^2)^2 - 9^2 = (4x^2+9)(4x^2-9)$$
$$= (4x^2+9)[(2x)^2 - 3^2]$$
$$= (4x^2+9)(2x+3)(2x-3)$$

(xxiii) We have,

$$x^4 - y^4 = (x^2)^2 - (y^2)^2$$
$$= (x^2+y^2)(x^2-y^2)$$
$$= (x^2+y^2)(x+y)(x-y)$$

(xxiv) We have,

$$y^4 - 81 = (y^2)^2 - (9)^2 = (y^2+9)(y^2-9)$$
$$= (y^2+9)[(y)^2 - (3)^2]$$
$$= (y^2+9)(y+3)(y-3)$$

(xxv) We have,

$$16x^4 - 625y^4 = (4x^2)^2 - (25y^2)^2 = (4x^2+25y^2)(4x^2-25y^2)$$
$$= (4x^2+25y^2)[(2x)^2 - (5y)^2]$$
$$= (4x^2+25y^2)(2x+5y)(2x-5y)$$

(xxvi) We have,

$$(a-b)^2 - (b-c)^2 = (a-b+b-c)(a-b-b+c)(a-c)(a-2b+c)$$

(xxvii) We have,

$$(x+y)^4 - (x-y)^4 = [(x+y)^2]^2 - [(x-y)^2]^2$$
$$= [(x+y)^2 + (x-y)^2][(x+y)^2 - (x-y)^2]$$

$$= (x^2 + y^2 + 2xy + x^2 + y^2 - 2xy)(x + y + x - y)(x + y - x + y)$$
$$= (2x^2 + 2y^2)(2x)(2y) = 2(x^2 + y^2)(2x)(2y) = 8xy(x^2 + y^2)$$

(xxviii) We have,

$$x^4 - y^4 + x^2 - y^2 = (x^2)^2 - (y^2)^2 + (x^2 - y^2) = (x^2 + y^2)(x^2 - y^2) + (x^2 - y^2)$$
$$= (x^2 - y^2)(x^2 + y^2 + 1) = (x + y)(x - y)(x^2 + y^2 + 1)$$

(xxix) We have,

$$8a^3 - 2a = 2a\,(4a^2 - 1)$$
$$= 2a\,[(2a)^2 - (1)^2] = 2a\,(2a + 1)(2a - 1)$$

(xxx) We have,

$$x^2 - \frac{y^2}{100} = x^2 - \left(\frac{y}{10}\right)^2 = \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$$

(xxxi) We have,

$$9x^2 - (3y + z)^2 = (3x)^2 - (3y + z)^2 = (3x + 3y + z)(3x - 3y - z)$$

Question. 93 The following expressions are the areas of rectangles. Find the possible lengths and breadths of these rectangles.

(i) $x^2 - 6x + 8$

(ii) $x^2 - 3x + 2$

(iii) $x^2 - 7x + 10$

(iv) $x^2 + 19x - 20$

(v) $x^2 + 9x + 20$

Solution.

(i) Given, area of a rectangle $= x^2 - 6x + 8$

Now, we have to find the possible length and breadth of the rectangle.

So, we factorise the given expression.

i.e. $x^2 - 6x + 8 = x^2 - (4+2)x + 8 = x^2 - 4x - 2x + 8$

$$= x(x - 4) - 2(x - 4) = (x - 4)(x - 2)$$

Since, area of a rectangle = Length × Breadth. Hence, the possible length and breadth are $(x - 4)$ and $(x - 2)$.

(ii) We have,

Area of rectangle $= x^2 - 3x + 2$

$$= x^2 - (2 + 1)x + 2 = x^2 - 2x - x + 2$$
$$= x(x - 2) - 1(x - 2) = (x - 2)(x - 1)$$

∴ The possible length and breadth are $(x - 2)$ and $(x - 1)$.

(iii) We have,

Area of rectangle $= x^2 - 7x + 10$

$$= x^2 - (5 + 2)x + 10 = x^2 - 5x - 2x + 10$$
$$= x(x - 5) - 2(x - 5) = (x - 5)(x - 2)$$

∴ The possible length and breadth are $(x - 5)$ and $(x - 2)$.

(iv) We have,

Area of rectangle $= x^2 + 19x - 20$

$$= x^2 + (20 - 1)x - 20 = x^2 + 20x - x - 20$$
$$= x(x + 20) - 1(x + 20) = (x + 20)(x - 1)$$

∴ The possible length and breadth are $(x + 20)$ and $(x - 1)$.

(v) We have, area of rectangle

$$= x^2 + 9x + 20$$
$$= x^2 + (5 + 4)x + 20 = x^2 + 5x + 4x + 20$$
$$= x(x + 5) + 4(x + 5) = (x + 5)(x + 4)$$

∴ The possible length and breadth are $(x + 5)$ and $(x + 4)$.

Question. 94 Carry out the following divisions:

(i) $51x^3y^2z \div 17xyz$

(ii) $76x^3yz^3 \div 19x^2y^2$

(iii) $17ab^2c^3 \div (-abc^2)$

(iv) $-121p^3q^3r^3 \div (-11xy^2z^3)$

Solution.

(i) We have,

$$51x^3y^2z \div 17xyz = \frac{51x^3y^2z}{17xyz}$$
$$= \frac{17 \times 3 \times x \times x \times x \times y \times y \times z}{17 \times x \times y \times z} = 3x^2y$$

(ii) We have,

$$76x^3yz^3 \div 19x^2y^2 = \frac{76x^3yz^3}{19x^2y^2}$$
$$= \frac{4 \times 19 \times x \times x \times x \times y \times z \times z \times z}{19 \times x \times x \times y \times y} = \frac{4xz^3}{y}$$

(iii) We have,

$$17ab^2c^3 \div (-abc^2) = \frac{17ab^2c^3}{-abc^2} = \frac{17 \times a \times b \times b \times c \times c \times c}{-a \times b \times c \times c} = -17bc$$

(iv) We have,

$$-121p^3q^3r^3 \div (-11xy^2z^3) = \frac{-121p^3q^3r^3}{-11xy^2z^3}$$
$$= \frac{-11 \times 11 \times p \times p \times p \times q \times q \times q \times r \times r \times r}{-11 \times x \times y \times y \times z \times z \times z} = \frac{11p^3q^3r^3}{xy^2z^3}$$

Question. 95 Perform the following divisions:

(i) $(3pqr - 6p^2q^2r^2) \div 3pq$

(ii) $(ax^3 - bx^2 + cx) \div (-dx)$

(iii) $(x^3y^3 + x^2y^3 - xy^4 + xy) \div xy$

(iv) $(-qrxy + pryz - rxyz) \div (-xyz)$

Solution.

(i) We have,

$$(3pqr - 6p^2q^2r^2) \div 3pq = \frac{3pqr - 6p^2q^2r^2}{3pq} = \frac{3pqr}{3pq} - \frac{6p^2q^2r^2}{3pq}$$
$$= r - \frac{2 \times 3 \times p \times p \times q \times q \times r \times r}{3 \times p \times q}$$
$$= r - 2pqr^2$$

(ii) We have,

$$(ax^3 - bx^2 + cx) \div (-dx) = \frac{ax^3 - bx^2 + cx}{-dx}$$

$$= \frac{ax^3}{-dx} + \frac{bx^2}{dx} + \frac{cx}{-dx} = \frac{a \times x \times x \times x}{-d \times x} + \frac{b \times x \times x}{d \times x} + \frac{c \times x}{-d \times x}$$

$$= -\frac{a}{d}x^2 + \frac{b}{d}x - \frac{c}{d}$$

(iii) We have,

$$(x^3y^3 + x^2y^3 - xy^4 + xy) \div xy$$

$$= \frac{x^3y^3 + x^2y^3 - xy^4 + xy}{xy} = \frac{x^3y^3}{xy} + \frac{x^2y^3}{xy} - \frac{xy^4}{xy} + \frac{xy}{xy}$$

$$= \frac{x \times x \times x \times y \times y \times y}{x \times y} + \frac{x \times x \times y \times y \times y}{x \times y} - \frac{x \times y \times y \times y \times y}{x \times y} + \frac{x \times y}{x \times y}$$

$$= x^2y^2 + xy^2 - y^3 + 1$$

(iv) We have,

$$(-qrxy + pryz - rxyz) \div (-xyz)$$

$$= \frac{-qrxy + pryz - rxyz}{-xyz} = \frac{-qrxy}{-xyz} + \frac{pryz}{-xyz} - \frac{rxyz}{-xyz} = \frac{qr}{z} - \frac{pr}{x} + r$$

Question. 96 Factorise the expressions and divide them as directed.

(i) $(x^2 - 22x + 117) \div (x - 13)$

(ii) $(x^3 + x^2 - 132x) \div x(x - 11)$

(iii) $(2x^3 - 12x^2 + 16x) \div (x - 2)(x - 4)$

(iv) $(9x^2 - 4) \div (3x + 2)$

(v) $(3x^2 - 48) \div (x - 4)$

(vi) $(x^4 - 16) \div x^3 + 2x^2 + 4x + 8$

(vii) $(3x^4 - 1875) \div (3x^2 - 75)$

Solution.

(i) We have,

$$(x^2 - 22x + 117) \div (x - 13)$$

$$= \frac{x^2 - 22x + 117}{x - 13} = \frac{x^2 - 13x - 9x + 117}{x - 13} = \frac{x(x - 13) - 9(x - 13)}{x - 13}$$

$$= \frac{(x - 13)(x - 9)}{x - 13} = x - 9$$

(ii) We have,

$$(x^3 + x^2 - 132x) \div x(x - 11)$$

$$= \frac{x^3 + x^2 - 132x}{x(x - 11)} = \frac{x(x^2 + x - 132)}{x(x - 11)} = \frac{x^2 + 12x - 11x - 132}{x - 11}$$

$$= \frac{x(x + 12) - 11(x + 12)}{x - 11} = \frac{(x + 12)(x - 11)}{x - 11}$$

$$= x + 12$$

(iii) We have,

$(2x^3 - 12x^2 + 16x) \div (x-2)(x-4)$

$$= \frac{2x^3 - 12x^2 + 16x}{(x-2)(x-4)} = \frac{2x(x^2 - 6x + 8)}{(x-2)(x-4)}$$

$$= \frac{2x(x^2 - 4x - 2x + 8)}{(x-2)(x-4)}$$

$$= \frac{2x[x(x-4) - 2(x-4)]}{(x-2)(x-4)} = \frac{2x(x-4)(x-2)}{(x-2)(x-4)} = 2x$$

(iv) We have,

$$(9x^2 - 4) \div (3x+2) = \frac{9x^2 - 4}{3x+2} = \frac{(3x)^2 - (2)^2}{3x+2}$$

$$= \frac{(3x+2)(3x-2)}{3x+2} \qquad [\because a^2 - b^2 = (a+b)(a-b)]$$

$$= 3x - 2$$

(v) We have,

$$(3x^2 - 48) \div (x-4) = \frac{3x^2 - 48}{x-4} = \frac{3(x^2 - 16)}{x-4}$$

$$= \frac{3(x^2 - 4^2)}{x-4}$$

$$= \frac{3(x+4)(x-4)}{x-4} \qquad [\because a^2 - b^2 = (a+b)(a-b)]$$

$$= 3(x+4)$$

(vi) We have,

$(x^4 - 16) \div x^3 + 2x^2 + 4x + 8$

$$= \frac{x^4 - 16}{x^3 + 2x^2 + 4x + 8} = \frac{(x^2)^2 - 4^2}{x^2(x+2) + 4(x+2)}$$

$$[\because a^2 - b^2 = (a+b)(a-b)]$$

$$= \frac{(x^2+4)(x^2-4)}{(x^2+4)(x+2)} = \frac{x^2 - 2^2}{x+2} = \frac{(x+2)(x-2)}{x+2} = x - 2$$

(vii) We have,

$$(3x^4 - 1875) \div (3x^2 - 75) = \frac{3x^4 - 1875}{3x^2 - 75} = \frac{x^4 - 625}{x^2 - 25} = \frac{(x^2)^2 - (25)^2}{x^2 - 25}$$

$$= \frac{(x^2+25)(x^2-25)}{(x^2-25)} = x^2 + 25$$

Question. 97 The area of a square is given by $4x^2$+ 12xy + $9y^2$. Find the side of the square.

Solution.

We have,

Area of square $= 4x^2 + 12xy + 9y^2$

So, we factorise the given expression.

$$\therefore \quad 4x^2 + 12xy + 9y^2 = (2x)^2 + 2 \times 2x \times 3y + (3y)^2 \qquad [\because a^2 + 2ab + b^2 = (a+b)^2]$$

$$= (2x + 3y)^2$$

Since, area of a square having side length a is a^2. Hence, side of the given square is $2x + 3y$.

Question. 98 The area of a square is $9x^2$ + 24xy + $16y^2$. Find the side of the square.

Solution.

We have,

Area of a square $= 9x^2 + 24xy + 16y^2 = (3x)^2 + 2 \times 3x \times 4y + (4y)^2$

$[\because a^2 + 2ab + b^2 = (a+b)^2]$

$= (3x + 4y)^2$

$\therefore$ The side of the square $= 3x + 4y$ $\quad [\because$ area of square $= (\text{side})^2]$

Question. 99 The area of a rectangle is x^2 + 7x + 12. If its breadth is (x + 3), then find its length.

Solution.

Let the length of the rectangle be l.

Given, area of a rectangle $= x^2 + 7x + 12$

and breadth $= x + 3$

We know that,

Area of rectangle = Length × Breadth

$\Rightarrow \quad x^2 + 7x + 12 = l \times (x + 3)$

$$\Rightarrow \quad l = \frac{x^2 + 7x + 12}{x + 3} = \frac{x^2 + 4x + 3x + 12}{x + 3} = \frac{x(x + 4) + 3(x + 4)}{x + 3} = \frac{(x + 4)(x + 3)}{x + 3} = x + 4$$

Hence, the length of rectangle $= x + 4$

Question. 100 The curved surface area of a cylinder is $2\pi(y^2 - 7y + 12)$ and its radius is (y – 3). Find the height of the cylinder (CSA of cylinder = Formula does not parse)

Solution.

Let the height of cylinder be h.

Given, the curved surface area of a cylinder $= 2\pi(y^2 - 7y + 12)$

and radius of cylinder $= y - 3$

We know that,

Curved surface area of cylinder $= 2\pi rh$

$\therefore \quad 2\pi rh = 2\pi(y^2 - 7y + 12)$

$\Rightarrow \quad 2\pi rh = 2\pi(y^2 - 4y - 3y + 12) = 2\pi[y(y - 4) - 3(y - 4)] = 2\pi(y - 3)(y - 4)$

$\Rightarrow \quad 2\pi rh = 2\pi r(y - 4)$ $\quad [\because r = (y - 3)$, given$]$

On comparing the both sides, we get $h = y - 4$

Hence, the height of the cylinder is $y - 4$.

Question. 101 The area of a circle is given by the expression $\pi x^2 + 6\pi x + 9\pi$. Find the radius of the circle.

Solution.

We have,

Area of a circle $= \pi x^2 + 6\pi x + 9\pi = \pi(x^2 + 6x + 9)$

$\Rightarrow \quad \pi r^2 = \pi(x^2 + 3x + 3x + 9)$ $\quad [\because$ area of a circle $= \pi r^2$, where r is the radius$]$

$\Rightarrow \quad \pi r^2 = \pi[x(x + 3) + 3(x + 3)] = \pi(x + 3)(x + 3) = \pi(x + 3)^2$

$\Rightarrow \quad \pi r^2 = \pi(x + 3)^2$

On comparing both sides, $r^2 = (x + 3)^2 \quad \Rightarrow \quad r = x + 3$

Hence, the radius of circle is $x + 3$.

Question.102 The sum of first n natural numbers is given by the expression $\frac{n^2}{2} + \frac{n}{2}$ Factorise this expression.

Solution.

We have, the sum of first n natural numbers

$$= \frac{n^2}{2} + \frac{n}{2}$$

Factorisation of given expression $= \frac{1}{2}(n^2 + n) = \frac{1}{2}n(n+1)$ [taking n as common]

Question.103 The sum of (x + 5) observations is x^4 – 625. Find the mean of the observations.

Solution.

We have, the sum of $(x + 5)$ observations $= x^4 - 625$

We know that, the mean of the n observations $x_1, x_2, \dots x_n$ is given by $\frac{x_1 + x_2 + \dots x_n}{n}$.

$\therefore$ The mean of $(x + 5)$ observations

$$= \frac{\text{Sum of } (x+5) \text{ observations}}{x+5} = \frac{x^4 - 625}{x+5} = \frac{(x^2)^2 - (25)^2}{x+5}$$

$$= \frac{(x^2+25)(x^2-25)}{x+5} \quad [\because a^2 - b^2 = (a+b)(a-b)]$$

$$= \frac{(x^2+25)[(x)^2 - (5)^2]}{x+5}$$

$$= \frac{(x^2+25)(x+5)(x-5)}{(x+5)} = (x^2+25)(x-5)$$

Question.104 The height of a triangle is $x^4 + y^4$ and its base is 14xy. Find the area of the triangle.

Solution.

Given, the height of a triangle and its base are $x^4 + y^4$ and $14xy$, respectively.

We know that, the area of a triangle $= \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 14xy \times (x^4 + y^4)$

$$= 7xy(x^4 + y^4)$$

Question.105 The cost of a chocolate is Rs (x + 4) and Rohit bought (x + 4) chocolates. Find the total amount paid by him in terms of x. If x = 10, find the amount paid by him.

Solution.

Given, cost of a chocolate = ₹ $(x + 4)$

Rohit bought $(x + 4)$ chocolates.

$\therefore$ The cost of $(x + 4)$ chocolates

= Cost of one chocolate × Number of chocolates $= (x+4)(x+4) = (x+4)^2$

$$= x^2 + 8x + 16 \quad [\because (a+b)^2 = a^2 + 2ab + b^2]$$

$\therefore$ The total amount paid by Rohit = ₹$(x^2 + 8x + 16)$

Now, if $x = 10$. Then, the amount paid by Rohit $= 10^2 + 8 \times 10 + 16 = 100 + 80 + 16 =$ ₹196

Question.106 The base of a parallelogram is (2x + 3) units and the corresponding height is (2x – 3) units. Find the area of the parallelogram in terms of x. What will be the area of a parallelogram of x = 30 units?

Solution.

We have, the base and the corresponding height of a parallelogram are $(2x + 3)$ units and $(2x - 3)$ units, respectively.

$\because$ Area of a parallelogram = Base × Height

$$= (2x+3) \times (2x-3) = (2x)^2 - (3)^2 \quad [\because (a+b)(a-b) = a^2 - b^2]$$

$$= (4x^2 - 9) \text{ sq units}$$

Now, if $x = 10$. Then, the area of the parallelogram $= 4 \times (10)^2 - 9 = 400 - 9 = 391$ sq units

Question.107 The radius of a circle is 7ab – 7be – 14ac . Find the circumference of the circle,$(\pi = \frac{22}{7})$

Solution.

We have, radius of the circle = $7ab - 7bc - 14ac = r$ [say]

We know that,

∵ The circumference of the circle = $2\pi r$

$$= 2 \times \frac{22}{7} \times (7ab - 7bc - 14ac)$$

$$= \frac{44}{7} \times 7\,(ab - bc - 2ac)$$

$$= 44[ab - c(b + 2a)]$$

Question.108 If p + q = 12 and pq = 22, then find $p^2 + q^2$.

Solution.

Given, $p + q = 12$ and $pq = 22$

Since,

$(p+q)^2 = p^2 + q^2 + 2pq$ [using the identity, $(a+b)^2 = a^2 + b^2 + 2ab$]

∴ $(12)^2 = p^2 + q^2 + 2 \times 22$

⇒ $p^2 + q^2 = (12)^2 - 44$

⇒ $p^2 + q^2 = 144 - 44 = 100$

Question.109 If a + b = 25 and $a^2 + b^2$ then find ab.

Solution.

Given, $a + b = 25$ and $a^2 + b^2 = 225$

We know that,

$(a + b)^2 = a^2 + b^2 + 2ab$ [an algebraic identity]

⇒ $(25)^2 = 225 + 2ab$

⇒ $2ab = (25)^2 - 225$

⇒ $2ab = 625 - 225$

⇒ $2ab = 400$

⇒ $ab = \frac{400}{2}$

⇒ $ab = 200$

Question.110 If x – y = 13 and xy = 28, then find $x^2 + y^2$.

Solution.

Given, $x - y = 13$ and $xy = 28$

Since, $(x - y)^2 = x^2 + y^2 - 2xy$

[using the identity, $(a-b)^2 = a^2 + b^2 - 2ab$]

∴ $(13)^2 = x^2 + y^2 - 2 \times 28$

⇒ $x^2 + y^2 = (13)^2 + 56$

⇒ $x^2 + y^2 = 169 + 56$

⇒ $x^2 + y^2 = 225$

Question.111 If m – n = 16 and $m^2 + n^2$ = 400, then find mn.

Solution.

Given, $m - n = 16$ and $m^2 + n^2 = 400$.

Since, $(m-n)^2 = m^2 + n^2 - 2mn$

[using the identity, $(a-b)^2 = a^2 + b^2 - 2ab$]

$\therefore \quad (16)^2 = 400 - 2mn$

$\Rightarrow \quad 2mn = 400 - (16)^2$

$\Rightarrow \quad 2mn = 400 - 256$

$\Rightarrow \quad 2mn = 144$

$\Rightarrow \quad mn = \frac{144}{2}$

$\Rightarrow \quad mn = 72$

Question.112 If $a^2 + b^2 = 74$ and ab = 35, then find a + b ?

Solution.

Given, $a^2 + b^2 = 74$ and $ab = 35$

Since, $(a+b)^2 = a^2 + b^2 + 2ab$

[using the identity, $(a+b)^2 = a^2 + b^2 + 2ab$]

$\therefore \quad (a+b)^2 = 74 + 2 \times 35$

$\therefore \quad (a+b)^2 = 74 + 2 \times 35$

$\Rightarrow \quad (a+b)^2 = 144$

$\Rightarrow \quad a + b = \sqrt{144}$ [taking square root]

$\Rightarrow \quad a + b = 12$ [rejecting –ve sign]

Question.113 Verify the following:

(i) $(ab + bc)(ab - bc) + (bc + ca)(bc - ca) + (ca + ab)(ca - ab) = 0$

(ii) $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$

(iii) $(p - q)(p^2 + pq + q^2) = p^3 - q^3$

(iv) $(m + n)(m^2 - mn + n^2) = m^3 + n^3$

(v) $(a + b)(a + b)(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$

(vi) $(a - b)(a - b)(a - b) = a^3 - 3a^2b + 3ab^2 - b^3$

(vii) $(a^2 - b^2)(a^2 + b^2) + (b^2 - c^2)(b^2 + c^2) + (c^2 - a^2)(c^2 + a^2) = 0$

(viii) $(5x + 8)^2 - 160x = (5x - 8)^2$

(ix) $(7p - 13q)^2 + 364pq = (7p + 13q)^2$

(x) $\left(\frac{3p}{7} + \frac{7}{6p}\right)^2 - \left(\frac{3p}{7} - \frac{7}{6p}\right)^2 = 2$

Solution.

(i) Taking LHS $= (ab + bc)(ab - bc) + (bc + ca)(bc - ca) + (ca + ab)(ca - ab)$

$= [(ab)^2 - (bc)^2] + [(bc)^2 - (ca)^2] + [(ca)^2 - (ab)^2]$

[using the identity, $(a + b)(a - b) = a^2 - b^2$]

$= a^2b^2 - b^2c^2 + b^2c^2 - c^2a^2 + c^2a^2 - a^2b^2 = 0$

= RHS [cancelling the like terms having opposite signs]

Hence verified.

(ii) Taking LHS $= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= a(a^2 + b^2 + c^2 - ab - bc - ca) + b(a^2 + b^2 + c^2 - ab - bc - ca)$
$+ c(a^2 + b^2 + c^2 - ab - bc - ca)$

[distributive law]

$= a^3 + ab^2 + ac^2 - a^2b - abc - a^2c + ba^2 + b^3 + bc^2$
$- b^2a - b^2c - bca + ca^2 + cb^2 + c^3 - cab - c^2b - c^2a$

$= a^3 + b^3 + c^3 - 3abc =$ RHS **Hence verified.**

(iii) Taking LHS $= (p - q)(p^2 + pq + q^2)$

$= p(p^2 + pq + q^2) - q(p^2 + pq + q^2)$

$= p^3 + p^2q + pq^2 - qp^2 - pq^2 - q^3 = p^3 - q^3 =$ RHS **Hence verified.**

(iv) Taking LHS $= (m + n)(m^2 - mn + n^2)$

$= m(m^2 - mn + n^2) + n(m^2 - mn + n^2)$

$= m^3 - m^2n + mn^2 + nm^2 - mn^2 + n^3 = m^3 + n^3 =$ RHS **Hence verified.**

(v) Taking LHS $= (a + b)(a + b)(a + b)$

$= (a + b)(a + b)^2$

$= (a + b)(a^2 + b^2 + 2ab)$ [using the identity, $(a + b)^2 = a^2 + 2ab + b^2$]

$= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2)$

$= a^3 + 2a^2b + ab^2 + ba^2 + 2ab^2 + b^3$

$= a^3 + 3a^2b + 3ab^2 + b^3$ [adding like terms]

= RHS **Hence verified.**

(vi) Taking LHS $= (a - b)(a - b)(a - b)$

$= (a - b)(a - b)^2$

$= (a - b)(a^2 - 2ab + b^2)$ [using the identity, $(a - b)^2 = a^2 - 2ab + b^2$]

$= a(a^2 - 2ab + b^2) - b(a^2 - 2ab + b^2)$

$= a^3 - 2a^2b + ab^2 - ba^2 + 2ab^2 - b^3$

$= a^3 - 3a^2b + 3ab^2 - b^3$ [adding like terms]

= RHS **Hence verified.**

(vii) Taking LHS $= (a^2 - b^2)(a^2 + b^2) + (b^2 - c^2)(b^2 + c^2) + (c^2 - a^2)(c^2 + a^2)$

$= (a^4 - b^4 + b^4 - c^4 + c^4 - a^4)$ [using the identity, $(a - b)(a + b) = a^2 - b^2$]

$= 0 =$ RHS **Hence verified.**

(viii) Taking LHS $= (5x + 8)^2 - 160x$

$= (5x)^2 + (8)^2 + 2 \times 5x \times 8 - 160x$ [using the identity, $(a + b)^2 = a^2 + 2ab + b^2$]

$= (5x)^2 + (8)^2 + 80x - 160x$

$= (5x)^2 + (8)^2 - 80x$

$= (5x)^2 + (8)^2 - 2 \times 5x \times 8$

$= (5x - 8)^2$ [$\because a^2 + b^2 - 2ab = (a - b)^2$]

$=$ RHS **Hence verified.**

(ix) Taking LHS $= (7p - 13q)^2 + 364pq$

$= (7p)^2 + (13q)^2 - 2 \times 7p \times 13q + 364pq$

$= (7p)^2 + (13q)^2 - 182pq + 364pq$

$= (7p)^2 + (13q)^2 + 182pq$

$= (7p)^2 + (13q)^2 + 2 \times 7p \times 13q = (7p + 13q)^2 =$ RHS **Hence verified.**

(x) Taking LHS $= \left(\frac{3p}{7} + \frac{7}{6p}\right)^2 - \left(\frac{3p}{7} - \frac{7}{6p}\right)^2$

$= \left[\left(\frac{3p}{7} + \frac{7}{6p}\right) + \left(\frac{3p}{7} - \frac{7}{6p}\right)\right]\left[\left(\frac{3p}{7} + \frac{7}{6p}\right) - \left(\frac{3p}{7} - \frac{7}{6p}\right)\right]$

[using the identity, $a^2 - b^2 = (a + b)(a - b)$]

$= \left(\frac{3p}{7} + \frac{7}{6p} + \frac{3p}{7} - \frac{7}{6p}\right)\left(\frac{3p}{7} + \frac{7}{6p} - \frac{3p}{7} + \frac{7}{6p}\right) = \frac{6p}{7} \times \frac{14}{6p} = 2 =$ RHS

Hence verified.

Question.114 Find the value of a, if

(i) $8a = 35^2 - 27^2$

(ii) $9a = 76^2 - 67^2$

(iii) $pqa = (3p + q)^2 - (3p - q)^2$

(iv) $pq^2a = (4pq + 3q)^2 - (4pq - 3q)^2$

Solution.

(i) We have,

$8a = 35^2 - 27^2$

$\Rightarrow$ $8a = (35 + 27)(35 - 27)$ [using the identity, $a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow$ $8a = 62 \times 8$

$\Rightarrow$ $a = \frac{62 \times 8}{8}$

$\Rightarrow$ $a = 62$

(ii) We have, $9a = (76)^2 - (67)^2$

$\Rightarrow$ $9a = (76 + 67)(76 - 67)$ [using the identity, $a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow$ $9a = 143 \times 9$

$\Rightarrow$ $a = \frac{143 \times 9}{9}$

$\Rightarrow$ $a = 143$

(iii) We have, $pqa = (3p + q)^2 - (3p - q)^2$

$\Rightarrow$ $pqa = [(3p + q) + (3p - q)][(3p + q) - (3p - q)]$

[using the identity, $a^2 - b^2 = (a + b)(a - b)$]

$\Rightarrow$ $pqa = [(3p + q + 3p - q)][3p + q - 3p + q]$

$\Rightarrow$ $pqa = 6p \times 2q$

$\Rightarrow$ $a = \frac{6p \times 2q}{pq} = \frac{(6 \times 2)pq}{pq}$

$\Rightarrow$ $a = 12$

(iv) We have,

$$pq^2a = (4pq + 3q)^2 - (4pq - 3q)^2$$

$$\Rightarrow \quad = [(4pq + 3q) + (4pq - 3q)][(4pq + 3q) - (4pq - 3q)]$$

[using the identity, $a^2 - b^2 = (a+b)(a-b)$]

$$= (4pq + 3q + 4pq - 3q)(4pq + 3q - 4pq + 3q)$$

$$= 8pq \times 6q$$

$$\Rightarrow \quad pq^2a = 48pq^2$$

$$\Rightarrow \quad a = \frac{48pq^2}{pq^2}$$

$$\Rightarrow \quad a = 48$$

Question.115 What should be added to 4c (-a + b + c) to obtain 3a(a + b + c) – 2b (a – b + c)?

Solution.

Let x be added to the given expression $4c(-a + b + c)$ to obtain $3a(a + b + c) - 2b(a - b + c)$

i.e. $x + 4c(-a + b + c) = 3a(a + b + c) - 2b(a - b + c)$

$$\Rightarrow \quad x = 3a(a + b + c) - 2b(a - b + c) - 4c(-a + b + c)$$

$$= 3a^2 + 3ab + 3ac - 2ba + 2b^2 - 2bc + 4ca - 4cb - 4c^2$$

$$\Rightarrow \quad x = 3a^2 + ab + 7ac + 2b^2 - 6bc - 4c^2 \quad \text{[adding the like terms]}$$

Question.116 Subtract b(b^2 + b – 7) + 5 from 3b^2 – 8 and find the value of expression obtained for b = – 3.

Solution.

We have,

Required difference $= (3b^2 - 8) - [b(b^2 + b - 7) + 5]$

$$= 3b^2 - 8 - b(b^2 + b - 7) - 5$$

$$= 3b^2 - 8 - b^3 - b^2 + 7b - 5 = -b^3 + 2b^2 + 7b - 13$$

Now, if $b = -3$

The value of above expression $= -(-3)^3 + 2(-3)^2 + 7(-3) - 13$

$$= -(-27) + 2 \times 9 - 21 - 13$$

$$= 27 + 18 - 21 - 13$$

$$= 45 - 34 = 11$$

Question.117 If x – $\frac{1}{x}$ = 1, then find the value of $x^2 + \frac{1}{x^2}$.

Solution.

Given, $x - \frac{1}{x} = 7$.

Since, $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2 \cdot x \cdot \frac{1}{x}$ [using the identity, $(a-b)^2 = a^2 + b^2 - 2ab$]

$$\therefore \quad 7^2 = x^2 + \frac{1}{x^2} - 2$$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} = 49 + 2$$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} = 51$$

Question.118 Factorise $x^2 + \frac{1}{x^2} + 2 - 3x - \frac{3}{x}$.

Solution.

We have, $x^2 + \frac{1}{x^2} + 2 - 3x - \frac{3}{x}$

$$= x^2 + \frac{1}{x^2} + 2 \cdot x \cdot \frac{1}{x} - 3\left(x + \frac{1}{x}\right)$$

$$= \left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right)$$

[using the identity, $a^2 + b^2 + 2ab = (a + b)^2$]

$$= \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x} - 3\right) \qquad \left[\text{taking } \left(x + \frac{1}{x}\right) \text{as common}\right]$$

Question.119 Factorise $p^4 + q^4 + p^2q^2$.

Solution.

We have, $p^4 + q^4 + p^2q^2$

$= p^4 + q^4 + 2p^2q^2 - 2p^2q^2 + p^2q^2$ [adding and subtracting $2p^2q^2$]

$= p^4 + q^4 + 2p^2q^2 - p^2q^2$

$= [(p^2)^2 + (q^2)^2 + 2p^2q^2] - p^2q^2$

[using the identity, $a^2 + b^2 + 2ab = (a + b)^2$]

$= (p^2 + q^2)^2 - (pq)^2$

$= (p^2 + q^2 + pq)(p^2 + q^2 - pq)$ [using the identity, $a^2 - b^2 = (a + b)(a - b)$]

Question.120 Find the value of

(i) $\frac{6.25 \times 6.25 - 1.75 \times 1.75}{4.5}$ **(ii)** $\frac{198 \times 198 - 102 \times 102}{96}$

Solution.

(i) We have, $\frac{6.25 \times 6.25 - 1.75 \times 1.75}{4.5} = \frac{(6.25)^2 - (1.75)^2}{4.5}$

$= \frac{(6.25 + 1.75)(6.25 - 1.75)}{4.5}$ [using the identity, $a^2 - b^2 = (a + b)(a - b)$]

$= \frac{8 \times 4.5}{4.5} = 8$

(ii) We have,

$\frac{198 \times 198 - 102 \times 102}{96} = \frac{(198)^2 - (102)^2}{96} = \frac{(198 + 102)(198 - 102)}{96}$

$= \frac{300 \times 96}{96} = 300$ [using the identity, $a^2 - b^2 = (a - b)(a + b)$]

Question.121 The product of two expressions is $x^5 + x^3 + x$. If one of them is $x^2 + x + 1$, find the other.

Solution.

We have, product of two expressions $x^5 + x^3 + x$ and one is $x^2 + x + 1$.
Let the other expression be A. Then,

$$A \cdot (x^2 + x + 1) = x^5 + x^3 + x$$

$$\Rightarrow \quad A = \frac{x^5 + x^3 + x}{x^2 + x + 1} = \frac{x(x^4 + x^2 + 1)}{x^2 + x + 1}$$

$$\Rightarrow \quad A = \frac{x(x^4 + 2x^2 - x^2 + 1)}{x^2 + x + 1} = \frac{x(x^4 + 2x^2 + 1 - x^2)}{x^2 + x + 1}$$

[adding and subtracting x^2 in numerator term]

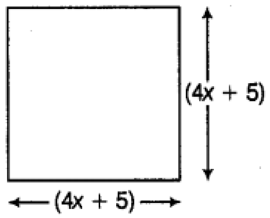
$$= \frac{x[(x^4 + 2x^2 + 1) - x^2]}{x^2 + x + 1} = \frac{x[(x^2 + 1)^2 - x^2]}{x^2 + x + 1}$$

$$= \frac{x(x^2 + 1 + x)(x^2 + 1 - x)}{x^2 + x + 1}$$ [using the identity, $a^2 - b^2 = (a + b)(a - b)$]

$$= x(x^2 + 1 - x)$$

Hence, the other expression is $x(x^2 - x + 1)$.

Question.122 Find the length of the side of the given square, if area of the square is 625sq units and then find the value of x.



Solution.

We have, a square having length of a side $(4x + 5)$ units and area is 625 sq units.

$\because$ Area of a square = $(\text{Side})^2$

$$(4x + 5)^2 = 625$$

$$\Rightarrow \quad (4x + 5)^2 = (25)^2$$ [taking square root both sides and neglecting (–ve) sign]

$$\Rightarrow \quad 4x + 5 = 25$$

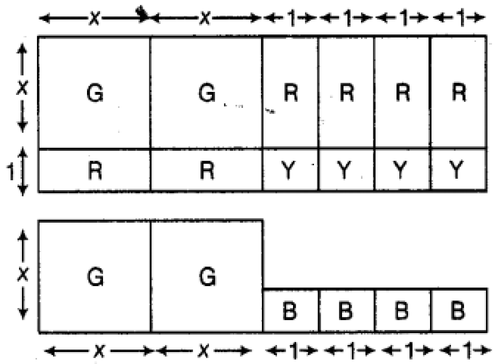
$$\Rightarrow \quad 4x = 25 - 5$$

$$\Rightarrow \quad 4x = 20$$

$$\Rightarrow \quad x = 5$$

Hence, side = $4x + 5 = 4 \times 5 + 5 = 25$ units

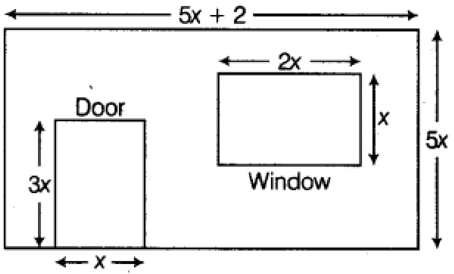
Question.123 Take suitable number of cards given in the adjoining diagram [G(x x x) representing x^2, R (x x 1) representing x and Y (1 x 1) representing 1] to factorise the following expressions, by arranging to cards in the form of rectangles: (i) $2x^2$ + 6x + 4 (ii) x^2 + 4x + 4. Factorise $2x^2$ + 6x + 4 by using the figure.



Calculate the area of figure.

Solution. The given information is incomplete for solution of this question.

Question.124 The figure shows the dimensions of a wall having a window and a door of a room. Write an algebraic expression for the area of the wall to be painted.



Solution.

We have a wall of dimension $5x \times (5x + 2)$ having a window and a door of dimension $(2x \times x)$ and $(3x \times x)$, respectively.

Then, area of the window $= 2x \times x = 2x^2$ sq units

Area of the door $= 3x \times x = 3x^2$ sq units

and area of wall $= (5x + 2) \times 5x = (25x^2 + 10x)$ sq units

Now, area of the required part of the wall to be painted

= Area of the wall – (Area of the window + Area of the door)

$= 25x^2 + 10x - (2x^2 + 3x^2)$

$= 25x^2 + 10x - 5x^2 = 20x^2 + 10x$

$= 2 \times 2 \times 5 \times x \times x + 2 \times 5 \times x$

$= 2 \times 5 \times x(2x + 1) = 10x(2x + 1)$ sq units

Question.125 Match the expressions of column I with that of column II

	Column I		Column II
(i)	$(21x + 13y)^2$	(a)	$441x^2 - 169y^2$
(ii)	$(21x - 13y)^2$	(b)	$441x^2 + 169y^2 + 546xy$
(iii)	$(21x - 13y)(21x + 13y)$	(c)	$441x^2 + 169y^2 - 546xy$
		(d)	$441x^2 - 169y^2 + 546xy$

Solution.

(i) We have,

$(21x + 13y)^2 = (21x)^2 + (13y)^2 + 2 \times 21x \times 13y$

[using the identity, $(a + b)^2 = a^2 + b^2 + 2ab$]

$= 441x^2 + 169y^2 + 546xy$

(ii) $(21x - 13y)^2 = (21x)^2 + (13y)^2 - 2 \times 21x \times 13y$

[using the identity, $(a - b)^2 = a^2 + b^2 - 2ab$]

$= 441x^2 + 169y^2 - 546xy$

(iii) $(21x - 13y)(21x + 13y)$

$= (21x)^2 - (13y)^2 = 441x^2 - 169y^2$

[using the identity, $(a - b)(a + b) = a^2 - b^2$]

Hence, (i) → (a), (ii) → (c), (iii) → (a)