1. NUMBER SYSTEM

TYPES OF NUMBERS

1. **Natural Numbers:** Numbers which are used for counting are called Natural Number. The set of natural number is denoted by 'N'.

   For example: Set of Natural Numbers, N = {1, 2, 3, ...}

2. **Whole Numbers:** All set of natural numbers together with 0 form whole numbers. The set of whole number is denoted by 'W'.

   For example: Set of whole numbers, W. = {0, 1, 2, 3, ...}

3. **Integers:** The collection of all whole numbers i.e. positive, zero and negative numbers, are called integers. The set of integers is denoted by Z or I.

   For example: Set of Integers, Z or I = {..., –3, –2, –1, 0, 1, 2, 3, ... }

4. **Rational numbers**: A number '*r*' is called a rational number, if it can be written in the form *p*/*q*, where *p* and *q* are integers and $q \neq 0$. The set of rational numbers is denoted by Q.

   For example: Set of rational numbers, $Q=\left\{\frac{p}{q}: p, q \in I, q \neq 0\right\}$

NOTE:

The rational numbers also include the natural numbers, whole numbers and integers.

Equivalent Rational Numbers

Two rational numbers are said to be equivalent, if both the numerators and denominators are in proportion or they are reducible to be equal. The rational numbers do not have a unique representation in the form *p*/*q*, where *p* and *q* are integers and $q \neq 0$.

For example: $\frac{1}{2}=\frac{2}{4}=\frac{10}{20}=\frac{25}{50}=\frac{47}{94}$, and so on.

5. **Irrational numbers**: A number which is not rational or cannot be represented in the form $\frac{p}{q}$, is called an irrational number.

   For example: $\sqrt{2}, \sqrt{3}, \pi$ etc.

ILLUSTRATION

Q.1 Are the following statements true or false? Give reasons for your answers.

(i) Every whole number is a natural number.

(ii) Every integer is a rational number.

(iii) Every rational number is an integer.

Sol. (i) False, because zero is a whole number but not a natural number.

(ii) True, because very integer m can be expressed in the form $\frac{m}{1}$, and so it is a rational number.

(iii) False, because $\frac{3}{5}$ is not an integer.

PRACTICE PROBLEMS

1. State whether the following statements are true or false? Give reasons for your answers.
(i) Every natural number is a whole number. (ii) Every integer is a whole number
(iii) Every rational number is a whole number.

• REAL NUMBER AND THEIR DECIMAL EXPANSIONS •

1. **Real Numbers:** The set of rational numbers and irrational numbers form a set of real numbers. The set of real numbers is denoted by R.

NOTE:

Every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.

2. **Decimal Expansions of Real Numbers:** The decimal expansions of real numbers can be used to distinguish between terminating and non terminating (recurring and non-recurring) numbers.

(a) Terminating Decimal Expansions: In this case, the decimal expansion terminates or ends after a finite number of steps. We call such a decimal expansion as terminating.

For example: Decimal expansions of $\frac{7}{8} = 0.875$

$$\begin{array}{r|l} & 0.875 \\ \hline 8 & 7.0 \\ & 64 \\ \hline & \;\;60 \\ & \;\;56 \\ \hline & \;\;\;\;40 \\ & \;\;\;\;40 \\ \hline & \;\;\;\;\;\;0 \end{array}$$

(b) Non-terminating Recurring Decimal Expansions: In this case, we have a repeating block of digits in the quotient. We say that this expansion is non-terminating recurring.

For example: (i) Decimal expansion of $\frac{10}{3} = 3.33333....$ $\frac{1}{7} = 0.142857142857142857.....$

$$\begin{array}{r|l} & 3.33333...... \\ \hline 3 & 10 \\ & \;9 \\ \hline & \;\;10 \\ & \;\;\;9 \\ \hline & \;\;\;10 \\ & \;\;\;\;9 \\ \hline & \;\;\;\;10 \\ & \;\;\;\;\;9 \\ \hline & \;\;\;\;\;\;1 \end{array}
\qquad
\begin{array}{r|l} & 0.142857 \\ \hline 7 & 1.0 \\ & \;7 \\ \hline & \;30 \\ & \;28 \\ \hline & \;\;20 \\ & \;\;14 \\ \hline & \;\;\;60 \\ & \;\;\;56 \\ \hline & \;\;\;40 \\ & \;\;\;35 \\ \hline & \;\;\;\;50 \\ & \;\;\;\;49 \\ \hline & \;\;\;\;\;1 \end{array}$$

How to Write Non-terminating Recurring Expansions in Short

The usual way of showing that 3 repeats in the quotient of $\frac{1}{3}$ is to write it as $0.\overline{3}$. Similarly, since blocks of digits 142857 repeats in the quotient of $\frac{1}{7}$, we write $\frac{1}{7}$ as $0.\overline{142857}$, where the bar above the digits indicates the block of digits that repeats.

(c) Non-terminating Non Recurring Decimal Expansions: In this case, we have a non repeating block of digits after the decimal.

For example: 5.2345654345678........., 0.298456712345.........

NOTE:

(a) The decimal expansion of a rational number is either terminating or non-terminating recurring. Moreover, a number whose decimal expansion is terminating or non-terminating recurring is rational.

(b) The decimal expansion of an irrational number is non-terminating non-recurring. Moreover, a number whose decimal expansion is non-terminating non-recurring is irrational.

ILLUSTRATION

Q.2 Check whether $7\sqrt{5}, \frac{7}{\sqrt{5}}, \sqrt{2}+21, \pi-2$ are irrational numbers or not using their decimal expansions.

Sol. $\sqrt{5} = 2.236..., \sqrt{2} = 1.4142..., \pi = 3.1415......$

Then, $7\sqrt{5} = 15.652..., \frac{7}{\sqrt{5}} = \frac{7\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{7\sqrt{5}}{5} = 3.1304....,\ \sqrt{2} + 21 = 22.4142....,\ \pi - 2 = 1.1415....$

All these are non-terminating non-recurring decimals. So, all these are irrational numbers.

NUMBERS BETWEEN ANY TWO GIVEN NUMBERS

1. **Rational number between two given rational number:** In general, there are infinitely many rational numbers between any two given rational numbers.

Method I: To find a rational number between s and t, add the two number and divide the result by 2, i.e. $\frac{s+t}{2}$ lies between s and t. Proceeding in this manner, we may find more rational numbers between s and t.

Method II: We can write the decimal equivalent of the numbers and can find numbers between them.

ILLUSTRATION

Q.3 Prove that between two distinct rational numbers a and b, there exists another rational number.

Sol. Since $a \neq b$, therefore, let us assume that $a < b$.

Now, $a < b$

By adding 'a' on both side, we get $a + a < b + a$

$\Rightarrow\ 2a < a + b\ \Rightarrow\ a < \frac{a+b}{2}$ ...(1)

Again from $a < b$, by adding 'b' on both sides, we get $\frac{a+b}{2} < b$ ...(2)

Combining (1) and (2), we get $a < \frac{a+b}{2} < b$

Since, a, b and 2 ($\neq 0$) are rational numbers.

$\therefore\ \frac{a+b}{2}$ is also a rational number.

Thus, there exists another rational number between two distinct rational numbers a and b.

Q.4 Find a rational number lying between $\frac{4}{5}$ and $\frac{5}{7}$.

Sol. A rational number lying between $\frac{4}{5}$ and $\frac{5}{7}$ is $\frac{\frac{4}{5}+\frac{5}{7}}{2} = \frac{\frac{53}{35}}{2} = \frac{53}{70}$.

Alternatively: $\frac{4}{5} = 0.8$ and $\frac{5}{7} = 0.714285$, so a rational number lying between them is 0.72 or $\frac{72}{100}$ or $\frac{18}{25}$

PRACTICE PROBLEMS

2. Find three rational numbers between $\frac{2}{3}$ and $\frac{3}{2}$.

3. Find five rational numbers between $0.\overline{45}$ and $0.\overline{569}$.

2. Irrational Numbers Between Two Given Rational Numbers

ILLUSTRATION

Q.5 Find two irrational numbers between 0.14 and 0.15.

Sol. The two irrational numbers between 0.14 and 0.15 can be taken as

0.14101001000100001...... and 0.14201001000100001......

Q.6 Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Sol. $\frac{5}{7} = 0.714285\ ...... = 0.\overline{714285}$; $\frac{9}{11} = 0.8181...... = 0.\overline{81}$

$\therefore$ Three different irrational numbers between the rational numbers are

0.75075007500075000075......., 0.7670767000767.........., 0.808008000800008...........,

PRACTICE PROBLEMS

4. Find three irrational numbers between $\frac{3}{5}$ and $\frac{5}{3}$.

5. Find five irrational numbers between $0.\overline{23}$ and $0.\overline{249}$.

3. Conversion Of A Decimal Number Into Rational Number Of The Form $\frac{p}{q}$.

I. When the decimal expansion is terminating.

In order to convert a rational number having finite number of digits after the decimal point, follow the steps:

Step I: Determine the digits in the decimal part of given decimal number.

Step II: Remove decimal point from the numerator. Write 1 in the denominator and put as many zeroes on the right side of 1 as the number of digits in the decimal part of the given rational number.

Step III: Express the number in its simplest form.

ILLUSTRATION

Q.7 Express the numbers in the form $\frac{p}{q}$. (i) 0.12 (ii) 0.545 (iii) 0.00034

Sol. (i) We have $0.12 = \frac{12}{100} = \frac{3}{25}$ (ii) $0.545 = \frac{545}{1000} = \frac{109}{200}$ (iii) $0.00034 = \frac{34}{100000} = \frac{17}{50000}$

II. When the decimal expansion is non-terminating recurring.

In a non terminating repeating decimal, there are two types of decimal representations.

(i) A decimal in which all the digits after the decimal point are repeated are known as pure recurring decimals.

For example: $0.\overline{4}, 0.\overline{32}, 0.\overline{754}$

(ii) A decimal in which atleast one of the digits after the decimal point is not repeated and then some digit or digits are repeated are known as mixed recurring decimals.

For example: $0.3\overline{4}, 0.5\overline{17}, 0.23\overline{732}$

II A. Conversion Of A Pure Recurring Decimal Number Into The Form $\frac{p}{q}$ (The X-Method)

In order to convert a pure recurring decimal to the form $\frac{p}{q}$, follow the steps:

Step I : Assume the given decimal number as x.

Step II: Write the number in decimal form by removing bar from the top of repeating digits and listing repeating digits atleast twice. For example, write x = $0.\overline{4}$ as x = 0.444..... or x = $0.\overline{23}$ as x = 0.232323......

Step III: Determine the number of digits which are repeating.

Step IV: If the repeating decimal has 1 place repetition, multiply equation in Step II by 10; a two place repetition, multiply by 100: a three place repetition, multiply it by 1000 ans so on.

Step V: Subtract the equation in step II from the equation obatined in step IV.

Step VI: Divide both side of the equation by the coefficient of x.

Step VII: Write the rational number in its simplest form.

ILLUSTRATION

Q.8 Convert $0.\overline{3}$ in the form $\frac{p}{q}$.

Sol. Let x = 0.3333.....(= $0.\overline{3}$) .... (i)

Since one digit repeats, we multiply, Eq (i) by 10 to get,

10x = 10 × (0.3333.....) = 3.333.... .... (ii)

Subtracting Eq (i) from Eq (ii), we get

$\Rightarrow$ 10x – x = 3.3333...... – 0.3333.....

$\Rightarrow 9x = 3 \quad \Rightarrow x = \frac{3}{9} \quad \Rightarrow x = \frac{1}{3}$

Q.9 Convert $0.\overline{27}$ in the form $\frac{p}{q}$.

Sol. Let x = 0.272727.....(= $0.\overline{27}$) .... (i)

Since two digit repeats, we multiply, Eq (i) by 100 to get,

100x = 100 × (0.272727....) = 27.2727...... ....(ii)

Subtracting Eq (i) from Eq (ii), we get

$\Rightarrow$ 100x – x = 27.2727...... – 0.272727.....

$\Rightarrow 99x = 27 \Rightarrow x = \frac{27}{99} \quad \Rightarrow x = \frac{3}{11}$

II B. Conversion of a mixed recurring decimal number into the form $\frac{p}{q}$ (The Y-Method)

In order to convert a mixed recurring decimal to the form $\frac{p}{q}$, follow the steps:

Step I : Assume the given decimal number as y.

Step II: Write the number in decimal form by removing bar from the top of repeating digits and listing repeating digits atleast twice. For example, write x = $2.1\overline{3}$ as x = 2.1333..... or x = $34.7\overline{26}$ as x = 34.7262626......

Step III: Determine the number of digits which are in between decimal and repeating first digit.

Step IV: If the number of digit in between is 1, multiply equation in Step II by 10, number of digits in between is 2, multiply equation by 100 and so on to bring the decimal place adjacent to the repeating number.

Step V: Determine the number of digits which are repeating.

Step VI: If the repeating decimal has 1 place repetition, multiply equation in Step IV by 10; a two place repetition, multiply by 100, a three place repetition, multiply it by 1000 ans so on.

Step VII: Subtract the equation in step IV from the equation obatined in step VI.

Step VIII: Divide both side of the equation by the coefficient of x.

Step IX: Write the rational number in its simplest form.

ILLUSTRATION

Q.10 Convert $0.4\overline{7}$ in the form $\frac{p}{q}$.

Sol. Let y = $0.4\overline{7}$ = 0.47777........

Since one digit is in between decimal and repeating first digit, therefore multiplying both sides by 10, we get

10y = 4.7777....... ....(i)

Since one digit repeats, therefore we again multiplying both sides of Eq (i) by 10, to get

100y = 47.777....... ....(ii)

Subtracting Eq (i) from Eq (ii), we get

100y – 10y = 47.777...... – 4.7777.......

$90y = 43 \Rightarrow y = \frac{43}{90}$.

Q.11 Convert $2.3\overline{65}$ in the form $\frac{p}{q}$.

Sol. Let y = $2.3\overline{65}$ = 2.3656565......

since one digit is in between decimal and repeating first digit, therefore multiplying both sides by 10, we get

10y = 23.656565....... ....(i)

since two digits repeats, therefore we again multiplying both sides of Eq (i) by 100, to get

1000y = 2365.6565..... ....(ii)

Subtracting Eq (i) from Eq (ii), we get

1000y – 10y = 2365.6565..... – 23.656565.......

$990y = 2342 \Rightarrow y = \frac{2342}{990}$.

4. Representation of Irrational Numbers on the Number Line

As we know that, every real number can be represented on number line, so along with rational numbers, irrational numbers can also be represented on number line.

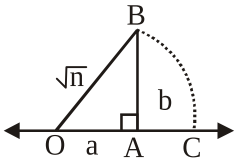
We can locate some of the irrational numbers on the number line e.g., $\sqrt{2}, \sqrt{3}, \sqrt{3.5}, \sqrt{9.5},$ ... etc.

5. Representation of $\sqrt{n}$, where n is a whole number

For representing, $\sqrt{n}$, where n is a whole number, on the number line, we use the following steps

Step I: Write the given number (without root) as the sum of the squares of two natural numbers (say a and b).

Step II : Take the distance equal to these two natural numbers (a and b) on the number line starting from O (say OA and AB) in such a way that one is perpendicular to other (say $AB \perp OA$).



Step III: Use Pythagoras theorem to find the distance OB.

Step IV: Take O as centre and OB as radius, draw an arc, which cut the number line at C (say).

Thus, the point C will represent the location of $\sqrt{n}$ on the number line.

ILLUSTRATION

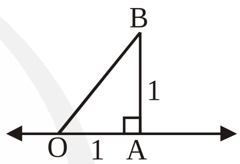
Q.12 Locate $\sqrt{2}$ on the number line.

Sol. ***Step I:*** Write the given number (without root) as the sum of the square of two natural numbers.

Here, $2 = 1 + 1 = 1^2 + 1^2$

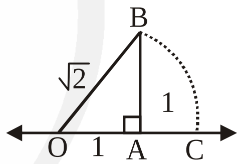
Step II: Draw these two natural numbers on the number line, in which one is perpendicular to other.

Draw OA = 1 units and AB = 1 units, such that $AB \perp OA$



Step III: By using Pythagoras theorem, find OB. $OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$

Step IV: Take O as centre and draw an arc of radius OB.



Having O as centre & radius equal to OB, draw an arc, which cuts the number line at C. OC corresponds to $\sqrt{2}$.

Hence, OC represent $\sqrt{2}$.

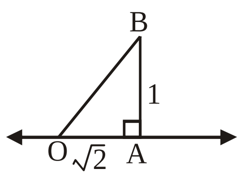
Q.13 Locate $\sqrt{3}$ on the number line.

Sol. ***Step I:*** Write the given number (without root) as the sum of the square of two natural numbers.

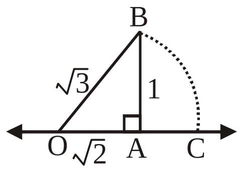
Here, $3 = (\sqrt{2})^2 + (1)^2$

Step II: Draw these two natural numbers on the number line, in which one is perpendicular to other.

Draw OA = $\sqrt{2}$ units and AB = 1 units, such that $AB \perp OA$



Step III: By using Pythagoras theorem, find OB. $OB = \sqrt{OA^2 + AB^2} = \sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{2+1} = \sqrt{3}$



Step IV: Take O as centre and draw an arc of radius OB.

Having O as centre and radius equal to OB, draw an arc, which cuts the number line at C. OC corresponds to $\sqrt{3}$.

Hence, OC represent $\sqrt{3}$.

PRACTICE PROBLEMS

6. Represent $\sqrt{5}, \sqrt{11}$ on number line.

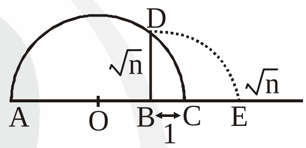
6. Representation of $\sqrt{n}$, where n is a decimal number

To represent $\sqrt{n}$, (where n is any positive number) on the number line, we use the following steps

Step I: Mark the distance n units from a fixed point (say) A on a given line to obtain a point B, so that AB = n units.

Step II: From point B, take a distance of 1 unit and mark the new point C.

Step III: Find the mid-point of AC and mark that point as O.



Step IV: Draw a semi-circle with centre O and radius OC.

Step V: Draw a line perpendicular to AC passing through B and intersecting the semi-circle at D. Then, BD = $\sqrt{n}$

Step VI: Produced the line BC as the number line with B as zero, C as 1 unit and so on.

Step VII: Draw an arc with B as centre and radius BD, which intersects the number line at E.

Hence, E represents $\sqrt{n}$.

ILLUSTRATI0N

Q.14 Represent $\sqrt{3.5}$ on the number line.

Sol. To represent $\sqrt{3.5}$ on number line we follow the steps below.

Step I: First, mark the distance 3.5 from point A to B on the number line.
Mark the distance 3.5 units from a fixed point A on a given line and take point B, such that AB=3.5 units.

Step II: Mark a distance 1 unit from Point B.
Take a distance 1 unit from B and mark the point C

Step III: Mark the mid-point of AC.
Mark the mid-point of AC as O.

Step IV: Draw a semi-circle.
Draw a semi-circle with centre O and radius OC

Step V: Draw a line from point D to AB.

Draw BD perpendicular to AC and intersecting the semi-circle at D. Then, BD = $\sqrt{3.5}$

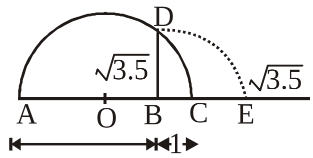
Step VI: Produce the line BC as the number line with B on as zero.

Now, treat BC as the number line with B as zero. C as 1 unit and so on.

Step VII: Draw an arc having centre B and radius BD.

Draw an arc with centre B and radius BD which intersects the number line at E.

Hence, E represent on the number line



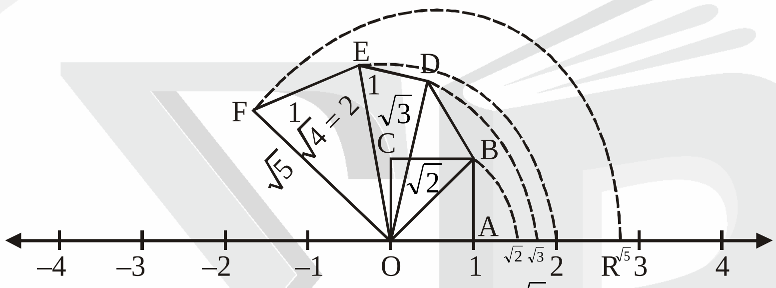
PRACTICE PROBLEMS

7. Represent $\sqrt{5.3}, \sqrt{9.4}$ on number line.

7. Representation of Irrational Number using Spiral Method

ILLUSTRATION

Q.15 Represent $\sqrt{5}$ on number line using spiral method

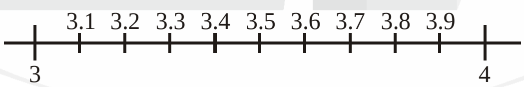


Representation of $\sqrt{5}$

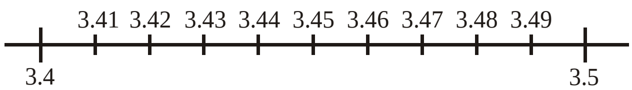
8. Representation of rational numbers on the number line through successive magnification

Let us try to represent 3.47 on the number line.

We know that 3.4 lies between 3 and 4. We divide the portion between 3 and 4 into 10 equal parts as below:



Now, 3.47 lies between 3.4 and 3.5. Again we divide the portion between 3.4 and 3.5 into 10 equal parts.



Now, we can easily locate 3.47 on the number line.

In the above method, we have successively magnified different portions to represent 3.47 on the number line.

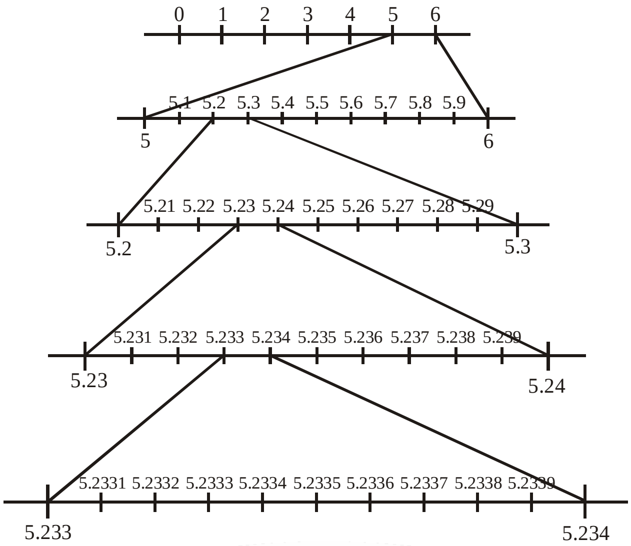
This method of representation of real number on the number line is known as method of successive magnification.

ILLUSTRATION

Q.16 Represent $5.2\overline{3}$ on the number line using successive magnification (upto 4 places of decimal).

Sol. $5.2\overline{3}$ lies between 5 and 6.To represent 5.2333

∴ We divide portion between 5 and 6 into 10 equal parts and go on successively magnifying as follows:



NOTE:

The numbers of times we successively magnify determines the level of accuracy of representation.

PRACTICE PROBLEMS

8. Visualise 3.765 on the number line, using successive magnification.

9. Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.

• OPERATIONS ON RATIONAL NUMBERS AND IRRATIONAL NUMBERS •

1. **Operations on Rational Numbers:** Rational numbers satisfy the commutative, associative and distributive laws for addition and multiplication.

2. **Operations on Irrational Numbers:** Irrational numbers also satisfy the commutative, associative and distributive laws for addition and multiplication.

Some useful facts :

(a) The sum or difference of a rational number and an irrational number is irrational.

(b) The product or quotient of a non-zero rational number with an irrational number is irrational.

(c) If we add, subtract, multiply or divide two irrationals, the result may be rational or irrational.

3. **Square Root of Real Numbers:** We know that, if a is a natural number, then $\sqrt{a} = b$ means $b^2 = a$ and $b \geq 0$. The same definition can be extended for positive real numbers.

Let $a > 0$ be a real number. Then $\sqrt{a} = b$ means $b^2 = a$ and $b \geq 0$.

n^{th} Root of a Real Number: Let $a > 0$ be a real number and n be a positive integer.

Then $\sqrt[n]{a} = b,$ if $b^n = a$ and $b > 0$.

For example: $\sqrt[3]{64} = 4 \text{ as } 4^3 = 64$ $\quad \sqrt[5]{243} = 3 \text{ as } 3^5 = 243$

4. **Some Identities Related to Square Roots:**

Let a and b be positive real numbers. Then

(i) $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$ (ii) $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

(iii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=a-b$

(iv) $(a+\sqrt{b})(a-\sqrt{b})=a^2-b$

(v) $(\sqrt{a}+\sqrt{b})(\sqrt{c}-\sqrt{d})=\sqrt{ac}-\sqrt{ad}+\sqrt{bc}-\sqrt{bd}$

(vi) $(\sqrt{a}+\sqrt{b})^2=a+2\sqrt{ab}+b.$

5. Simplest Form of Irrational Numbers (Square roots)

A number having square root can be simplified (if possible) by using its prime factors.

For example:

$\sqrt{8}=\sqrt{2\times2\times2}=2\sqrt{2}$ (from the square root we can take out one factor of a pair and remove the pair)

$\sqrt{12}=\sqrt{2\times2\times3}=2\sqrt{3}$

$\sqrt{18}=\sqrt{2\times3\times3}=3\sqrt{2}$

$\sqrt{28}=\sqrt{2\times2\times7}=2\sqrt{7}$

$\sqrt{40}=\sqrt{2\times2\times2\times5}=2\sqrt{10}$

$\sqrt{80}=\sqrt{2\times2\times2\times2\times5}=2\times2\sqrt{5}=4\sqrt{5}$

PRACTICE PROBLEMS

10. Rewrite the simplest form of the given irrational number:

(i) $\sqrt{50}$ (ii) $\sqrt{60}$ (iii) $\sqrt{24}$ (iv) $\sqrt{32}$ (v) $\sqrt{200}$ (vi) $\sqrt{486}$

6. Addition and Subtraction of Irrational numbers

Step I: Simplify all the terms of the given question.

Step II: Identify the Like terms. (Like terms have same numbers in their radical sign. Eg. $\sqrt{2}, 3\sqrt{2}, 7\sqrt{2}, 9\sqrt{2}$)

Step III: Add/Subtract Like terms together.

ILLUSTRATI0N

Q.17 Add $2\sqrt{2}+5\sqrt{3}$ and $\sqrt{2}-3\sqrt{3}.$

Sol. $(2\sqrt{2}+5\sqrt{3})+(\sqrt{2}-3\sqrt{3})$

Step I: Simplify all the terms, here all the terms are in their simplest form.

Step II: Identify Like terms: $2\sqrt{2}$, $\sqrt{2}$ are Like terms, $5\sqrt{3}, -3\sqrt{3}$ are Like terms

Step III: $2\sqrt{2}+\sqrt{2}+5\sqrt{3}-3\sqrt{3} = 3\sqrt{2}+2\sqrt{3}$

Q.18 Add $5\sqrt{2}-\sqrt{18}+\sqrt{12}+6\sqrt{3}$

Sol. $\sqrt{12}=\sqrt{2\times2\times3}=2\sqrt{3}$ and $\sqrt{18}=\sqrt{2\times3\times3}=3\sqrt{2}$

So, $5\sqrt{2}-3\sqrt{2}+2\sqrt{3}+6\sqrt{3} = 2\sqrt{2}+8\sqrt{3}$

PRACTICE PROBLEMS

11. **Simplify** $3\sqrt{45}-\sqrt{125}+\sqrt{200}-\sqrt{50}$

12. **Simplify** $5\sqrt{2}-\sqrt{18}+\sqrt{12}+6\sqrt{3}$

7. Multiplication/Division of Irrational Number

Step I: Simplify all the terms.

Step II: Multiply/Divide rational part with rational part and irrational part with irrational part.

ILLUSTRATION

Q.19 Multiply $7\sqrt{2}\times 5\sqrt{3}$

Sol. $7\sqrt{2}\times 5\sqrt{3} = 35\sqrt{6}$ (Multiply the rational parts 7 with 5 = 35, and irrational parts $\sqrt{2}$ with $\sqrt{3} = \sqrt{6}$)

Q.20 Multiply $12\sqrt{5}\times 3\sqrt{7}$

Sol. $12\sqrt{5}\times 3\sqrt{7} = 36\sqrt{35}$

Q.21 Multiply $4\sqrt{2}\times 3\sqrt{6}$

Sol. $4\sqrt{2}\times 3\sqrt{6} = 12\sqrt{12} = 12\sqrt{3\times 2\times 2} = 12\times 2\sqrt{3} = 24\sqrt{3}$

Q.22 Divide $8\sqrt{15}$ by $2\sqrt{3}$.

Sol. $8\sqrt{15}\div 2\sqrt{3} = \dfrac{8\sqrt{3}\times\sqrt{5}}{2\sqrt{3}} = 4\sqrt{5}$

Q.23 Multiply $5\sqrt{2}(3\sqrt{7}+8\sqrt{6})$

Sol. $5\sqrt{2}(3\sqrt{7}+8\sqrt{6}) = 5\sqrt{2}\times 3\sqrt{7}+5\sqrt{2}\times 8\sqrt{6} = 15\sqrt{14}+40\sqrt{12}$

$= 15\sqrt{14}+40\sqrt{2\times 2\times 3}$

$= 10\sqrt{14}+40\sqrt{12}$

$= 10\sqrt{14}+40\sqrt{2\times 2\times 3} = 10\sqrt{14}+40\times 2\sqrt{3}$

$= 10\sqrt{14}+80\sqrt{3}$

Q.24 Multiply $7\sqrt{2}(3\sqrt{2}+9\sqrt{3})$

Sol. $7\sqrt{2}(3\sqrt{2}+9\sqrt{3})$

$= 7\sqrt{2}\times 3\sqrt{2}+7\sqrt{2}\times 9\sqrt{3}$

$= 21\sqrt{4}+63\sqrt{6}$

$= 21\sqrt{2\times 2}+63\sqrt{6}$

$= 42+63\sqrt{6}$

Q.25 Multiply $(3+\sqrt{3})(2+\sqrt{2})$

Sol. $(3+\sqrt{3})(2+\sqrt{2})$

$= 3\times 2+3\times\sqrt{2}+\sqrt{3}\times 2+\sqrt{3}\times\sqrt{2}$

$= 6+3\sqrt{2}+2\sqrt{3}+\sqrt{3\times 2} = 6+3\sqrt{2}+2\sqrt{3}+6$

$= 12+3\sqrt{2}+2\sqrt{3}$

Q.26 Multiply $(5+\sqrt{5})(5-\sqrt{5})$

Sol. $(5+\sqrt{5})(5-\sqrt{5}) = (5)^2-(\sqrt{5})^2 = 25-5 = 20$

Q.27 Multiply $(3+2\sqrt{2})(3-2\sqrt{2})$

Sol. $(3+2\sqrt{2})(3-2\sqrt{2}) = 3^2-(2\sqrt{2})^2 = 9-4\times(\sqrt{2})^2 = 9-4\times 2 = 9-8 = 1$

PRACTICE PROBLEMS

13. Multiply the given expressions:

a. $2\sqrt{5}\times 5\sqrt{2}$ **b.** $3\sqrt{2}\times 7\sqrt{2}$ **c.** $3\sqrt{2}\times(5\sqrt{3}+\sqrt{7})$ **d.** $7\sqrt{3}\times(2\sqrt{2}+3\sqrt{5})$

e. $(4+\sqrt{7})(3+\sqrt{2})$ **f.** $(3+\sqrt{3})(5-\sqrt{2})$ **g.** $(11+\sqrt{11})(11-\sqrt{11})$ **h.** $(5+\sqrt{7})(5-\sqrt{7})$

• RATIONALISING THE DENOMINATOR •

When the denominator of an expression contains a term with a square root, the procedure of converting it to an equivalent expression whose denominator is a rational number is called rationalising the denominator.

For example: $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ and $\frac{1}{\sqrt{5}+1} = \frac{1}{\sqrt{5}+1}\times\frac{\sqrt{5}-1}{\sqrt{5}-1} = \frac{\sqrt{5}-1}{4}$

I. Rationalising the expression having one term in its Denominator (One Term Denominator)

Step I: Multiply and divide the given expression by the irrational part of the denominator.

Step II: Simplify the expression to find an equivalent expression.

ILLUSTRATION

Q.28 Rationalise the denominator of $\frac{5}{\sqrt{3}}$

Sol. $\frac{5\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}} = \frac{5\sqrt{3}}{3}$

Q.29 Rationalise the denominator of $\frac{16}{\sqrt{5}}$

Sol. $\frac{16\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}} = \frac{16\sqrt{5}}{5}$

Q.30 Rationalise the denominator of $\frac{3+\sqrt{2}}{3\sqrt{5}}$

Sol. $\frac{(3+\sqrt{2})\times\sqrt{5}}{3\sqrt{5}\times\sqrt{5}} = \frac{3\sqrt{5}+\sqrt{10}}{3\times 5} = \frac{3\sqrt{5}+\sqrt{10}}{15}$

II. Rationalising the expression having two terms in its Denominator (Two Terms denominator)

Step I: Find the conjugate to make rationalising factor of the denominator. Conjugate is found by changing the sign between the two given terms. For example: conjugate of $\sqrt{3}+\sqrt{2}$ is $\sqrt{3}-\sqrt{2}$**,** conjugate of $\sqrt{7}-3$ is $\sqrt{7}+3$

Step II: Multiply and divide the given expression by the conjugate of the denominator.

Step III: Simplify the expression to find an equivalent expression.

Q.31 Rationalise the denominator of $\frac{5}{\sqrt{3}-\sqrt{5}}$

Sol. $\frac{5}{\sqrt{3}-\sqrt{5}} = \frac{5}{\sqrt{3}+\sqrt{5}}\times\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}} = \frac{5(\sqrt{3}+\sqrt{5})}{3-5} = \left(\frac{-5}{2}\right)(\sqrt{3}+\sqrt{5})$

Q.32 Rationalise the denominator of $\frac{1}{7+3\sqrt{2}}$.

Sol. $\frac{1}{7+3\sqrt{2}} = \frac{1}{7+3\sqrt{2}} \times \left(\frac{7-3\sqrt{2}}{7-3\sqrt{2}}\right) = \frac{7-3\sqrt{2}}{49-18} = \frac{7-3\sqrt{2}}{31}$

Q.33 Rationalise the denominator of $\frac{5}{\sqrt{3}-\sqrt{5}}$.

Sol. We have, $\frac{5}{\sqrt{3}-\sqrt{5}}$, So, multiplying the numerator & denominator by rationalisation factor (conjugate) of denominator,

$$\frac{5}{\sqrt{3}-\sqrt{5}} = \frac{5}{\sqrt{3}-\sqrt{5}} \times \frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}} = \frac{5(\sqrt{3}+\sqrt{5})}{3-5} = -\frac{5}{2}(\sqrt{3}+\sqrt{5})$$

Q.34 Simplify each of the following by rationalising the denominator:

(i) $\frac{5+\sqrt{6}}{5-\sqrt{6}}$ (ii) $\frac{7+3\sqrt{5}}{7-3\sqrt{5}}$ (iii) $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$

Sol. (i) Multiplying the numerator & denominator by rationalisation factor (conjugate) $5+\sqrt{6}$ of denominator, we get

$$\frac{5+\sqrt{6}}{5-\sqrt{6}} = \frac{5+\sqrt{6}}{5-\sqrt{6}} \times \frac{5+\sqrt{6}}{5+\sqrt{6}} = \frac{(5+\sqrt{6})^2}{5^2-(\sqrt{6})^2} = \frac{25+6+2\times 5\times\sqrt{6}}{25-6} = \frac{31\times 10\sqrt{6}}{19}$$

(ii) Multiplying and dividing by the rationalisation factor (conjugate) of denominator, we have

$$\frac{7+3\sqrt{5}}{7-3\sqrt{5}} = \frac{7+3\sqrt{5}}{7-3\sqrt{5}} \times \frac{7+3\sqrt{5}}{7+3\sqrt{5}} = \frac{(7+3\sqrt{5})^2}{72-(3\sqrt{5})^2} = \frac{49+(3\sqrt{5})^2+2\times 7\times 3\sqrt{5}}{49-45}$$

$$= \frac{49+45+42\sqrt{5}}{4} = \frac{94+42\sqrt{5}}{4} = \frac{47+21\sqrt{5}}{2}$$

(iii) Multiplying and dividing by the rationalisation factor of denominator, we have

$$\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} = \frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}} \times \frac{2\sqrt{2}-3\sqrt{3}}{2\sqrt{2}-3\sqrt{3}} = \frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{3})}$$

$$= \frac{2\sqrt{3}\times 2\sqrt{2}-2\sqrt{3}\times 3\sqrt{3}-\sqrt{5}\times 2\sqrt{2}+\sqrt{5}\times 3\sqrt{3}}{(2\sqrt{2})^2-(3\sqrt{3})^2} = \frac{4\sqrt{3\times 2}-6\sqrt{3\times 3}-2\sqrt{5\times 2}+3\sqrt{5\times 3}}{4\times 2-9\times 3}$$

$$= \frac{4\sqrt{6}-6\times 3-2\sqrt{10}+3\sqrt{15}}{8-27} = \frac{4\sqrt{6}-18-2\sqrt{10}+3\sqrt{15}}{-19} = \frac{18+2\sqrt{10}-4\sqrt{6}-3\sqrt{15}}{19}$$

III. Applications of Rationalising the denominator

Q.35 Find the value of a and b in the given expressions

(i) $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$ (ii) $\frac{3+\sqrt{7}}{3-\sqrt{7}} = a+b\sqrt{7}$ (iii) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = a+b\sqrt{15}$ (iv) $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a-b\sqrt{6}$

Sol. (i) Rationalising the denominator, we have

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}=\frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}=\frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-1^2}$$

$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1}=\frac{(\sqrt{3})^2+1-2\sqrt{3}}{(\sqrt{3})^2-1}=\frac{3+1-2\sqrt{3}}{3-1}=\frac{4-2\sqrt{3}}{2}=2-\sqrt{3}$$

$$\therefore \frac{\sqrt{3}-1}{\sqrt{3}+1}=a+b\sqrt{3} \Rightarrow 2-\sqrt{3}=a+b\sqrt{3} \Rightarrow a+b\sqrt{3}=2+(-1)\sqrt{3} \Rightarrow a = 2 \text{ and } b = -1$$

[On equating rational and irrational parts]

(ii) Rationalising the denominator, we have

$$\frac{3+\sqrt{7}}{3-\sqrt{7}}=\frac{3+\sqrt{7}}{3-\sqrt{7}}\times\frac{3+\sqrt{7}}{3+\sqrt{7}}=\frac{(3+\sqrt{7})(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}=\frac{(3+\sqrt{7})^2}{3^2-(\sqrt{7})^2}$$

$$\Rightarrow \frac{3+\sqrt{7}}{3-\sqrt{7}}=\frac{3^2+(\sqrt{7})^2+2\times3\times\sqrt{7}}{9-7}=\frac{9+7+6\sqrt{7}}{9-7}=\frac{16+6\sqrt{7}}{2}=8+3\sqrt{7}$$

$$\therefore \frac{3+\sqrt{7}}{3-\sqrt{7}}=a+b\sqrt{7} \Rightarrow 8+3\sqrt{7}=a+b\sqrt{7} \Rightarrow a+b\sqrt{7}=8+3\sqrt{7} \Rightarrow a = 8 \text{ and } b = 3$$

(iii) Rationalising the denominator, we get

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2} \Rightarrow \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{(\sqrt{5})^2+(\sqrt{3})^2+2\sqrt{5}+\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$\Rightarrow \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{5+3+2\sqrt{5\times3}}{5-3}=\frac{8+2\sqrt{15}}{2}=4+\sqrt{15}$$

$$\therefore \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=a+b\sqrt{15} \Rightarrow 4+\sqrt{15}=a+b\sqrt{15} \Rightarrow a = 4 \text{ and } b = 1$$

(iv) Rationalising the denominator, we have

$$\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}\times\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}=\frac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})}$$

$$\Rightarrow \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=\frac{\sqrt{2}\times3\sqrt{2}+\sqrt{2}\times2\sqrt{3}+\sqrt{3}\times3\sqrt{2}+\sqrt{3}\times2\sqrt{3}}{(3\sqrt{2})^2-(2\sqrt{3})^2}$$

$$\Rightarrow \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=\frac{3\sqrt{2\times2}+2\sqrt{3\times2}+3\sqrt{3\times2}+2\sqrt{3\times3}}{9(\sqrt{2})^2-4(\sqrt{3})^2}$$

$$\Rightarrow \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=\frac{3\times2+2\sqrt{6}+3\sqrt{6}+2\times3}{9\times2-4\times3}$$

$$\Rightarrow \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=\frac{6+(2+3)\sqrt{6}+6}{18-12}=\frac{12+5\sqrt{6}}{6}=2+\frac{5}{6}\sqrt{6}$$

$$\therefore \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a - b\sqrt{6} \Rightarrow \frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a - b\sqrt{6} \Rightarrow a - b\sqrt{6} = 2-(-5/6)\sqrt{6} \Rightarrow a = 2 \text{ and } b = -5/6$$

Q.36 Prove that: $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}=2$

Sol. Rationalising the denominator of each term on LHS, we have

$$\text{LHS} = \frac{1-\sqrt{2}}{1-2}+\frac{\sqrt{2}-\sqrt{3}}{2-3}+\frac{\sqrt{3}-\sqrt{4}}{3-4}+\frac{\sqrt{4}-\sqrt{5}}{4-5}+\frac{\sqrt{5}-\sqrt{6}}{5-6}+\frac{\sqrt{6}-\sqrt{7}}{6-7}+\frac{\sqrt{7}+\sqrt{8}}{7-8}+\frac{\sqrt{8}-\sqrt{9}}{8-9}$$

$$= -1+\sqrt{2}-\sqrt{2}+\sqrt{3}-\sqrt{3}+\sqrt{4}-\sqrt{4}+\sqrt{5}-\sqrt{5}+\sqrt{6}-\sqrt{6}+\sqrt{7}-\sqrt{7}+\sqrt{8}-\sqrt{8}+\sqrt{9}$$

$$= -1+\sqrt{9} = -1+3 = 2 = \text{RHS}.$$

Q.37 Evaluate $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$, is being given that $\sqrt{5}=2.236$ and $\sqrt{10} = 3.162$

Sol. We have, $\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}$

$$= \sqrt{10}+\sqrt{2^2\times 5}+\sqrt{2^2\times 10}-\sqrt{5}-\sqrt{2^4\times 5} \quad = \sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-2^2\sqrt{5}$$

$$= \sqrt{10}+2\sqrt{10}+2\sqrt{5}-\sqrt{5}-4\sqrt{5} \quad = (1+2)\sqrt{10}+(2-1-4)\sqrt{5} = 3\sqrt{10}-3\sqrt{5} = 3(\sqrt{10}-\sqrt{5})$$

$$\therefore \frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$$

$$= \frac{15}{3(\sqrt{10}-\sqrt{5})} = \frac{5}{\sqrt{10}-\sqrt{5}} \quad = \frac{5(\sqrt{10}+\sqrt{5})}{(\sqrt{10}-\sqrt{5})(\sqrt{10}-\sqrt{5})}$$ [Multiplying and dividing by $\sqrt{10}+\sqrt{5}$]

$$= \frac{5(\sqrt{10}+\sqrt{5})}{10-5} = \sqrt{10}+\sqrt{5} = 3.162+2.236 = 5.398$$

PRACTICE PROBLEMS

14. Rationalise the denominator of the given expressions

a. $\frac{3}{2\sqrt{5}}$ **b.** $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$ **c.** $\frac{1}{3+\sqrt{2}}$ **d.** $\frac{1}{\sqrt{6}-\sqrt{5}}$

e. $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$ **f.** $\frac{b^2}{\sqrt{a^2+b^2}+a}$ **g.** $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ **h.** $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

i. $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$ **j.** $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}$

k. $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = a - b\sqrt{77}$ **l.** If $a = \frac{2-\sqrt{5}}{2+\sqrt{5}}$ and $b = \frac{2+\sqrt{5}}{2-\sqrt{5}}$, find $a^2 - b^2$

• LAWS OF EXPONENTS FOR REAL NUMBERS •

Let $a > 0$ be a real number and m, n are rational numbers, Then

(i) $a^m \cdot a^n = a^{m+n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $\frac{a^m}{a^n} = a^{m-n}, m > n$

(iv) $a^m b^m = (ab)^m$ where a is called the base and m and n are the exponents.

(v) Value of $(a)^0$: We have $(a)^0 = 1$. (vi) $a^{-n} = \frac{1}{a^n}$.

For example: (i) $17^2 . 17^5 = 17^7$ (ii) $(5^2)^7 = 5^{14}$ (iii) $\frac{23^{10}}{23^7} = 23^3$ (iv) $7^3 . 9^3 = 63^3$.

Explanation of negative exponents through some examples

(i) $17^2 . 17^{-5} = 17^{-3} = \frac{1}{17^3}$ (ii) $(5^2)^{-7} = 5^{-14}$ (iii) $\frac{23^{-10}}{23^7} = 23^{-17}$ (iv) $(7)^{-3} . (9)^{-3} = (63)^{-3}$

Meaning of $\sqrt[n]{a}$ in the Language of Exponents: we define $\sqrt[n]{a} = a^{1/n}$.

For example: $\sqrt[3]{2} = 2^{1/3}$.

Thus, $4^{3/2} = (4^{1/2})^3 = 2^3 = 8$. Also, $4^{3/2} = (4^3)^{1/2} = (64)^{1/2} = 8$.

ILLUSTRATION

Q.38 Simplify

(i) $2^{\frac{2}{3}} . 2^{\frac{1}{3}}$ (ii) $\left(3^{\frac{1}{5}}\right)^4$ (iii) $\frac{7^{\frac{1}{5}}}{7^{\frac{1}{3}}}$ (iv) $13^{\frac{1}{5}} . 17^{\frac{1}{5}}$

Sol. (i) $2^{\frac{2}{3}} . 2^{\frac{1}{3}} = 2^{\left(\frac{2}{3}+\frac{1}{3}\right)} = 2^{\frac{3}{3}} = 2^1 = 2$ (ii) $\left(3^{\frac{1}{5}}\right)^4 = 3^{\frac{4}{5}}$

(iii) $\frac{7^{\frac{1}{5}}}{7^{\frac{1}{3}}} = 7^{\left(\frac{1}{5}-\frac{1}{3}\right)} = 7^{\frac{3-5}{15}} = 7^{\frac{-2}{15}}$ (iv) $13^{\frac{1}{5}} . 17^{\frac{1}{5}} = (13 \times 17)^{\frac{1}{5}} = 221^{\frac{1}{5}}$

Q.39 Simplify : $5^{1/5} . 5^{\frac{1}{10}}$

Sol. $5^{1/5} . 5^{1/10} = 5^{1/5 + 1/10} = 5^{3/10}$.

Q.40 Simplify : $256^{3/4}$.

Sol. $256^{3/4} = (4^4)^{3/4} = 4^{4\times3/4} = 4^3 = 64$.

Q.41 Simplify : $36^{1/3} . 6^{1/3}$.

Sol. $36^{1/3} . 6^{1/3} = (36 \times 6)^{1/3} = (6^3)^{1/3} = 6^{3 \times 1/3} = 6^1 = 6$.

Q.42 Simplify: $\left(\frac{1}{5^3}\right)^4$.

Sol. $\left(\frac{1}{5^3}\right)^4 = \frac{1^4}{(5^3)^4} = \frac{1}{5^{3\times4}} = \frac{1}{5^{12}}$

Q.43 Simplify: $(x^{-2/3} y^{-1/2})^2$

Sol. We have, $(x^{-2/3} y^{-1/2})^2 = (x^{-2/3})^2 (y^{-1/2})^2 = x^{-2/3x2} y^{-1/2\times2} = x^{-4/3} y^{-1} = \frac{1}{x^{4/3} y}$

Q.44 $\sqrt[3]{xy^2} \div x^2 y$

Sol. We have, $\sqrt[3]{xy^2} \div x^2 y = \frac{\sqrt[3]{xy^2}}{x^2 y} = \frac{(xy^2)^{1/3}}{x^2 y} = \frac{x^{1/3} y^{2/3}}{x^2 y} = x^{1/3-2} y^{2/3-1} = x^{-5/3} y^{-1/3}$

Q.45 Simplify:

(i) $\dfrac{(25)^{3/2}\times(243)^{3/5}}{(16)^{5/4}\times(8)^{4/3}}$ **(ii)** $\dfrac{16\times 2^{n+1}-4\times 2^{n}}{16\times 2^{n+2}-2\times 2^{n+2}}$

Sol. (i) We have, $\dfrac{(25)^{3/2}\times(243)^{3/5}}{(16)^{5/4}\times(8)^{4/3}}=\dfrac{(5^2)^{3/2}\times(3^5)^{3/5}}{(2^4)^{5/4}\times(2^3)^{4/3}}=\dfrac{5^{2\times3/2}\times3^{5\times3/5}}{2^{4\times5/4}\times2^{3\times4/3}}=\dfrac{5^3\times3^3}{2^5\times2^4}=\dfrac{125\times27}{32\times16}=\dfrac{3375}{512}$

(ii) We have, $\dfrac{16\times 2^{n+1}-4\times 2^{n}}{16\times 2^{n+2}-2\times 2^{n+2}}=\dfrac{2^4\times 2^{n+1}-2^2\times 2^{n}}{2^4\times 2^{n+2}-2\times 2^{n+2}}=\dfrac{2^{n+5}-2^{n+2}}{2^{n+6}-2^{n+3}}=\dfrac{2^{n+5}-2^{n+2}}{2.2^{n+5}-2.2^{n+2}}=\dfrac{2^{n+5}-2^{n+2}}{2(2^{n+5}-2^{n+2})}=\dfrac{1}{2}$

Q.46 Simplify: $\left(\dfrac{81}{16}\right)^{-3/4}\times\left[\left(\dfrac{25}{9}\right)^{-3/2}\div\left(\dfrac{5}{2}\right)^{-3}\right]$

Sol. We have, $\left(\dfrac{81}{16}\right)^{-3/4}\times\left[\left(\dfrac{25}{9}\right)^{-3/2}\div\left(\dfrac{5}{2}\right)^{-3}\right]=\left(\dfrac{3^4}{2^4}\right)^{-3/4}\times\left[\left(\dfrac{5^2}{3^2}\right)^{-3/2}\div\left(\dfrac{5}{2}\right)^{-3}\right]$

$=\left[\left(\dfrac{3}{2}\right)^{4}\right]^{-3/4}\times\left[\left(\dfrac{5}{3}\right)^{2}\right]^{-3/2}\div\left[\left(\dfrac{5}{2}\right)^{-3}\right]=\left(\dfrac{3}{2}\right)^{4\text{x}-3/4}\times\left[\left(\dfrac{5}{3}\right)^{2\text{x}-3/2}\div\left(\dfrac{5}{2}\right)^{-3}\right]=\left(\dfrac{3}{2}\right)^{-3}\times\left[\left(\dfrac{5}{3}\right)^{-3}\div\left(\dfrac{5}{2}\right)^{-3}\right]$

$=\left(\dfrac{2}{3}\right)^{3}\times\left[\left(\dfrac{3}{5}\right)^{-3}\div\left(\dfrac{2}{5}\right)^{3}\right]=\dfrac{2^3}{3^3}\times\left[\dfrac{3^3}{5^3}+\dfrac{2^3}{5^3}\right]=\dfrac{2^3}{3^3}\times\left[\dfrac{3^3}{5^3}\times\dfrac{5^3}{2^3}\right]=1$

Q.47 If x, y, z are positive real numbers show that: $\sqrt{x^{-1}y}\,.\,\sqrt{y^{-1}z}\,.\,\sqrt{z^{-1}x}=1$

Sol. We have, $\sqrt{x^{-1}y}\,.\,\sqrt{y^{-1}z}\,.\,\sqrt{z^{-1}x}=\sqrt{\dfrac{y}{x}}.\sqrt{\dfrac{z}{y}}.\sqrt{\dfrac{x}{z}}=\left(\dfrac{y}{x}\right)^{1/2}\left(\dfrac{z}{y}\right)^{1/2}\left(\dfrac{x}{z}\right)^{1/2}=\dfrac{y^{1/2}}{x^{1/2}}.\dfrac{z^{1/2}}{y^{1/2}}.\dfrac{x^{1/2}}{z^{1/2}}=1$

Q.48 Show that : $\dfrac{(x^{a+b})^2\,(x^{b+c})^2\,(x^{c+a})^2}{(x^a\,x^b\,x^c)^4}=1$

Sol. We have, $\dfrac{(x^{a+b})^2\,(x^{b+c})^2\,(x^{c+a})^2}{(x^a\,x^b\,x^c)^4}=\dfrac{x^{2(a+b)}.x^{2(b+c)}\,x^{2(c+a)}}{(x^a)^4(x^b)^4(x^c)^4}=\dfrac{x^{2a+2b}.x^{2b+2c}.x^{2c+2a}}{x^{4a}.x^{4b}.x^{4c}}=\dfrac{x^{2a+2b+2b+2c+2a}}{x^{4a+4b+4c}}=\dfrac{x^{4a+4b+4c}}{x^{4a+4b+4c}}=1$

Q.49 Find the value of x, if $5^{x-3}\,.\,3^{2x-8}=225$

Sol. We have, $5^{x-3}\times3^{2x-8}=225$ $\Rightarrow$ $5^{x-3}\times3^{2x-8}=5^2\times3^2$

$\Rightarrow x-3=2$ and $2x-8=2$ [On equating the exponents] $\Rightarrow$ $x=5$

Q.50 Prove that $\dfrac{2^{30}+2^{29}+2^{28}}{2^{31}+2^{30}-2^{29}}=\dfrac{7}{10}$.

Sol. LHS $=\dfrac{2^{30}+2^{29}+2^{28}}{2^{31}+2^{30}-2^{29}}=\dfrac{2^{28}(2^2+2^1+1)}{2^{29}(2^2+2^1-1)}=\dfrac{4+2+1)}{2(4+2-1)}=\dfrac{7}{2\times5}=\dfrac{7}{10}=$ RHS Hence proved.

PRACTICE PROBLEMS

15. $5^{x-2}\times3^{2x-3}=135$ **16.** $2^{x-5}\times5^{x-4}=5$ **17.** If $27^x=\dfrac{9}{3x}$, find x

18. Show that

(i) $9^{\frac{3}{2}} - 3\times 5^{\circ} - \left(\frac{1}{81}\right)^{\frac{1}{2}} = 15$ (ii) $\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c = 1$ (iii) $\frac{16\times 2^{n+1} - 4\times 2^n}{16\times 2^{n+2} - 2\times 2^{n+2}} = \frac{1}{2}$

(iv) $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n} = \frac{3}{2}$ (v) $\frac{(0.6)^0 - (0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3 + \left(\frac{-1}{3}\right)^{-1}} = -\frac{3}{2}$ (vi) $\frac{3^{30} + 3^{29} + 3^{28}}{3^{31} + 3^{30} - 3^{29}} + \frac{2^{30} + 2^{29} + 2^{28}}{2^{31} + 2^{30} - 2^{29}} = \frac{361}{330}$

PRACTICE PROBLEMS ANSWERS

1. (i) Yes (ii) No (iii) No.

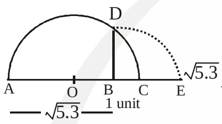
2. $\frac{21}{24}, \frac{26}{24}, \frac{31}{24}$

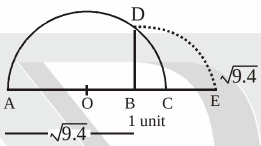
3. 0.45454545...., 0.569569569...., 0.46464646...., 0.474747...., 0.4848...., 0.494949...., 0.505050....

4. 0.6 and 0.666...., 0.61234125...., 0.6263452135...., 0.625315215....

5. 0.2345172...., 0.23521752...., 0.2315263....

6. OB = $\sqrt{5}$ units, OB = $\sqrt{11}$ unit

7. $\sqrt{5.3}$ 

, $\sqrt{9.4}$ 

10. (i) $5\sqrt{2}$ (ii) $\sqrt{2\times 2\times 3\times 5} = 2\sqrt{15}$ (iii) $\sqrt{2\times 2\times 2\times 3} = 2\sqrt{6}$ (iv) $\sqrt{2\times 2\times 2\times 2\times 2} = 4\sqrt{2}$

(v) $\sqrt{2\times 2\times 5\times 5\times 2} = 2\times 5\sqrt{2} = 10\sqrt{2}$ (vi) $\sqrt{486} = \sqrt{2\times 3\times 3\times 3\times 3\times 3} = 9\sqrt{6}$

11. $3\sqrt{3\times 3\times 5} - \sqrt{5\times 5\times 5} + \sqrt{2\times 2\times 5\times 5\times 2} - \sqrt{5\times 5\times 2} = 9\sqrt{5} - 5\sqrt{5} + 10\sqrt{2} - 5\sqrt{2} = 4\sqrt{5} + 5\sqrt{2}$

12. $5\sqrt{2} - \sqrt{3\times 3\times 2} + \sqrt{2\times 2\times 3} + 6\sqrt{3} = 5\sqrt{2} - 3\sqrt{2} + 2\sqrt{3} + 6\sqrt{3} = 2\sqrt{2} + 8\sqrt{3}$

13. a. $10\sqrt{10}$ b. 42 c. $15\sqrt{6} + 3\sqrt{14}$ d. $14\sqrt{6} + 21\sqrt{15}$

e. $12 + 4\sqrt{2} + 3\sqrt{7} + \sqrt{14}$ f. $15 - 3\sqrt{2} + 5\sqrt{3} - \sqrt{6}$ g. 110 h. 18

14. a. $\frac{3}{10}\sqrt{5}$ b. $\frac{\sqrt{6} + \sqrt{15}}{3}$ c. $\frac{3 - \sqrt{2}}{7}$ d. $\sqrt{6} + \sqrt{5}$

e. $17 - 12\sqrt{2}$ f. $\sqrt{a^2 + b^2} - a$ g. $5 - 2\sqrt{6}$ h. $\frac{9 + 4\sqrt{30}}{21}$

i. 11 j. 0 k. $a = \frac{9}{2}, b = \frac{1}{2}$ l. $-144\sqrt{5}$

15. x = 3 **16.** x = 5 **17.** $x = \frac{1}{2}$

EXERCISE

TYPE I : FUNDAMENTALS OF NUMBER SYSTEM

1. Examine, whether the following numbers are rational or an irrational :

i. $\sqrt{7}$ **ii.** $\sqrt{4}$ **iii.** $2+\sqrt{3}$ **iv.** $\sqrt{3}+\sqrt{2}$

v. $(\sqrt{2}-2)^2$ **vi.** $(\sqrt{2}+\sqrt{3})^2$ **vii.** $-2\sqrt{8}$ **viii.** $(2-\sqrt{2})(2+\sqrt{2})$

2. Give an example each, of two irrational numbers, whose

i. difference is a rational number
ii. difference is an irrational number
iii. sum is a rational number
iv. sum is an irrational number
v. product is an irrational number
vi. product is a rational number
vii. quotient is a rational number
vii. quotient is an irrational number

3. **i.** Let 'a' be a rational number and 'b' be an irrational number. Is 'ab' necessarily an irrational number? Justify your answer with an example.

ii. Let x and y be rational and irrational numbers, respectively. Is x + y necessarily an irrational number? Give an example in support of your answer.

4. Find three rational number between

i. 4 and 5 **ii.** 20 and 21 **iii.** $\frac{3}{4}$ and $\frac{4}{3}$ **iv.** $\frac{15}{9}$ and $\frac{18}{4}$

v. $\sqrt{2}$ and $\sqrt{3}$ **vi.** $\sqrt{15}$ and $\sqrt{20}$ **vii.** 18 and 18.5 **viii.** 0.15 and 0.16

5. Find three irrational number between

i. 6 and 7 **ii.** 11 and 12 **iii.** $\frac{1}{7}$ and $\frac{2}{7}$ **iv.** $\frac{21}{15}$ and $\frac{14}{4}$

v. $\sqrt{2}$ and $\sqrt{3}$ **vi.** $\sqrt{3}$ and $\sqrt{5}$ **vii.** 0.5 and 0.55 **viii.** 0.1 and 0.12

6. Find two irrational number between

i. 0.202002000200002... and 0.203003000300003...
ii. 0.1001000100001... and 0.101001000100001....
iii. 1.333333333333..... and 1.444444444444444....
iv. 2.666666666666.... and 2.7

7. Find the decimal expansion of the follwing rational numbers:

i. $\frac{10}{3}$ **ii.** $\frac{7}{8}$ **iii.** $\frac{1}{7}$ **iv.** $\frac{17}{4}$

8. Find the decimal expansion of the following irrational numbers:

i. $\sqrt{2}$ **ii.** $\sqrt{3}$ **iii.** $\sqrt{5}$ **iv.** $\sqrt{7}$

TYPE II : CONVERT INTO P/Q FORM

9. Find the p/q form of

i. 5.7 **ii.** 17.9 **iii.** 0.3 **iv.** $0.\overline{37}$

v. $0.\overline{05}$ **vi.** $1.\overline{3}$ **vii.** $0.\overline{621}$ **viii.** $3.\overline{14}$

ix. $0.1\overline{63}$ **x.** $2.3\overline{42}$ **xi.** $1.\overline{324}$ **xii.** $32.\overline{7}$

xiii. $0.\overline{6}+0.4\overline{7}$ **xiv.** $0.\overline{45}-0.3\overline{42}$

TYPE III : NUMBER LINE

10. Mark $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{10}, \sqrt{11}$ on a number line. (Use separate lines).

11. Mark $\sqrt{2.1}$, $\sqrt{3.4}$, $\sqrt{4.3}$, $\sqrt{5.1}$, $\sqrt{6.4}$ on a number line.

12. Visualise the following real numbers on a number line:

i. $5.3\overline{7}$ **ii.** 3.765 **iii.** $4.\overline{26}$ **iv.** $7.\overline{521}$

TYPE IV : OPERATIONS ON REAL NUMBERS

13. Simplify and express in simplest form :

i. $\sqrt{5}+\sqrt{20}+\sqrt{45}$ **ii.** $\sqrt{27}+3\sqrt{12}$ **iii.** $21\sqrt{384}+8\sqrt{96}$ **iv.** $\sqrt{50}-\sqrt{98}+\sqrt{162}$

v. $\sqrt{147}-\sqrt{108}-\sqrt{3}$ **vi.** $7\sqrt{6}-\sqrt{252}-\sqrt{294}+6\sqrt{7}$

14. Simplify, and specify which of the following is rational or irrational number:

i. $(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})$ **ii.** $(4\sqrt{3}-3\sqrt{5})^2$

iii. $(\sqrt{6}+\sqrt{7})(\sqrt{6}-\sqrt{7})$ **iv.** $(3\sqrt{18}+2\sqrt{12})(\sqrt{50}-\sqrt{27})$

v. $4\sqrt{28}\div 3\sqrt{7}$ **vi.** $\dfrac{\sqrt{3}\times\sqrt{8}\times\sqrt{39}}{\sqrt{24}\times\sqrt{26}}$

vii. $\dfrac{\sqrt{75}\times\sqrt{60}\times\sqrt{63}}{\sqrt{40}\times\sqrt{200}}$ **viii.** $\dfrac{2\sqrt{7}\times\sqrt{8}\times 5\sqrt{3}}{3\sqrt{3}\times\sqrt{15}\times\sqrt{42}}$

TYPE V : RATIONALISE THE DENOMINATOR

15. Rationalise the denominator of (One term denominator):

i. $\dfrac{1}{\sqrt{5}}$ **ii.** $\dfrac{5\sqrt{3}}{\sqrt{2}}$ **iii.** $\dfrac{4}{\sqrt{8}}$ **iv.** $\dfrac{6+\sqrt{2}}{\sqrt{5}}$

v. $\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}}$ **vi.** $\dfrac{\sqrt{12}+\sqrt{18}}{\sqrt{8}}$

16. Rationalise the denominator of (two term denominator):

i. $\dfrac{1}{3+\sqrt{2}}$ **ii.** $\dfrac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}$ **iii.** $\dfrac{1}{2\sqrt{5}-\sqrt{3}}$ **iv.** $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{7}+\sqrt{3}}$

v. $\dfrac{\sqrt{2}+\sqrt{3}}{5+\sqrt{2}}$ **vi.** $\dfrac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

17. Rationalise the denominator of (three term denominator):

i. $\dfrac{1}{\sqrt{3}-\sqrt{2}-1}$ **ii.** $\dfrac{1}{3+\sqrt{5}-2\sqrt{2}}$ **iii.** $\dfrac{1}{\sqrt{6}+\sqrt{5}-\sqrt{11}}$

18. Simplify by rationalising the denominator:

i. $\frac{3}{5-\sqrt{3}}+\frac{2}{5+\sqrt{3}}$ **ii.** $\frac{4\sqrt{3}}{2-\sqrt{2}}-\frac{30}{4\sqrt{3}-\sqrt{18}}-\frac{\sqrt{18}}{3+2\sqrt{3}}$ **iii.** $\frac{3\sqrt{3}-2\sqrt{5}}{3\sqrt{3}+2\sqrt{5}}+\frac{\sqrt{12}}{\sqrt{5}-\sqrt{3}}$

TYPE VI : APPLICATIONS OF RATIONALISE THE DENOMINATOR

19. If $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a+b\sqrt{3}$, find the value of a and b.

20. Find the values of a and b if $\frac{7+3\sqrt{5}}{3+\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}$ = a + $\sqrt{5}$ b

21. If a = $\frac{3-\sqrt{5}}{3+\sqrt{5}}$ and b = $\frac{3+\sqrt{5}}{3-\sqrt{5}}$, find $a^2 - b^2$.

22. If $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$ = a –b $\sqrt{6}$, find the value of a and b.

23. If x = $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ and y = $\frac{3+2\sqrt{2}}{3-2\sqrt{2}}$, then find the value of x + y.

24. Find the value of a, if $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5}-\frac{19}{11}$

25. If a = 2 + $\sqrt{3}$, then find the value of a + $\frac{1}{a}$.

26. If x = 2 + $\sqrt{3}$, find the value $x^2+\frac{1}{x^2}$.

27. If a = $\frac{3+\sqrt{7}}{2}$, then find the value of $a^2+\frac{1}{a^2}$.

28. If x = $\sqrt{3}-2$, find the value of $\left(x+\frac{1}{x}\right)^3$.

29. If x = 2 + $\sqrt{5}$, find the value of $\left(x^2-\frac{1}{x^2}\right)$.

30. If a = 7 –4 $\sqrt{3}$, find the value of $\sqrt{a}+\frac{1}{\sqrt{a}}$.

31. If a = $\frac{4}{3-\sqrt{5}}$. Find the value of $a+\frac{4}{a}$.

32. If x = $\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ and y = $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$, find the value of $x^2 + xy + y^2$.

33. If x = 3 + $2\sqrt{2}$, find $\sqrt{x}+\frac{1}{\sqrt{x}}$.

34. If x = 3 + $2\sqrt{2}$, check whether x + $\frac{1}{x}$ is rational or irrational.

35. If a = $1-\sqrt{2}$, find $\left(a-\frac{1}{a}\right)^3$.

36. If x = $1-\sqrt{2}$, find value of $\left(x-\frac{1}{x}\right)^{1/4}$.

37. If $\sqrt{2}$ = 1.414, $\sqrt{3}$ = 1.732, then find the value of $\frac{6}{3\sqrt{2}-2\sqrt{3}}$.

38. Prove: $\frac{1}{2+\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}=0$.

TYPE VII : RATIONAL EXPONENTS

39. Evalaute the following exponents:

i. $\left(\frac{64}{125}\right)^{-2/3}$ **ii.** $\left(\frac{81}{16}\right)^{-3/4}\times\left(\frac{25}{9}\right)^{-3/2}$ **iii.** $\left(\frac{15^{1/3}}{9^{1/4}}\right)^{-6}$ **iv.** $\left(\frac{12^{1/5}}{27^{1/5}}\right)^{5/2}$

v. $\sqrt[4]{81}-8\sqrt[3]{216}+15\sqrt[5]{32}+\sqrt{225}$ **vi.** $\left\{5\left(8^{1/3}+27^{1/3}\right)^3\right\}^{1/4}$ **vii.** $\frac{9^{1/3}\times 27^{1/2}}{3^{-1/4}\times 3^{1/3}}$

viii. $\frac{(25)^{3/2}\times(343)^{5/3}}{16^{5/4}\times(125)^{4/3}\times 7^5}$ **ix.** $(32)^{1/5} + (-7)^{\circ} + (64)^{1/2}$ **x.** $\sqrt[3]{(343)^{-2}}$

xi. $\frac{4}{(216)^{-2/3}}-\frac{1}{(256)^{-3/4}}$ **xii.** $\left(\frac{81}{16}\right)^{-3/4}\times\left\{\left(\frac{25}{9}\right)^{-3/2}\div\left(\frac{5}{2}\right)^{-3}\right\}$

40. Simplify : $\sqrt[4]{\sqrt[3]{x^2}}$ and express the result in the exponential form of x.

41. Find the value of x : $\left(\frac{3}{4}\right)^3 \left(\frac{4}{3}\right)^{-7} = \left(\frac{3}{4}\right)^{2x}$.

42. Find the value of *x* if $2^4 \times 2^5 = (2^3)^x$.

43. Show that : $(x^{a-b})^{a+b} . (x^{b-c})^{b+c} . (x^{c-a})^{c+a} = 1$.

44. If a = 2, b = 3, then find the values of the following : **i.** $(a^b + b^a)^{-1}$ **ii.** $(a^a + b^b)^{-1}$

45. Prove that : $\frac{2^{30} + 2^{29} + 2^{28}}{2^{31} + 2^{30} - 2^{29}} = \frac{7}{10}$.

46. Show that $\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c = 1$.

47. If $(5)^{x-3} \times (3)^{2x-8} = 225$, then find the value of x.

48. Prove $\left(\frac{2^a}{2^b}\right)^{a+b} \times \left(\frac{2^b}{2^c}\right)^{b+c} \times \left(\frac{2^c}{2^a}\right)^{c+a} = 1$

49. Prove that : $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} = 1$

50. Arrange in ascending order : $\sqrt[4]{3}, \sqrt[3]{2}, \sqrt[3]{4}$

51. Arrange in descending order : $\sqrt[3]{2}, \sqrt{3}, \sqrt[6]{5}$

TYPE VIII : HOTS AND VALUE BASED QUESTIONS

52. Show that : $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2} = 5$

53. Evaluate : $\sqrt{5+2\sqrt{6}} + \sqrt{8-2\sqrt{15}}$.

54. Simplify : $10\sqrt[3]{40} - 4\sqrt[3]{320} - \sqrt[3]{5}$.

55. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{2x-8}$, then find value of x.

56. Evaluate : $\frac{\left(\frac{9}{4}\right)^{-\frac{3}{2}} \times \left(\frac{125}{27}\right)^{-\frac{2}{3}} \times \left(\frac{3}{5}\right)^{-2}}{\left(\sqrt{2}\right)^4}$.

57. Evaluate : $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}$.

58. If $x^a = y$, $y^b = z$ and $z^c = x$, then prove that $abc = 1$.

59. Simplify : $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{8}+\sqrt{9}}$

60. Write in the simplest form : $12\sqrt{18} + 6\sqrt{20} - 6\sqrt{147} + 3\sqrt{50}$

61. If $a + 8\sqrt{5}\,b = \frac{8+\sqrt{5}}{8-\sqrt{5}} + \frac{8-\sqrt{5}}{8+\sqrt{5}}$. Find a and b.

62. Simplify $\frac{16\times 2^{n+1} - 4\times 2^{n}}{16\times 2^{n+2} - 2\times 2^{n+2}}$

63. Evaluate $\frac{(243)^{\frac{n}{5}}\times 3^{2n+1}}{9^{n}\times 3^{n-1}}$

64. If $x = \frac{1}{2-\sqrt{3}}$, then show that the value of $(x^3 - 2x^2 - 7x + 5)$ is 3.

65. Simplify: $\frac{3^{30}+3^{29}+3^{28}}{3^{31}+3^{30}-3^{29}} + \frac{2^{30}+2^{29}+2^{28}}{2^{31}+2^{30}-2^{29}}$

66. In a school, 5 out of every 7 children participated in 'SAVE WILDLIFE' campaign organised by the school authorities.

i. What fraction of the student participated in the campaign.

ii. Find what kind of decimal expansion it has?

iii. What values do the participating students possess?

67. While discussing rationalising factor, teacher asked the students to find the reciprocal of $\sqrt{6}+\sqrt{5}$. Ramesh answered $\sqrt{6}-\sqrt{5}$.

i. Is he correct? Justify your answer.

ii. Which values are depicted in this question?

68. Teacher asked the students, "Can we write $0.4\overline{7}$ in $\frac{p}{q}$ form as $\frac{47}{100}$? Sonia answered, "No, it is $\frac{43}{90}$.

i. Is Sonia correct? Justify your answer.

ii. Which values of Sonia are depicted in this question?

ANSWERS

1. Rational ii, viii

2. (i) $6-\sqrt{2}$, $5-\sqrt{2}$ (ii) $4\sqrt{3}$, $2\sqrt{2}$

(iii) $2+\sqrt{2}$, $2-\sqrt{2}$ (iv) $\sqrt{3}$, $\sqrt{2}$

(v) $4\sqrt{3}$, $3\sqrt{2}$ (vi) $5\sqrt{2}$, $2\sqrt{2}$

(vii) $7\sqrt{3}$, $2\sqrt{3}$ (viii) $6\sqrt{5}$, $3\sqrt{2}$

3. (i) No, a = 0, b= $\sqrt{3}+\sqrt{2}$ (ii) Yes, x=5, b= $\sqrt{3}+2$

4. (i) 9/2, 17/4, 19/4 (ii) 81/4, 82/4, 83/4
(iii) 10/12, 11/12, 12/12 (iv) 11/6, 12/6, 13/6
(v) 1.5, 1.6, 1.7 (vi) 3.9, 4.0, 4.1
(vii) 18.1, 18.2, 18.3 (viii) 0.151, 0.152, 0.153

5. (i) $\sqrt{42},\sqrt{43},\sqrt{44}$ (ii) $\sqrt{132},\sqrt{133},\sqrt{134}$

(iii) $\sqrt{2/49},\sqrt{2.5/49},\sqrt{3/49}$

(iv) $\sqrt{5},\sqrt{6},\sqrt{7}$ (v) $\sqrt{2.1},\sqrt{2.2},\sqrt{2.3}$

(vi) $\sqrt{3.1},\sqrt{3.2},\sqrt{3.3}$

(vii) 0.5151151115.., 0.5252252225.., 0.5353353335..
(viii) 0.1010010001., 0.11010010001., 0.111010010001..

6. (i) 0.2020030003..... (ii)0.100202002.......
(iii) 1.343343334...... (iv) 2.676676667......

7. (i) $3.\overline{3}$, (ii) 0.875, (iii) $0.\overline{142857}$
(iv) 4.25

8. (i) 1.4142....... (ii) 1.7320.......
(iii) 2.2360...., (iv) 2.6457.........

9. (i) 57/10 (ii) 179/10 (iii) 3/10 (iv) 37/99
(v) 5/99 (vi) 4/3 (vii) 23/37 (viii)311/99
(ix) 9/55 (x) 773/330 (xi) 323/999 (xii) 295/9
(xiii) 103/90 (xiv) 37 / 330

13. (i) $6\sqrt{5}$ (ii) $9\sqrt{3}$ (iii) $200\sqrt{6}$

(iv) $7\sqrt{2}$ (v) 0 (vi) 0.

14. (i) 2 (ii) $93-24\sqrt{15}$ (iii) –1 (iv) $54-7\sqrt{6}$

(v) 8/3 (vi) $\sqrt{3/2}$ (vii) $(9/4)\times\sqrt{7}$

(viii) $20/9\sqrt{5}$

15. (i) $\sqrt{5}/5$ (ii) $\frac{5\sqrt{6}}{2}$ (iii) $\sqrt{2}$

(iv) $(6\sqrt{5}+\sqrt{10})/5$ (v) $(2+\sqrt{6})/2$

(vi) $(\sqrt{6}+3)/2$

16. (i) $(3-\sqrt{2})/7$ (ii) $3\sqrt{2}+2\sqrt{3}$ (iii) $(2\sqrt{5}+\sqrt{3})/17$

(iv) $(\sqrt{21}-3+\sqrt{14}-\sqrt{6})/4$ (v) $(5\sqrt{2}-2+5\sqrt{3}-\sqrt{6})/23$

(vi) $(114-41\sqrt{6})/30$

17. (i) $-(2+\sqrt{2}+\sqrt{6})/4$ (ii) $(1+\sqrt{10})12$

(iii) $(6\sqrt{3}+5\sqrt{6}+\sqrt{330})/60$

18. (i) $(25+\sqrt{3})/22$ (ii) 0 (iii) $\frac{68-5\sqrt{15}}{7}$

19. a = 2, b = –1 **20.** a = 0, b = 1

21. $-21\sqrt{5}$ **22.** a = 2, b = $-5/6$

23. $19-\sqrt{3}-12\sqrt{2}$ **24.** a = 9 / 11

25. 4 **26.** 14 **27.** $\frac{40-9\sqrt{7}}{2}$

28. – 64 **29.** $8\sqrt{5}$ **30.** 4

31. 6 **32.** 187 / 9 **33.** $2\sqrt{2}$ or $\sqrt{8}$

34. Rational Number, 8 **35.** 8

36. $2^{\frac{1}{4}}$ **37.** 0.778

39. (i) 25 / 16 (ii) 8 / 125 (iii) 27 / 225

(iv) 2 / 3 (v) 0 (vi) 5 (vii) $3^{\frac{25}{12}}$

(viii) 1/160 (ix) 11 (x) 1/49 (xi) 80

(xii) 2^6

40. $x^{\frac{1}{6}}$ **41.** 5 **42.** 3

44. (i) 1/17 (ii) 1/31 **47.** x = 5

50. $\sqrt[3]{2},\sqrt[4]{3},\sqrt[3]{4}$ **51.** $\sqrt{3},\sqrt[6]{5},\sqrt[3]{2}$ **53.** $\sqrt{2}+\sqrt{5}$

54. $3\sqrt[3]{5}$ **55.** 3 **56.** 2 / 27

57. 1 **59.** 2

60. $51\sqrt{2}+12\sqrt{5}-42\sqrt{3}$ **61.** a = 138 / 59, b = 0

62. 1 / 2 **63.** 9 **65.** $\frac{361}{330}$

66. i. $0.\overline{714285}$, ii. Non terminating, repeating,
iii. Helpful, Caring, Social and Environmental concern.

67. Yes, knowledge and curiosity.

68. Yes, knowledge and curiosity.