NCERT Solutions for Class 9 Maths Chapter 1- Number System

Exercise 1.1 Page: 5

1. Is zero a rational number? Can you write it in the form p/q where p and q are integers and q ≠ 0?

Solution:

We know that, a number is said to be rational if it can be written in the form p/q , where p and q are integers and q ≠ 0.

Taking the case of '0',

Zero can be written in the form 0/1, 0/2, 0/3 … as well as , 0/1, 0/2, 0/3 ..

Since it satisfies the necessary condition, we can conclude that 0 can be written in the p/q form, where q can either be positive or negative number.

Hence, 0 is a rational number.

2. Find six rational numbers between 3 and 4.

Solution:

There are infinite rational numbers between 3 and 4.

As we have to find 6 rational numbers between 3 and 4, we will multiply both the numbers, 3 and 4, with 6+1 = 7 (or any number greater than 6)

i.e., 3×(7/7) = 21/7

and, 4×(7/7) = 27/7 . ∴The numbers in between 21/7 and 28/7 will be rational and will fall between 3 and 4.

Hence, 22/7, 23/7, 24/7, 25/7, 26/7, 27/7 are the 6 rational numbers between 3 and 4.

3 Find five rational numbers between 3/5 and 4/5.

Solution:

There are infinite rational numbers between 3/5 and 4/5.

To find out 5 rational numbers between 3/5 and 4/5, we will multiply both the numbers 3/5 and 4/5 with 5+1=6 (or any number greater than 5)

i.e., (3/5)×(6/6) = 18/30

and, (4/5)×(6/6) = 24/30

∴The numbers in between18/30 and 24/30 will be rational and will fall between 3/5 and 4/5.

Hence,19/30, 20/30, 21/30, 22/30, 23/30 are the 5 rational numbers between 3/5 and 4/5.

4. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

Solution:

True

Natural numbers- Numbers starting from 1 to infinity (without fractions or decimals)

i.e., Natural numbers= 1,2,3,4…

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3…

Or, we can say that whole numbers have all the elements of natural numbers and zero.

∴ Every natural number is a whole number, however, every whole number is not a natural number.

(ii) Every integer is a whole number.

Solution:

False

Integers- Integers are set of numbers that contain positive, negative and 0; excluding fractional and decimal numbers.

i.e., integers= {…-4,-3,-2,-1,0,1,2,3,4…}

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3….

Hence, we can say that integers include whole numbers as well as negative numbers.

∴ Every whole number is an integer, however, every integer is not a whole number.

(iii) Every rational number is a whole number.

Solution:

False

Rational numbers- All numbers in the form p/q, where p and q are integers and q≠0.

i.e., Rational numbers = 0, 19/30, 2, 9/-3, -12/7…

Whole numbers- Numbers starting from 0 to infinity (without fractions or decimals)

i.e., Whole numbers= 0,1,2,3….

Hence, we can say that integers include whole numbers as well as negative numbers.

∴ Every whole numbers are rational, however, every rational numbers are not whole numbers.

Exercise 1.2 Page: 8

1.State whether the following statements are true or false. Justify your answers.
(i) Every irrational number is a real number.

Solution:
True
Irrational Numbers - A number is said to be irrational, if it **cannot** be written in the p/q, where p and q are integers and $q \neq 0$.
i.e., Irrational numbers = 0, 19/30, 2, 9/-3, -12/7, $\sqrt{2}$, $\sqrt{5}$, π, 0.102….

Real numbers - The collection of both rational and irrational numbers are known as real numbers.
i.e., Real numbers = $\sqrt{2}$, $\sqrt{5}$, π, 0.102…
∴ Every irrational number is a real number, however, every real numbers are not irrational numbers.

(ii) Every point on the number line is of the form $\sqrt{m}$ where m is a natural number.

Solution:
False
The statement is false since as per the rule, a negative number cannot be expressed as square roots.
E.g., $\sqrt{9}$ =3 is a natural number.
But $\sqrt{2}$ = 1.414 is not a natural number.
Similarly, we know that there are negative numbers on the number line but when we take the root of a negative number it becomes a complex number and not a natural number.
E.g., $\sqrt{-7}$ = 7i, where i = $\sqrt{-1}$
∴ The statement that every point on the number line is of the form $\sqrt{m}$, where m is a natural number is false.

(iii) Every real number is an irrational number.

Solution:
False
The statement is false, the real numbers include both irrational and rational numbers. Therefore, every real number cannot be an irrational number.
Real numbers - The collection of both rational and irrational numbers are known as real numbers.
i.e., Real numbers = $\sqrt{2}$, $\sqrt{5}$, π, 0.102…
Irrational Numbers - A number is said to be irrational, if it **cannot** be written in the p/q, where p and q are integers and $q \neq 0$.
i.e., Irrational numbers = 0, 19/30, 2, 9/-3, -12/7, $\sqrt{2}$, $\sqrt{5}$, π, 0.102….
∴ Every irrational number is a real number, however, every real number is not irrational.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Solution:
No, the square roots of all positive integers are not irrational.
For example,
$\sqrt{4}$ = 2 is rational.
$\sqrt{9}$ = 3 is rational.
Hence, the square roots of positive integers 4 and 9 are not irrational. (2 and 3, respectively).

3. Show how √5 can be represented on the number line.

Solution:

Step 1: Let line AB be of 2 unit on a number line.

Step 2: At B, draw a perpendicular line BC of length 1 unit.

Step 3: Join CA

Step 4: Now, ABC is a right-angled triangle. Applying Pythagoras theorem,

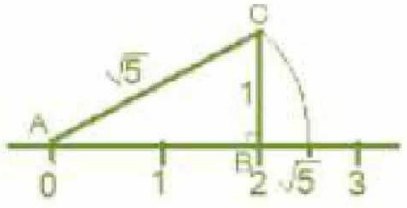
$AB^2+BC^2 = CA^2$

$2^2+1^2 = CA^2 \Rightarrow CA^2 = 5$

$\Rightarrow CA = \sqrt{5}$. Thus, CA is a line of length √5 unit.

Step 5: Taking CA as a radius and A as a center draw an arc touching the number line. The point at which number line get intersected by arc is at √5 distance from 0 because it is a radius of the circle whose center was A.

Thus, √5 is represented on the number line as shown in the figure.



4. Classroom activity (Constructing the 'square root spiral') : Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP1 of unit length. Draw a line segment P1P2 perpendicular to OP_1 of unit length (see Fig. 1.9). Now draw a line segment P_2P_3 perpendicular to OP_2. Then draw a line segment P_3P_4 perpendicular to OP_3. Continuing in Fig. 1.9 :

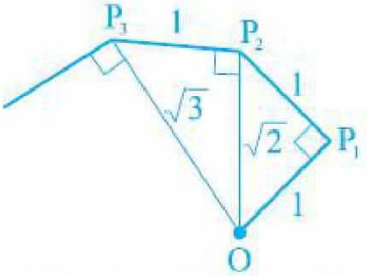
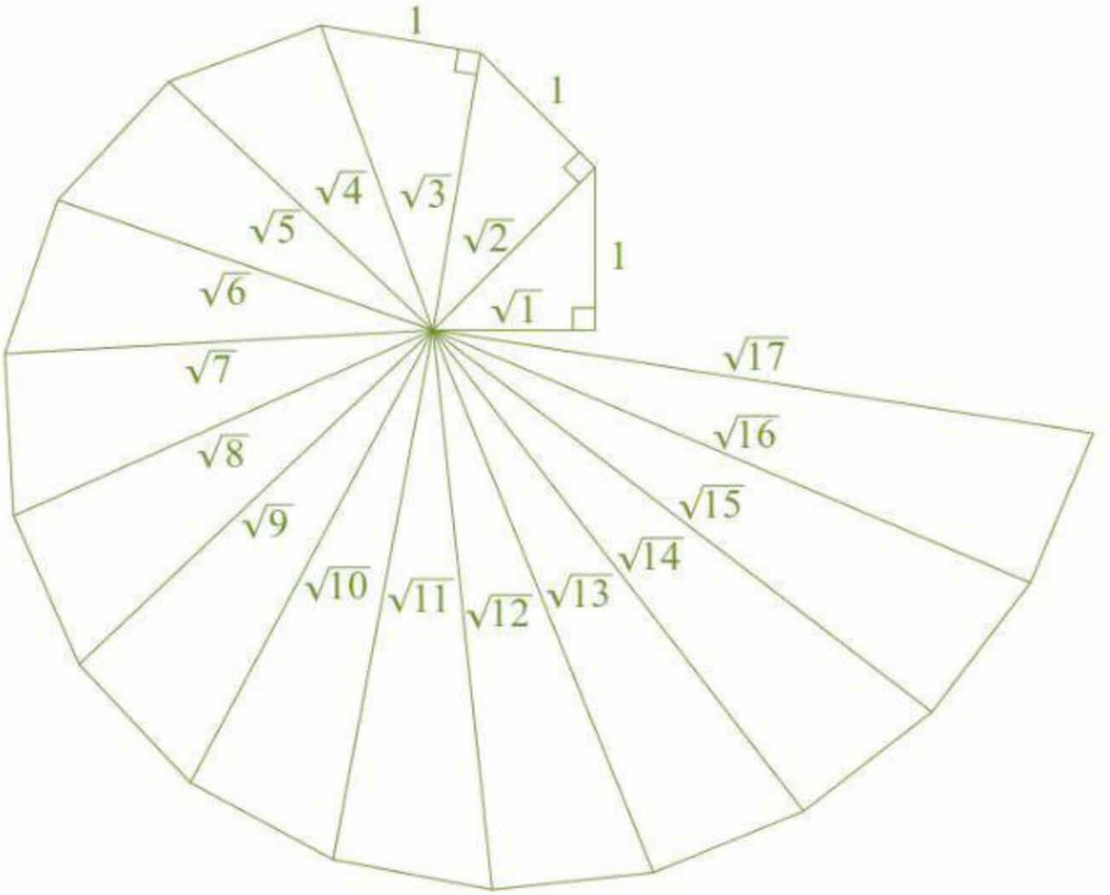


Fig. 1.9 : Constructing square root spiral

Constructing this manner, you can get the line segment $P_{n-1}Pn$ by square root spiral drawing a line segment of unit length perpendicular to OP_{n-1}. In this manner, you will have created the points P_2, P_3,....,Pn,... ., and joined them to create a beautiful spiral depicting √2, √3, √4, ...

Solution:



Step 1: Mark a point O on the paper. Here, O will be the center of the square root spiral.
Step 2: From O, draw a straight line, OA, of 1cm horizontally.
Step 3: From A, draw a perpendicular line, AB, of 1 cm.
Step 4: Join OB. Here, OB will be of $\sqrt{2}$
Step 5: Now, from B, draw a perpendicular line of 1 cm and mark the end point C.
Step 6: Join OC. Here, OC will be of $\sqrt{3}$
Step 7: Repeat the steps to draw $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$….

Exercise 1.3 Page: 14

1. Write the following in decimal form and say what kind of decimal expansion each has :

(i) 36/100

Solution:

```
         00.36
100 | 360-
      300
       600-
       600
        0
```

= 0.36 (Terminating)

(ii) 1/11

Solution:

```
     0.0909…
11 | 1
     0
     10
      0
     100
      99
       10
        0
       100
        99
         1
```

$= 0.0909\ldots = 0.\overline{09}$ (Non terminating and repeating)

(iii) $4\frac{1}{8}$

Solution:

$$4\frac{1}{8} = \frac{33}{8}$$

```
     4.125
  8 | 33
    | 32
    |  10
    |   8
    |   20
    |   16
    |    40
    |    40
    |     0
```

= 4.125 (Terminating)

(iii) 3/13

Solution:

```
     0.230769
 13 | 30
    | 26
    |  40
    |  39
    |   10
    |    0
    |   100
    |    91
    |     90
    |     78
    |     120
    |     117
    |       3
```

= 0.230769… = $0.\overline{230769}$

(iv)2/11

Solution:

```
     0.18
 11 | 2
    | 0
    | 20
    | 11
    |  90
    |  88
    |   2
```

= 0.181818181818… = $0.\overline{18}$ (Non terminating and repeating)

(iv) 329/400

Solution:

```
         0.8225
400  |  329
          0
        3290
        3200
          900
          800
          1000
           800
           2000
           2000
              0
```

= 0.8225 (Terminating)

2. You know that $1/7 = 0.\overline{142857}$. Can you predict what the decimal expansions of 2/7, 3/7, 4/7, 5/7, 6/7 are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of 1/7 carefully.]

Solution:

$1/7 = 0.\overline{142857}$

$\therefore 2\times1/7 = 2\times0.\overline{142857} = 0.\overline{285714}$

$3\times1/7 = 3\times0.\overline{142857} = 0.\overline{428571}$

$4\times1/7 = 4\times0.\overline{142857} = 0.\overline{571428}$

$5\times1/7 = 5\times0.\overline{142857} = 0.\overline{714285}$

$6\times1/7 = 6\times0.\overline{142857} = 0.\overline{857142}$

3. Express the following in the form p/q, where p and q are integers and q ≠ 0.

(i) $\mathbf{0.\overline{6}}$

Solution:

$0.\overline{6} = 0.666\ldots$

Assume that $x = 0.666\ldots$

Then, $10x = 6.666\ldots$

$10x = 6 + x$

$9x = 6$

$x = 2/3$

(ii) $0.4\overline{7}$

Solution:

$0.4\overline{7} = 0.4777\ldots$

= (4/10) + (0.777…./10)
Assume that x = 0.777…
Then, $10x$ = 7.777…
$10x = 7 + x$
$x = 7/9$
(4/10)+(0.777../10) = (4/10)+(7/90) (∴ x = 7/9 and x = 0.777…⇒0.777…/10 = 7/(9×10) = 7/90)
= (36/90)+(7/90) = 43/90

(iii) $0.\overline{001}$
Solution:

$0.\overline{001} = 0.001001\ldots$
Assume that x = 0.001001…
Then, $1000x$ = 1.001001…
$1000x = 1 + x$
$999x = 1$
$x = 1/999$

4. Express 0.99999.... in the form p/q . Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Solution:
Assume that x = 0.9999….. Eq (a)
Multiplying both sides by 10,
$10x$ = 9.9999…. Eq. (b)
Eq.(b) – Eq.(a), we get
$10x$ = 9.9999…
$- x$ = - 0.9999…
$9x = 9$
$x = 1$

The difference between 1 and 0.999999 is 0.000001 which is negligible.
Hence, we can conclude that, 0.999 is too much near 1, therefore, 1 as the answer can be justified.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of 1/17? Perform the division to check your answer.

Solution:
1/17

Dividing 1 by 17:

```
        0.0588235294117647
17 ) 1
     0
     10
     0
     100
     85
     150
     136
     140
     136
     40
     34
     60
     51
     90
     85
     50
     34
     160
     153
     70
     68
     20
     17
     30
     17
     130
     119
     110
     102
     80
     68
     120
     119
     1
```

$\frac{1}{17} = 0.\overline{0588235294117647}$

∴ There are 16 digits in the repeating block of the decimal expansion of 1/17.

6. Look at several examples of rational numbers in the form p/q (q ≠ 0), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Solution:

We observe that when q is 2, 4, 5, 8, 10… Then the decimal expansion is terminating. For example:

1/2 = 0. 5, denominator q = 2^1

7/8 = 0. 875, denominator q = 2^3

4/5 = 0. 8, denominator q = 5^1

We can observe that the terminating decimal may be obtained in the situation where prime factorization of the denominator of the given fractions has the power of only 2 or only 5 or both.

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Solution:

We know that all irrational numbers are non-terminating non-recurring. ∴ three numbers with decimal expansions that are non-terminating non-recurring are:

a) √3 = 1.732050807568

b) √26 = 5.099019513592

c) √101 = 10.04987562112

8. Find three different irrational numbers between the rational numbers 5/7 and 9/11.

Solution:

$\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

∴ Three different irrational numbers are:

a) 0.73073007300073000073...

b) 0.75075007300075000075…

c) 0.76076007600076000076...

9. Classify the following numbers as rational or irrational according to their type:

(i)√23

Solution:

√23 = 4.79583152331…

Since the number is non-terminating non-recurring therefore, it is an irrational number.

(ii)√225

Solution:

√225 = 15 = 15/1
Since the number can be represented in p/q form, it is a rational number.

(i) 0.3796

Solution:
Since the number,0.3796, is terminating, it is a rational number.

(ii) 7.478478

Solution:
The number,7.478478, is non-terminating but recurring, it is a rational number.

(iii) 1.101001000100001…

Solution:
Since the number,1.101001000100001…, is non-terminating non-repeating (non-recurring), it is an irrational number.