Exercise: 7.1 (Page No: 118)

1. In quadrilateral ACBD, AC = AD and AB bisect \angle A (see Fig. 7.16). Show that \triangle ABC \cong \triangle ABD. What can you say about BC and BD?

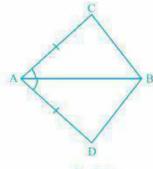


Fig. 7.16

Solution:

It is given that AC and AD are equal i.e. AC = AD and the line segment AB bisects \angle A. We will have to now prove that the two triangles ABC and ABD are similar i.e. \triangle ABC \cong \triangle ABD **Proof:**

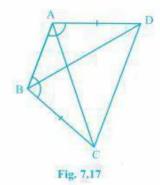
Consider the triangles ΔABC and ΔABD,

- (i) AC = AD (It is given in the question)
- (ii) AB = AB (Common)
- (iii) $\angle CAB = \angle DAB$ (Since AB is the bisector of angle A)

So, by **SAS** congruency criterion, $\triangle ABC \cong \triangle ABD$.

For the 2nd part of the question, BC and BD are of equal lengths by the rule of C.P.C.T.

- 2. ABCD is a quadrilateral in which AD = BC and \angle DAB = \angle CBA (see Fig. 7.17). Prove that
- (i) $\triangle ABD \cong \triangle BAC$
- (ii) BD = AC
- (iii) $\angle ABD = \angle BAC$.



Solution:

The given parameters from the questions are $\angle DAB = \angle CBA$ and AD = BC.

(i) \triangle ABD and \triangle BAC are similar by SAS congruency as AB = BA (It is the common arm) \angle DAB = \angle CBA and AD = BC (These are given in the question) So, triangles ABD and BAC are similar i.e. \triangle ABD \cong \triangle BAC. (Hence proved).

(ii) It is now known that $\triangle ABD \cong \triangle BAC$ so, BD = AC (by the rule of CPCT).

(iii) Since $\triangle ABD \cong \triangle BAC$ so, Angles $\angle ABD = \angle BAC$ (by the rule of CPCT).

3. AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.

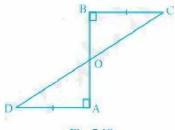


Fig. 7.18

Solution:

It is given that AD and BC are two equal perpendiculars to AB. We will have to prove that **CD** is the bisector of AB Now,

Triangles ΔAOD and ΔBOC are similar by AAS congruency since:

- (i) $\angle A = \angle B$ (They are perpendiculars)
- (ii) AD = BC (As given in the question)
- (iii) $\angle AOD = \angle BOC$ (They are vertically opposite angles)
- $\therefore \triangle AOD \cong \triangle BOC.$

So, AO = OB (by the rule of CPCT).

Thus, CD bisects AB (Hence proved).

4. I and m are two parallel lines intersected by another pair of parallel lines p and q (see Fig. 7.19). Show that $\triangle ABC \cong \triangle CDA$.

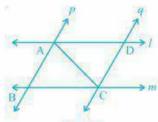


Fig. 7.19

Solution:

It is given that p II q and I II m

To prove:

Triangles ABC and CDA are similar i.e. \triangle ABC \cong \triangle CDA

Proof:

Consider the AABC and ACDA,

- (i) \angle BCA = \angle DAC and \angle BAC = \angle DCA Since they are alternate interior angles
- (ii) AC = CA as it is the common arm

So, by **ASA congruency criterion**, $\triangle ABC \cong \triangle CDA$.

- 5. Line I is the bisector of an angle $\angle A$ and B is any point on I. BP and BQ are perpendiculars from B to the arms of $\angle A$ (see Fig. 7.20). Show that:
- (i) $\triangle APB \cong \triangle AQB$
- (ii) BP = BQ or B is equidistant from the arms of $\angle A$.

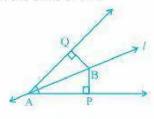


Fig. 7.20

Solution:

It is given that the line "l" is the bisector of angle $\angle A$ and the line segments BP and BQ are perpendiculars drawn from l.

(i) ΔAPB and ΔAQB are similar by AAS congruency because:

 $\angle P = \angle Q$ (They are the two right angles)

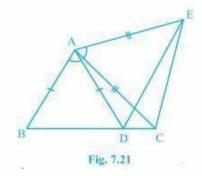
AB = AB (It is the common arm)

 \angle BAP = \angle BAQ (As line *l* is the bisector of angle A)

So, $\triangle APB \cong \triangle AQB$.

(ii) By the rule of CPCT, BP = BQ. So, it can be said the point B is equidistant from the arms of $\angle A$.

6. In Fig. 7.21, AC = AE, AB = AD and \angle BAD = \angle EAC. Show that BC = DE.



Solution:

It is given in the question that AB = AD, AC = AE, and \angle BAD = \angle EAC

To prove:

The line segment BC and DE are similar i.e. BC = DE

Proof:

We know that $\angle BAD = \angle EAC$

Now, by adding ∠DAC on both sides we get,

 $\angle BAD + \angle DAC = \angle EAC + \angle DAC$

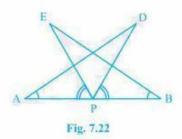
This implies, $\angle BAC = \angle EAD$

Now, ΔABC and ΔADE are similar by SAS congruency since:

- (i) AC = AE (As given in the question)
- (ii) $\angle BAC = \angle EAD$
- (iii) AB = AD (It is also given in the question)

∴ Triangles ABC and ADE are similar i.e. \triangle ABC \cong \triangle ADE. So, by the rule of CPCT, it can be said that BC = DE.

- 7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that \angle BAD = \angle ABE and \angle EPA = \angle DPB (see Fig. 7.22). Show that
- (i) $\Delta DAP \cong \Delta EBP$
- (ii) AD = BE



Solutions:

In the question, it is given that P is the mid-point of line segment AB. Also, \angle BAD = \angle ABE and \angle EPA = \angle DPB

(i) It is given that $\angle EPA = \angle DPB$

Now, add ∠DPE on both sides,

 $\angle EPA + \angle DPE = \angle DPB + \angle DPE$

This implies that angles DPA and EPB are equal i.e. \angle DPA = \angle EPB

Now, consider the triangles DAP and EBP.

 $\angle DPA = \angle EPB$

AP = BP (Since P is the mid-point of the line segment AB)

 \angle BAD = \angle ABE (As given in the question)

So, by **ASA congruency**, $\Delta DAP \cong \Delta EBP$.

- (ii) By the rule of CPCT, AD = BE.
- 8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig. 7.23). Show that:
- (i) $\triangle AMC \cong \triangle BMD$
- (ii) ∠DBC is a right angle.
- (iii) ΔDBC ≅ ΔACB

(iv) CM = 1/2 AB

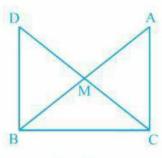


Fig. 7.23

Solution:

It is given that M is the mid-point of the line segment AB, $\angle C = 90^{\circ}$, and DM = CM

(i) Consider the triangles $\triangle AMC$ and $\triangle BMD$:

AM = BM (Since M is the mid-point)

CM = DM (Given in the question)

 \angle CMA = \angle DMB (They are vertically opposite angles)

So, by **SAS congruency criterion**, $\triangle AMC \cong \triangle BMD$.

(ii) $\angle ACM = \angle BDM$ (by CPCT)

: AC II BD as alternate interior angles are equal.

Now, \angle ACB + \angle DBC = 180° (Since they are co-interiors angles)

 \Rightarrow 90° + \angle B = 180°

∴ ∠DBC = 90°

(iii) In ΔDBC and ΔACB,

BC = CB (Common side)

 \angle ACB = \angle DBC (They are right angles)

DB = AC (by CPCT)

So, $\triangle DBC \cong \triangle ACB$ by **SAS congruency**.

(iv) DC = AB (Since \triangle DBC \cong \triangle ACB)

 \Rightarrow DM = CM = AM = BM (Since M the is mid-point)

So, DM + CM = BM + AM

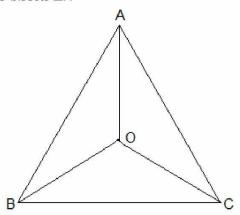
Hence, CM + CM = AB

⇒ CM = (½) AB

Exercise: 7.2 (Page No: 123)

1. In an isosceles triangle ABC, with AB = AC, the bisectors of \angle B and \angle C intersect each other at O. Join A to O. Show that:

(i)
$$OB = OC$$



Solution:

Given:

 \rightarrow AB = AC and

→ the bisectors of ∠B and ∠C intersect each other at O

(i) Since ABC is an isosceles with AB = AC,

 $\Rightarrow \angle B = \angle C$

 \Rightarrow ½ \angle B = ½ \angle C

 $\Rightarrow \angle OBC = \angle OCB$ (Angle bisectors)

: OB = OC (Side opposite to the equal angles are equal.)

(ii) In ΔAOB and ΔAOC,

AB = AC (Given in the question)

AO = AO (Common arm)

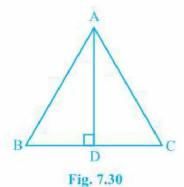
OB = OC (As Proved Already)

So, $\triangle AOB \cong \triangle AOC$ by SSS congruence condition.

 \angle BAO = \angle CAO (by CPCT)

Thus, AO bisects ∠A.

2. In \triangle ABC, AD is the perpendicular bisector of BC (see Fig. 7.30). Show that \triangle ABC is an isosceles triangle in which AB = AC.



Solution:

It is given that AD is the perpendicular bisector of BC

To prove:

AB = AC

Proof:

In ΔADB and ΔADC,

AD = AD (It is the Common arm)

 $\angle ADB = \angle ADC$

BD = CD (Since AD is the perpendicular bisector)

So, $\triangle ADB \cong \triangle ADC$ by **SAS** congruency criterion.

Thus,

AB = AC (by CPCT)

3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig. 7.31). Show that these altitudes are equal.

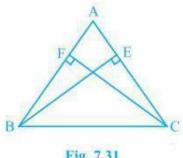


Fig. 7.31

Solution:

Given:

- (i) BE and CF are altitudes.
- (ii) AC = AB

To prove:

BE = CF

Proof:

Triangles ΔAEB and ΔAFC are similar by AAS congruency since

 $\angle A = \angle A$ (It is the common arm)

 $\angle AEB = \angle AFC$ (They are right angles)

AB = AC (Given in the question)

 $\therefore \triangle AEB \cong \triangle AFC$ and so, BE = CF (by CPCT).

4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig. 7.32). Show that

- (i) $\triangle ABE \cong \triangle ACF$
- (ii) AB = AC, i.e., ABC is an isosceles triangle.

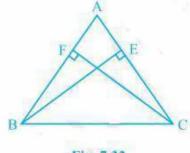


Fig. 7.32

Solution:

It is given that BE = CF

(i) In ΔABE and ΔACF,

 $\angle A = \angle A$ (It is the common angle)

 $\angle AEB = \angle AFC$ (They are right angles)

BE = CF (Given in the question)

 $\therefore \triangle ABE \cong \triangle ACF$ by **AAS** congruency condition.

(ii) AB = AC by CPCT and so, ABC is an isosceles triangle.

5. ABC and DBC are two isosceles triangles on the same base BC (see Fig. 7.33). Show that \angle ABD = \angle ACD.

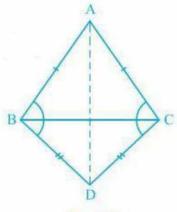


Fig. 7.33

Solution:

In the question, it is given that ABC and DBC are two isosceles triangles.

We will have to show that $\angle ABD = \angle ACD$

Proof:

Triangles ΔABD and ΔACD are similar by SSS congruency since

AD = AD (It is the common arm)

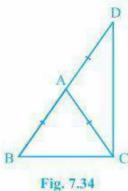
AB = AC (Since ABC is an isosceles triangle)

BD = CD (Since BCD is an isosceles triangle)

So, $\triangle ABD \cong \triangle ACD$.

 \therefore \angle ABD = \angle ACD by CPCT.

6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see Fig. 7.34). Show that ∠BCD is a right angle.



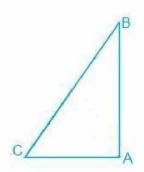
Solution:

```
It is given that AB = AC and AD = AB
We will have to now prove \angle BCD is a right angle.
Proof:
Consider AABC,
AB = AC (It is given in the question)
Also, \angle ACB = \angle ABC (They are angles opposite to the equal sides and so, they are equal)
Now, consider ΔACD,
AD = AB
Also, \angle ADC = \angle ACD (They are angles opposite to the equal sides and so, they are equal)
Now,
In AABC,
\angleCAB + \angleACB + \angleABC = 180°
So, \angleCAB + 2\angleACB = 180°
\Rightarrow \angleCAB = 180° - 2\angleACB --- (i)
Similarly, in ΔADC,
∠CAD = 180° - 2∠ACD --- (ii)
also,
\angleCAB + \angleCAD = 180° (BD is a straight line.)
Adding (i) and (ii) we get,
\angleCAB + \angleCAD = 180° - 2\angleACB+180° - 2\angleACD
\Rightarrow 180° = 360° - 2\angleACB-2\angleACD
```

7. ABC is a right-angled triangle in which $\angle A = 90^{\circ}$ and AB = AC. Find $\angle B$ and $\angle C$.

Solution:

 \Rightarrow \angle BCD = 90°



 \Rightarrow 2(\angle ACB+ \angle ACD) = 180°

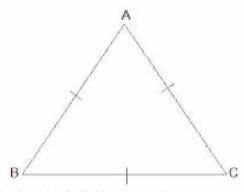
In the question, it is given that

$$\angle$$
A = 90° and AB = AC
AB = AC
 \Rightarrow \angle B = \angle C (They are angles opposite to the equal sides and so, they are equal)
Now,
 \angle A+ \angle B+ \angle C = 180° (Since the sum of the interior angles of the triangle)
 \therefore 90° + 2 \angle B = 180°
 \Rightarrow 2 \angle B = 90°
 \Rightarrow \angle B = 45°
So, \angle B = \angle C = 45°

8. Show that the angles of an equilateral triangle are 60° each.

Solution:

Let ABC be an equilateral triangle as shown below:



Here, BC = AC = AB (Since the length of all sides is same)

 \Rightarrow $\angle A = \angle B = \angle C$ (Sides opposite to the equal angles are equal.)

Also, we know that

$$\Rightarrow$$
 $\angle A = 60^{\circ}$

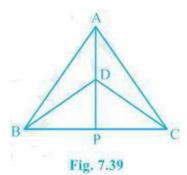
$$\therefore \angle A = \angle B = \angle C = 60^{\circ}$$

So, the angles of an equilateral triangle are always 60° each.

Exercise: 7.3 (Page No: 128)

1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to intersect BC at P, show that

- (i) $\triangle ABD \cong \triangle ACD$
- (ii) $\triangle ABP \cong \triangle ACP$
- (iii) AP bisects ∠A as well as ∠D.
- (iv) AP is the perpendicular bisector of BC.



Solution:

In the above question, it is given that $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles.

(i) ΔABD and ΔACD are similar by SSS congruency because:

AD = AD (It is the common arm)

AB = AC (Since $\triangle ABC$ is isosceles)

BD = CD (Since \triangle DBC is isosceles)

 $\therefore \triangle ABD \cong \triangle ACD.$

(ii) ΔABP and ΔACP are similar as:

AP = AP (It is the common side)

 $\angle PAB = \angle PAC$ (by CPCT since $\triangle ABD \cong \triangle ACD$)

AB = AC (Since $\triangle ABC$ is isosceles)

So, $\triangle ABP \cong \triangle ACP$ by SAS congruency condition.

(iii) $\angle PAB = \angle PAC$ by CPCT as $\triangle ABD \cong \triangle ACD$.

AP bisects ∠A. --- (i)

Also, \triangle BPD and \triangle CPD are similar by SSS congruency as

```
PD = PD (It is the common side)

BD = CD (Since \triangleDBC is isosceles.)

BP = CP (by CPCT as \triangleABP \cong \triangleACP)

So, \triangleBPD \cong \triangleCPD.

Thus, \angleBDP = \angleCDP by CPCT. --- (ii)

Now by comparing (i) and (ii) it can be said that AP bisects \angleA as well as \angleD.
```

(iv)
$$\angle$$
BPD = \angle CPD (by CPCT as \triangle BPD \cong \triangle CPD) and BP = CP --- (i) also,
 \angle BPD + \angle CPD = 180° (Since BC is a straight line.)
 \Rightarrow 2 \angle BPD = 180°
 \Rightarrow \angle BPD = 90° --- (ii)
Now, from equations (i) and (ii), it can be said that AP is the perpendicular bisector of BC.

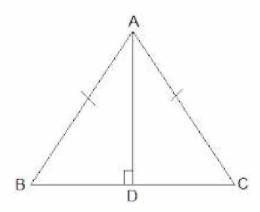
2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that

(i) AD bisects BC

(ii) AD bisects ∠A.

Solution:

It is given that AD is an altitude and AB = AC. The diagram is as follows:



```
    (i) In ΔABD and ΔACD,
    ∠ADB = ∠ADC = 90°
    AB = AC (It is given in the question)
    AD = AD (Common arm)
```

 \therefore $\triangle ABD \cong \triangle ACD$ by RHS congruence condition.

Now, by the rule of CPCT,

BD = CD.

So, AD bisects BC

(ii) Again, by the rule of CPCT, \angle BAD = \angle CAD Hence, AD bisects \angle A.

- 3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of Δ PQR (see Fig. 7.40). Show that:
- (i) $\triangle ABM \cong \triangle PQN$
- (ii) $\triangle ABC \cong \triangle PQR$

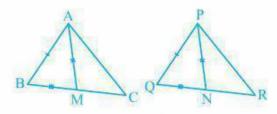


Fig. 7.40

Solution:

Given parameters are:

AB = PQ,

BC = QR and

AM = PN

(i) ½ BC = BM and ½ QR = QN (Since AM and PN are medians)

Also, BC = QR

So, 1/2 BC = 1/2 QR

 \Rightarrow BM = QN

In ΔABM and ΔPQN,

AM = PN and AB = PQ (As given in the question)

BM = QN (Already proved)

 \therefore $\triangle ABM \cong \triangle PQN$ by SSS congruency.

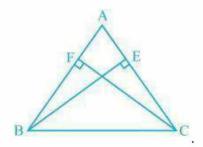
(ii) In ΔABC and ΔPQR,

AB = PQ and BC = QR (As given in the question)

 $\angle ABC = \angle PQR$ (by CPCT)

So, $\triangle ABC \cong \triangle PQR$ by SAS congruency.

4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.



Solution:

It is known that BE and CF are two equal altitudes.

Now, in ΔBEC and ΔCFB,

 \angle BEC = \angle CFB = 90° (Same Altitudes)

BC = CB (Common side)

BE = CF (Common side)

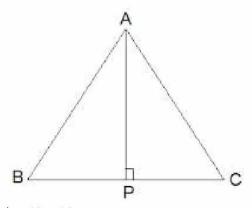
So, $\Delta BEC \cong \Delta CFB$ by RHS congruence criterion.

Also, $\angle C = \angle B$ (by CPCT)

Therefore, AB = AC as sides opposite to the equal angles is always equal.

5. ABC is an isosceles triangle with AB = AC. Draw AP \perp BC to show that \angle B = \angle C.

Solution:



In the question, it is given that AB = AC

Now, \triangle ABP and \triangle ACP are similar by RHS congruency as

 $\angle APB = \angle APC = 90^{\circ}$ (AP is altitude)

AB = AC (Given in the question)

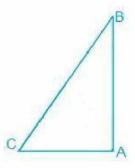
AP = AP (Common side)

So, $\triangle ABP \cong \triangle ACP$.

 $\therefore \angle B = \angle C$ (by CPCT)

Exercise: 7.4 (Page No: 132)

1. Show that in a right-angled triangle, the hypotenuse is the longest side.



Solution:

It is known that ABC is a triangle right angled at B.

We know that,

$$\angle A + \angle B + \angle C = 180^{\circ}$$

Now, if $\angle B+\angle C=90^{\circ}$ then $\angle A$ has to be 90°.

Since A is the largest angle of the triangle, the side opposite to it must be the largest. So, AB is the hypotenuse which will be the largest side of the above right-angled triangle i.e. \triangle ABC.

2. In Fig. 7.48, sides AB and AC of \triangle ABC are extended to points P and Q respectively. Also, \angle PBC < \angle QCB. Show that AC > AB.

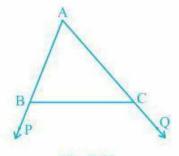


Fig. 7.48

Solution:

It is given that \angle PBC < \angle QCB We know that \angle ABC + \angle PBC = 180° So, \angle ABC = 180°- \angle PBC

Also,

 \angle ACB + \angle QCB = 180°

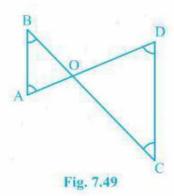
Therefore ∠ACB = 180° -∠QCB

Now, since $\angle PBC < \angle QCB$,

∴ ∠ABC > ∠ACB

Hence, AC > AB as sides opposite to the larger angle is always larger.

3. In Fig. 7.49, $\angle B < \angle A$ and $\angle C < \angle D$. Show that AD < BC.



Solution:

In the question, it is mentioned that angles B and angle C is smaller than angles A and D respectively i.e. \angle B < \angle A and \angle C < \angle D.

Now

Since the side opposite to the smaller angle is always smaller

AO < BO --- (i)

And OD < OC ---(ii)

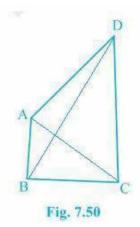
By adding equation (i) and equation (ii) we get

AO+OD < BO + OC

So, AD < BC

 AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see Fig. 7.50).

Show that $\angle A > \angle C$ and $\angle B > \angle D$.



Solution:

In ΔABD, we see that

AB < AD < BD

So, ∠ADB < ∠ABD --- (i) (Since angle opposite to longer side is always larger)

Now, in ΔBCD,

BC < DC < BD

Hence, it can be concluded that

∠BDC < ∠CBD --- (ii)

Now, by adding equation (i) and equation (ii) we get,

∠ADB + ∠BDC < ∠ABD + ∠CBD

⇒ ∠ADC < ∠ABC

 $\Rightarrow \angle B > \angle D$

Similarly, In triangle ABC,

∠ACB < ∠BAC --- (iii) (Since the angle opposite to the longer side is always larger)

Now, In ΔADC,

 $\angle DCA < \angle DAC --- (iv)$

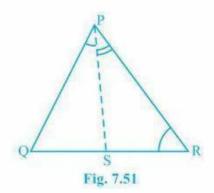
By adding equation (iii) and equation (iv) we get,

 \angle ACB + \angle DCA < \angle BAC+ \angle DAC

⇒ ∠BCD < ∠BAD

∴ ∠A > ∠C

5. In Fig 7.51, PR > PQ and PS bisect \angle QPR. Prove that \angle PSR > \angle PSQ.



Solution:

It is given that PR > PQ and PS bisects ∠QPR

Now we will have to prove that angle PSR is smaller than PSQ i.e. \angle PSR > \angle PSQ

Proof:

 $\angle QPS = \angle RPS --- (ii)$ (As PS bisects $\angle QPR$)

 $\angle PQR > \angle PRQ ---$ (i) (Since PR > PQ as angle opposite to the larger side is always larger)

 \angle PSR = \angle PQR + \angle QPS --- (iii) (Since the exterior angle of a triangle equals to the sum of opposite interior angles)

 \angle PSQ = \angle PRQ + \angle RPS --- (iv) (As the exterior angle of a triangle equals to the sum of opposite interior angles)

By adding (i) and (ii)

 $PQR + \angle QPS > \angle PRQ + \angle RPS$

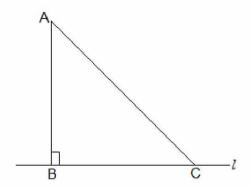
Thus, from (i), (ii), (iii) and (iv), we get

 $\angle PSR > \angle PSQ$

6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Solution:

First, let "l" be a line segment and "B" be a point lying on it. A line AB perpendicular to l is now drawn. Also, let C be any other point on l. The diagram will be as follows:



To prove:

AB < AC

Proof:

In $\triangle ABC$, $\angle B = 90^{\circ}$ Now, we know that

∠A+∠B+∠C = 180°

∴ ∠A +∠C = 90°

Hence, $\angle C$ must be an acute angle which implies $\angle C < \angle B$

So, AB < AC (As the side opposite to the larger angle is always larger)