EXERCISE 9.1

Write the correct answer in each of the following:

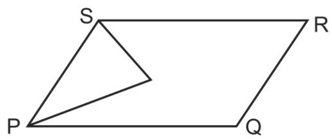
1. The median of a triangle divides it into two

(*a*) triangles of equal area. (*b*) congruent triangles

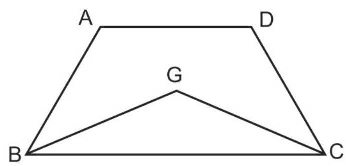
(*c*) right triangles (*d*) isosceles triangles.

Sol. The median of a triangle divides it into two triangles of equal area.
Hence, (*a*) is the correct answer.

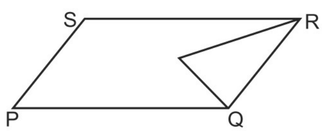
2. In which of the following figures, you find two polygons on the same base and between the same parallels?



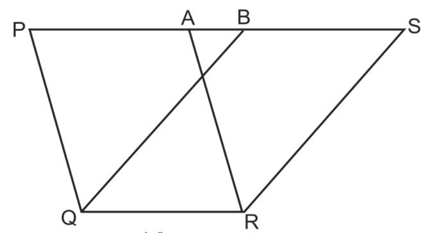
(*a*)



(*b*)



(*c*)



(*d*)

Sol. In figure (*d*), we find two polygons (parallelograms) on the same base and between the same parallels.

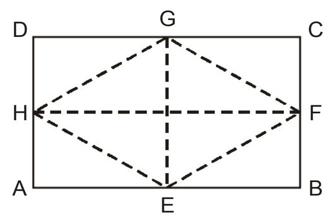
3. The figure obtained by joining the mid-points of the adjacent sides of a rectangle of sides 8 cm and 6 cm is:

(*a*) a rectangle of area 24 cm^2

(*b*) a square of area 25 cm^2

(*c*) a trapezium of area 24 cm^2

(*d*) a rhombus of area 24 cm^2

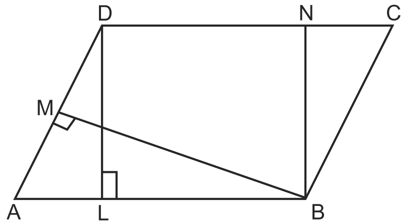


Sol. ABCD is a rectangle and E, F, G and H are the mid-points of the sides AB, BC, CD and DA respectively. The figure obtained is a rhombus whose area $= \frac{1}{2} \times EG \times FH = \frac{1}{2} \times 6\text{ cm} \times 8\text{ cm} = 24\text{ cm}^2$

Hence, (*d*) is the correct answer.

4. In the given figure, the area of parallelogram ABCD is:

(*a*) AB × BM (*b*) BC × BN (*c*) DC × DL (*d*) AD × DL



Sol. Area of a parallelogram = Base × Corresponding altitude

= AB × DL = DC × DL

[∵ AB = DC (Opposite sides of a || gm)]

Hence, (*c*) is the correct answer.

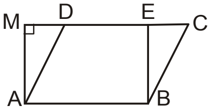
5. In the given figure, if parallelogram ABCD and rectangle ABEM are of equal area then

(*a*) perimeter of ABCD = perimeter of ABEM

(*b*) perimeter of ABCD < perimeter of ABEM

(*c*) perimeter of ABCD > perimeter of ABEM

(*d*) perimeter of ABCD $= \frac{1}{2}$ (perimeter of ABEM)



Sol. If parallelogram ABCD and rectangle ABEM are of equal area, then perimeter of ABCD > perimeter of ABEM because of all the line segments to a given line from a point outside it, the perpendicular is the least. Hence, (*c*) is the correct answer.

6. The mid-point of the sides of a triangle along with any of the vertices as the fourth point make a parallelogram of area equal to

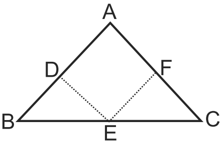
(*a*) $\frac{1}{2}$ ar (ΔABC) (*b*) $\frac{1}{3}$ ar (ΔABC)

(*c*) $\frac{1}{4}$ ar (ΔABC) (*d*) ar (ΔABC)

Sol. Since median of a triangle divides it into two triangles of equal area

∴ ar (ΔADE) = ar (ΔBDE) ...(1)

ar (ΔAEF) = ar (ΔEFC) ...(2)



Since AE is the diagonal of a parallelogram ADEF. It divides it into two triangles of equal area.

$\therefore$ ar (ΔADE) = ar (ΔAFE) ... (3)

From (1), (2) and (3), we get

$\therefore$ ar (ΔADE) = ar (ΔBDE) = ar (ΔAEF) = ar (ΔEFC)

Hence, ar (ADEF) = $\frac{1}{2}$ ar (ΔABC)

So, (*a*) is the correct answer.

7. Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is

(*a*) 1 : 2 (*b*) 1 : 1 (*c*) 2 : 1 (*d*) 3 : 1

Sol. We know that parallelograms on the same or equal bases and between the same parallels are equal in area .

So, the ratio of their areas is 1:1

Hence, (*b*) is the correct answer.

8. ABCD is a quadrilateral whose diagonal AC divides it in two parts, equal in area, then ABCD

(*a*) is a rectangle (*b*) is always a rhombus

(*c*) is a parallelogram (*d*) need not be any of (*a*), (*b*), or (*c*).

Sol. Since diagonal of a parallelogram divides it into two triangles of equal area and rectangle and a rhombus are also parallelograms. then ABCD need not be any of (*a*), (*b*) or (*c*).

Hence, (*d*) is the correct answer.

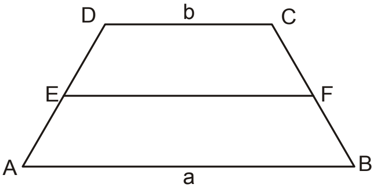
9. If a triangle and a parallelogram are on the same base and between the same parallels, then the ratio of the area of the triangle to the area of parallelogram is

(*a*) 1 : 3 (*b*) 1 : 2 (*c*) 3 : 1 (*d*) 1 : 4

Sol. We know that a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to half the area of the parallelogram. Hence the ratio of the area of the triangle to the area of parallelogram is 1:2.

Hence, (*b*) is the correct answer.

10. ABCD is a trapezium with parallel sides AB = *a* cm and DC = *b* cm (see fig.) E and F are the mid-points of non parallel sides. The ratio of ar (ABFE) and ar (EFCD) is



(*a*) $a : b$

(*b*) $(3a + b) : (a + 3b)$

(*c*) $(a + 3b) : (3a + b)$

(*d*) $(2a + b) : (3a + b)$

Sol. ABCD is a trapezium in which AB||DC. E and F are the mid-points of AD and BC, so

$$EF = \frac{1}{2}(a + b)$$

ABEF and EFCD are also trapeziums.

$$\text{ar}(\text{ABEF}) = \frac{1}{2}\left[\frac{1}{2}(a+b)+a\right] \times h = \frac{h}{4}(3a+b)$$

$$\text{ar}(\text{EFCD}) = \frac{1}{2}\left[b+\frac{1}{2}(a+b)\right] \times h = \frac{h}{4}(a+3b)$$

$$\therefore \quad \frac{\text{ar}(\text{ABEF})}{\text{ar}(\text{EFCD})} = \frac{\frac{h}{4}(3a+b)}{\frac{h}{4}(a+3b)} = \frac{(3a+b)}{(a+3b)}$$

So, the required ratio is $(3a + b) : (a + 3b)$.

Hence, (*b*) is the correct answer.

EXERCISE 9.2

Write True or False and justify your answer:

1. ABCD is a parallelogram and X is the mid-point of AB. If ar (AXCD) = 24 cm^2, then ar (Δ ABC) = 24 cm^2.

Sol. We have ABCD is a parallelogram and X is the mid-point of AB.

Now, $\quad \text{ar}(ABCD) = \text{ar}(AXCD) + \text{ar}(\Delta XBC) \quad$ ...(1)

$\because$ Diagonal AC of a parallelogram divides it into two triangles of equal area.

$\therefore \quad \text{ar}(ABCD) = 2\text{ar}(\Delta ABC) \quad$ ...(2)

Again X is the mid-point of AB, so

$$\text{ar}(\Delta CXB) = \frac{1}{2}\text{ar}(\Delta ABC) \quad ...(3)$$

[$\because$ Median divides the triangle in two triangles of equal area]

$$\therefore \quad 2\text{ar}(\Delta ABC) = 24 + \frac{1}{2}\text{ar}(\Delta ABC)$$

[Using (1), (2) and (3)]

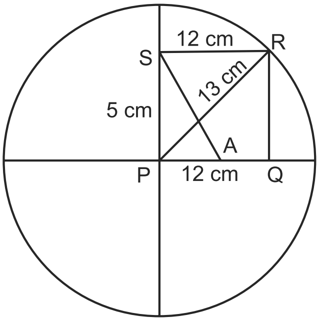
$$\therefore \quad 2\text{ar}(\Delta ABC) - \frac{1}{2}\text{ar}(\Delta ABC) = 24$$

$$\Rightarrow \quad \frac{3}{2}\text{ar}(\Delta ABC) = 24$$

$\Rightarrow \quad \text{ar}(\Delta ABC) = \dfrac{2 \times 24}{3} = 16 \text{ cm}^2$

Hence, the given statement is false.

2. PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on PQ. If PS = 5 cm, then ar (ΔPAS) = 30 cm^2.



Sol. It is given that A is any point on PQ, therefore, PA < PQ.

It is given that A is any point on PQ, therefore PA < PQ.

$$\text{Now, ar}(\Delta PQR) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Now, ar}(\Delta PQR) = \frac{1}{2} \times PQ \times QR = \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$$

[$\because$ PQRS is a rectangle $\therefore$ RQ = SP = 5 cm]

As PA < PQ (= 12 cm)

So ar (ΔPAS) < ar (ΔPQR)

or ar (ΔPAS) < 30 cm^2 [$\because$ ar (ΔPQR) = 30 cm^2]

Hence, the given statement is false.

3. PQRS is a parallelogram whose area is 180 cm^2 and A is any point on the diagonal QS. The area of Δ ASR = 90 cm^2.

Sol. PQRS is a parallelogram.

We know that diagonal (QS) of a parallelogram divides parallelogram into two triangles of equal area, so

$$\therefore \quad \text{ar}(\Delta QRS) = \frac{1}{2} \text{ar}(\parallel \text{gm } PQRS)$$

$$= \frac{1}{2} \times 180 = 90 \text{ cm}^2$$

$\because$ A is any point on SQ

$\therefore$ ar (ΔASR) < ar (ΔQRS)

Hence, ar (ΔASR) < 90 cm^2.

Hence, the given statement is false.

4. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Then, ar (ΔBDE) = $\dfrac{1}{4}$ ar (ΔABC).

Sol. ΔABC and ΔBDE are two equilateral triangles.

Let each side of triangle ABC be x.

Again, D is the mid-point of BC, so each side of triangle BDE is $\frac{x}{2}$.

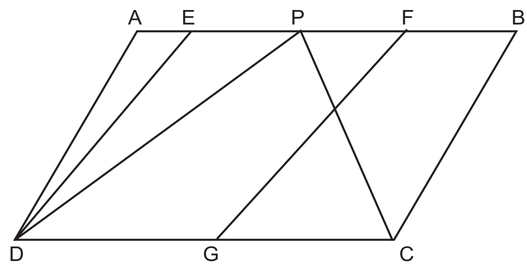
Now, $$\frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{\frac{\sqrt{3}}{4}\times\left(\frac{x}{2}\right)^2}{\frac{\sqrt{3}}{4}\times x^2} = \frac{x^2}{4x^2} = \frac{1}{4}$$

Hence, $ar(\Delta BDE) = \frac{1}{4} ar(\Delta ABC)$

∴ The given statement is true.

5. In the given figure, ABCD and EFGD are two parallelograms and G is the mid-point of CD. Then

$ar(\Delta DPC) = \frac{1}{2} ar(\| gm\ EFGD)$.



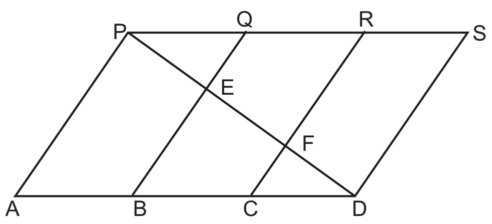
Sol. As ΔDPC and || gm ABCD are on the same base DC and between the same parallels AB and DC, so

$$ar(\Delta DPC) = \frac{1}{2} ar(\| gm.\ ABCD)$$
$$= ar(\| gm\ EFGD) \quad [\because \text{ G is the mid point of DC}]$$

Hence, the given statement is false.

EXERCISE 9.3

1. In given figure, PSDA is a parallelogram. Points Q and R are taken on PS such that PQ = QR = RS and PA || QB || RC. Prove that ar (ΔPQE) = ar (ΔCFD).



Sol. PSDA is a parallelogram. Points Q and R are taken on PS such that PQ = QR = RS and PA || QB || RC.

We have to prove that ar (ΔPQE) = ar (ΔCFD).

Now, PS = AD [Opp. sides of a || gm]

$\therefore \quad \frac{1}{3} PS = \frac{1}{3} AD \Rightarrow PQ = CD$ ...(1)

Again, PS || AD and QB cut them,

$\therefore \quad \angle PQE = \angle CBE$ [Alt. ∠s] ...(2)

Now, QB || RC and AD cuts them

$\therefore \quad \angle QBD = \angle RCD$ [Corres. ∠s]...(3)

so, $\angle PQE = \angle FCD$ ...(4)

[From (2) and (3), ∠CBE and ∠QBD are same and ∠RCD and ∠FCD are same]

Now, in ΔPQE and ΔCFD

$\angle QPE = \angle CDF$ [Alt. ∠s]

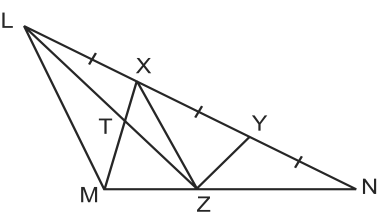
$PQ = CD$ [From (1)]

and $\angle PQE = \angle FCD$ [From (4)]

$\therefore \quad \Delta PQE \cong \Delta CFD$ [By ASA Congruence rule]

Hence, ar (ΔPQE) = ar (ΔCFD) [Congruent Δs are equal in area]

2. X and Y are points on the side LN of the triangle LMN such that LX = XY = YN. Through X, a line is drawn parallel to LM to meet MN at Z (See figure). Prove that ar (LZY) = ar (MZYX).



Sol. We have to prove that ar (ΔLZY) = ar (MZYX)

Since ΔLXZ and ΔXMZ are on the same base and between the same parallels LM and XZ, we have

ar (ΔLXZ) = ar (ΔXMZ) ...(1)

Adding ar (ΔXYZ) to both sides of (1), we get

ar (ΔLXZ) + ar (ΔXYZ) = ar (ΔXMZ) + ar (ΔXYZ)

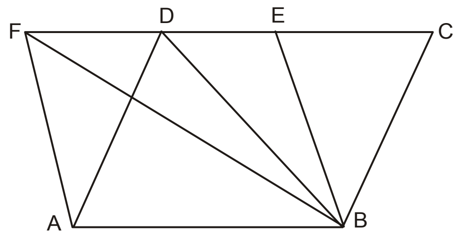
⇒ ar (ΔLZY) = ar (MZYX)

3. The area of the parallelogram ABCD is 90 cm^2 (See fig.). Find

(*i*) ar (ABEF)

(*ii*) ar (Δ ABD)

(*iii*) ar (Δ BEF)



Sol. (*i*) Since parallelograms on the same base and between the same parallels are equal in area, so we have ar (‖gm ABEF) = ar (‖gm ABCD)

Hence ar (‖gm ABEF) = ar (‖gm ABCD) = 90 cm^2

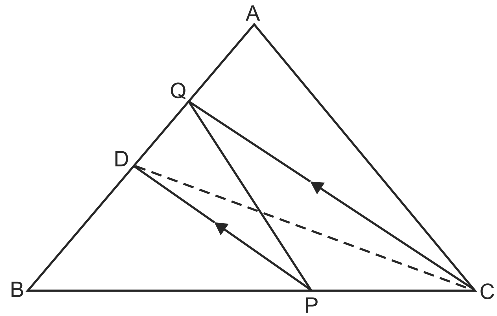
(*ii*) ar (ΔABD) = $\frac{1}{2}$ ar (‖gm ABCD)

[∵ A diagonal of a parallelogram divides the parallelogram in two triangles of equal area]

= $\frac{1}{2}$ × 90 cm^2 = 45 cm^2.

(*iii*) ar (ΔBEF) = $\frac{1}{2}$ ar (‖gm ABEF) = $\frac{1}{2}$ × 90 cm^2 = 45 cm^2.

4. In ΔABC, D is the mid-point of AB and P is any point on BC. If CQ ‖ PD meets AB in Q (See fig.), then prove that ar (Δ BPQ) = $\frac{1}{2}$ ar (Δ ABC).



Sol. D is the mid-point of AB and P is any point on BC of ΔABC. CQ ‖ PD meets AB in Q, we have to prove that ar (ΔBPQ) = $\frac{1}{2}$ ar (ΔABC).

Join CD. Since median of a triangle divides it into two triangles of equal area, so we have

ar (ΔBCD) = $\frac{1}{2}$ ar (ΔABC) ...(1)

Since triangles on the same base and between the same parallels are equal in area, so we have

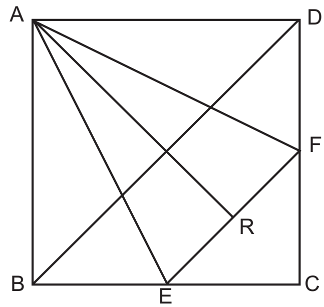
ar (ΔDPQ) = ar (ΔDPC) ...(2)

[∵ Triangle DPQ and DPC are on the same base DP and between the same parallels DP and CQ]

ar (ΔDPQ) + ar (ΔDPB) = ar (ΔDPC) + ar (ΔDPB)

Hence, ar (ΔBPQ)=ar (ΔBCD) = $\frac{1}{2}$ ar (ΔABC) [Using (1)]

5. ABCD is a square. E and F are respectively the mid-points of BC and CD. If R is the mid-point of EF (See fig.) prove that ar (Δ AER) = ar (Δ AFR).



Sol. ABCD is a square. E and F are respectively the mid-points of BC and CD. If R is the mid-point of EF, we have to prove that ar (ΔAER) = ar (ΔAFR).

In ΔABE and ΔADF, we have

AB = AD [Sides of a square are equal]

∠ABE = ∠ADF [Each 90°]

BE = DF

[∵ E is the mid-point of BC and F is the mid-point of CD. Also $\frac{1}{2}$ BC = $\frac{1}{2}$ CD]

ar (ΔABE) ≅ ar (ΔADF) [By SAS Congruence rule]

∴ AE = AF (CPCT) ...(1)

Now, in ΔAER and ΔAFR, we have

AE = AF [From (1)]

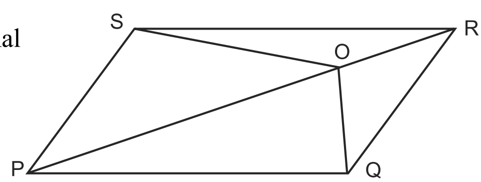
ER = RF [∵ R is mid-point of EF]

and AR = AR [Common side]

∴ ΔAER ≅ ΔAFR [By SSS rule of congruence]

Hence, ar(ΔAER) = ar(ΔAFR)[∵ Congruent triangles are equal in area]

6. In the given figure, O is any point on the diagonal PR of a parallelogram PQRS. Prove that ar (ΔPSO) = ar (ΔPQO)

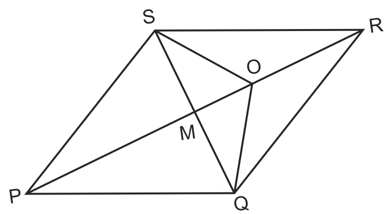


Sol. Join SQ, bisects the diagonal PR at M.

Since diagonals of a parallelogram bisect each other, so SM = MQ. Therefore, PM is a median of ΔPQS.

ar (ΔPSM) = ar (ΔPQM) ...(1)

[∵ Median divides a triangle into two triangles of equal area]



Again, as OM is the median of triangle ΔOSQ, so

$$ar(\Delta OSM) = ar(\Delta OQM) \quad ...(2)$$

Adding (1) and (2), we get

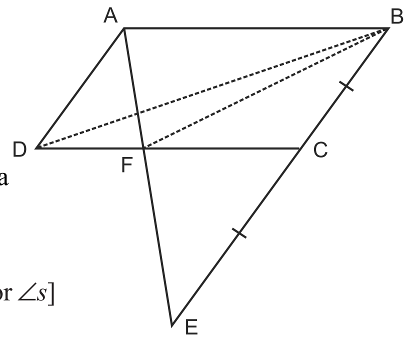
$$ar(\Delta PSM) + ar(\Delta OSM) = ar(\Delta PQM) + ar(\Delta OQM)$$

$$\Rightarrow \quad ar(\Delta PSO) = ar(\Delta PQO)$$

Hence, proved.

7. ABCD is a parallelogram in which BC is produced to E such that CE = BC in the given figure. AE intersects CD at F.

If ar (Δ DFB) = 3 cm^2, find the area of the parallelogram ABCD.



Sol. In ΔADF and ΔEFC, we have

$\angle DAF = \angle CEF$ [Alt. interior ∠s]

$AD = CE$ [∵ AD = BC = CE [Given]]

$\angle ADF = \angle FCE$ [Alt interior ∠s]

∴ $\Delta ADF \cong \Delta ECF$ [By SAS rule of congruence]

∴ $DF = CF$ [CPCT]

As BF is median of ΔBCD,

$$\therefore \quad ar(\Delta BDF) = \frac{1}{2} ar(\Delta BCD) \quad ...(1)$$

[∵ Median divides a triangle into two triangles of equal area]

Now, if a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangles is equal to half the area of the parallelogram.

$$\therefore \quad ar(\Delta BCD) = \frac{1}{2} ar(|| gm\ ABCD) \quad ...(2)$$

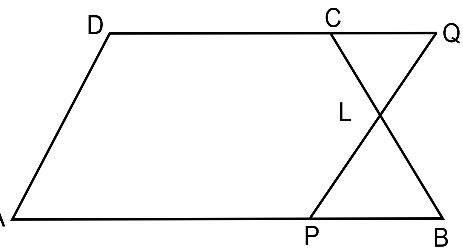
$$\therefore \text{ By (1), we have } ar(\Delta BDF) = \frac{1}{2}\left\{\frac{1}{2} ar(|| gm\ ABCD)\right\}$$

$$\Rightarrow \quad 3\ cm^2 = \frac{1}{4} ar(|| gm\ ABCD)$$

$$\Rightarrow \quad ar(|| gm\ ABCD) = 12\ cm^2$$

Hence, the area of the parallelogram ABCD is 12 cm^2.

8. In trapezium ABCD, AB||DC and L is the mid-point of BC. Through L, a line PQ||AD has been drawn which meets AB in P and DC produced in Q (See figure). Prove that ar(ABCD) = ar(APQD).



Sol. As AB || DC, so AB || DQ

In ΔCLQ and ΔBLP, we have

$\therefore \quad \angle QCL = \angle LBP$ [Alt. ∠s]

$CL = LB$ [∵ L is the mid-point of BC]

$\angle CLQ = \angle BLP$ [Vertically opposite ∠s]

$\therefore \quad \Delta CLQ \cong \Delta BLP$ [By ASA Congruence rule]

$\Rightarrow \quad ar(\Delta CLQ) = ar(\Delta BLP)$ ...(1) [Congruent Δs are equal in area]

Adding ar (APLCD) to both sides of (1), we get

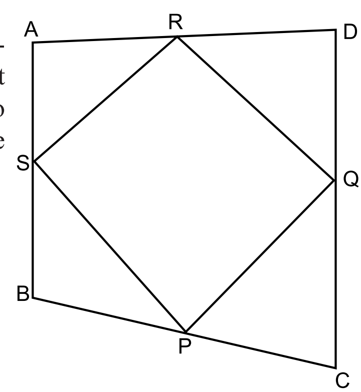
$ar(\Delta CLQ) + ar(APLCD) = ar(\Delta BLP) + ar(APLCD)$

$\Rightarrow \quad ar(APQD) = ar(ABCD)$

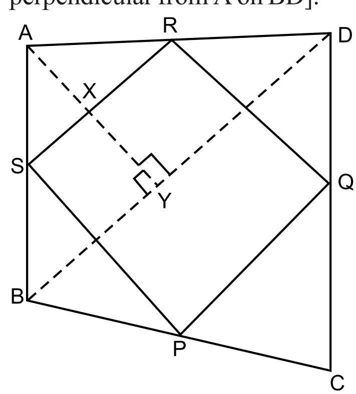
Hence, $ar(ABCD) = ar(APQD)$

9. If the mid-points of the sides of a quadrilateral are joined in order, prove that the area of the parallelogram so formed will be half of the area of the given quadrilateral. (figure)

[**Hint:** Join BD and draw perpendicular from A on BD].



Sol.



Given: A quadrilateral ABCD in which the mid-points of the sides of it are joined in order to form parallelogram PQRS.

To prove: ar(||gm PQRS) = $\frac{1}{2}$ ar (□ ABCD)

Construction: Join BD and draw perpendicular from A on BD which interect SR and BD at X and Y respectively.

Proof: In ΔABD, S and R are the mid-points of sides AB and AD respectively.

$\therefore$ SR || BD

$\Rightarrow$ SX || BY

$\Rightarrow$ X is the mid-point of AY [Converse of mid-point theorem]

$\Rightarrow$ AX = XY ...(1)

[$\because$ S is the mid-point of AB and SX || BY]

And, $SR = \frac{1}{2}BD$ ...(2) [$\because$ Mid-point theorem]

Now, $ar(\Delta ABD) = \frac{1}{2} \times BD \times AY$

and $ar(\Delta ASR) = \frac{1}{2} \times SR \times AX$

$\Rightarrow$ $ar(\Delta ASR) = \frac{1}{2} \times \left(\frac{1}{2}BD\right) \times \left(\frac{1}{2}AY\right)$ [Using (1) and (2)]

$\Rightarrow$ $ar(\Delta ASR) = \frac{1}{4} \times \left(\frac{1}{2} \times BD \times AY\right)$

$\Rightarrow$ $ar(\Delta ASR) = \frac{1}{4} ar(\Delta ABD)$ ...(3)

Similarly,

$$ar(\Delta CPQ) = \frac{1}{4} ar(\Delta CBD) \quad \ldots(4)$$

$$ar(\Delta BPS) = \frac{1}{4} ar(\Delta BAC) \quad \ldots(5)$$

$$ar(\Delta DRQ) = \frac{1}{4} ar(\Delta DAC) \quad \ldots(6)$$

Adding (3), (4), (5) and (6), we get

ar (ΔASR) + ar (ΔCPQ) + ar (ΔBPS) + ar (ΔDRQ)

$$= \frac{1}{4}ar(\Delta ABD) + \frac{1}{4}ar(\Delta CBD) + \frac{1}{4}ar(\Delta BAC) + \frac{1}{4}ar(\Delta DAC)$$

$$= \frac{1}{4}[ar(\Delta ABD) + ar(\Delta CBD)$$
$$+ar(\Delta BAC) + ar(\Delta DAC)]$$
$$= \frac{1}{4}[ar(\square ABCD) + ar(\square ABCD)]$$
$$= \frac{1}{4} \times 2\, ar(\square ABCD)$$
$$= \frac{1}{2}\, ar(\square ABCD)$$

$\therefore ar(\Delta ASR) + ar(\Delta CPQ) + ar(\Delta BPS) + ar(\Delta DRQ)$
$$= \frac{1}{2}\, ar(\square ABCD)$$

$\Rightarrow ar(\square ABCD) - ar(\| gm\ PQRS) = \frac{1}{2}\, ar(\square ABCD)$

$\Rightarrow ar(\| gm\ PQRS) = ar(\square ABCD) - \frac{1}{2}\, ar(\square ABCD)$

$\Rightarrow ar(\| gm\ PQRS) = \frac{1}{2}\, ar(\square ABCD)$

Hence, Proved

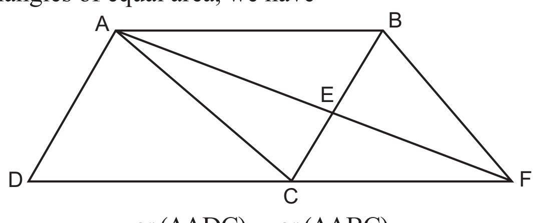
EXERCISE 9.4

1. A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F. Prove that ar (ΔADF) = ar (ABFC).

Sol. Given: ABCD is a parallelogram. A point E is taken on the side BC. AE and DC are produced to meet at F.

To prove: ar (ΔADF) = ar (ABFC)

Proof: Since ABCD is a parallelogram and diagonal AC divides it into two triangles of equal area, we have



$$\text{ar}(\Delta ADC) = \text{ar}(\Delta ABC) \qquad ...(1)$$

As DC∥AB, so CF∥ AB

Since triangles on the same base and between the same parallels are equal in area, so we have

$$\text{ar}(\Delta ACF) = \text{ar}(\Delta BCF) \qquad ...(2)$$

Adding (1) and (2), we get

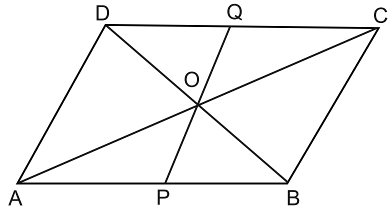
$$\text{ar}(\Delta ADC) + \text{ar}(\Delta ACF) = \text{ar}(\Delta ABC) + \text{ar}(\Delta BCF)$$

$$\Rightarrow \quad \text{ar}(\Delta ADF) = \text{ar}(ABFC)$$

Hence, proved.

2. The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. Show that PQ divides the parallelogram into two parts of equal area.

Sol. $\because$ AC is a diagonal of the ∥ gm ABCD



$$\therefore \quad \text{ar}(\Delta ACD) = \frac{1}{2}\text{ar}(\parallel \text{gm } ABCD) \qquad ...(1)$$

Now, in ΔAOP and ΔCOQ

$$AO = CO \qquad [\because \text{Diagonals of a} \parallel \text{gm bisect each other}]$$

$$\angle AOP = \angle COQ \qquad [\text{Vert. Opp. } \angle s]$$

$$\angle OAP = \angle OCQ \qquad [\text{Alt. } \angle s;\ AB \parallel CD]$$

$$\therefore \quad \Delta AOP \cong \Delta COQ \qquad [\text{By ASA Cong. rule}]$$

Hence, $\text{ar}(\Delta AOP) = \text{ar}(\Delta COQ)$ [Cong. area axiom] ...(2)

Adding ar (quad. AOQD) to both sides of (2), we get

$$\text{ar}(\text{quad. } AOQD) + \text{ar}(\Delta AOP) = \text{ar}(\text{quad. } AOQD) + \text{ar}(\Delta COQ)$$

$$\Rightarrow \quad \text{ar}(\text{quad. } APQD) = \text{ar}(\Delta ACD)$$

But, $\text{ar}(\Delta ACD) = \frac{1}{2}\text{ar}(\parallel \text{gm } ABCD)$ [From (1)]

Hence, $\text{ar}(\text{quad. } APQD) = \frac{1}{2}\text{ar}(\parallel \text{gm } ABCD)$.

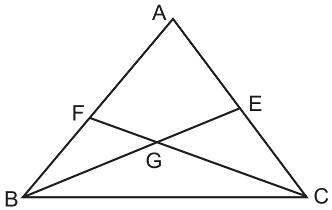
3. The medians BE and CF of a triangle ABC intersect at G. Prove that the area of ΔGBC = area of the quadrilateral AFGE.

Sol. BE and CF are medians of a triangle ABC intersect at G. We have to prove that the ar (ΔGBC) = area of the quadrilateral AFGE.

Since median (CF) divides a triangle into two triangles of equal area, so we have

$$\text{ar}(\Delta BCF) = \text{ar}(\Delta ACF)$$

$$\Rightarrow \quad \text{ar}(\Delta GBF) + \text{ar}(\Delta GBC) = \text{ar}(AFGE) + \text{ar}(\Delta GCE) \qquad ...(1)$$



Since median (BE) divides a triangle into two triangle of equal area, so we have

$$\Rightarrow \quad ar(\Delta GBF) + ar(AFGE) = ar(\Delta GCE) + ar(\Delta GBC) \quad ...(2)$$

Subtracting (2) from (1), we get

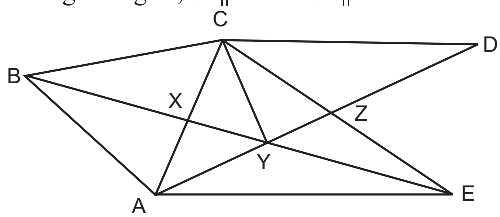
$$ar(\Delta GBC) - ar(AFGE) = ar(\Delta AFGE) - ar(\Delta GBC)$$

$$\Rightarrow \quad ar(\Delta GBC) + ar(\Delta GBC) = ar(AFGE) + ar(AFGE)$$

$$\Rightarrow \quad 2\,ar(\Delta GBC) = 2\,ar(AFGE)$$

Hence, $ar(\Delta GBC) = ar(AFGE)$

4. In the given figure, CD||AE and CY||BA. Prove that ar (Δ CBX) = ar (Δ AXY).



Sol. CD || AE and CY || BA. We have to prove that ar (ΔCBX) = ar (ΔAXY).

Since triangle on the same base and between the same parallels are equal in area, so we have

$$ar(\Delta ABC) = ar(\Delta ABY)$$

$$\Rightarrow \quad ar(\Delta CBX) + ar(\Delta ABX) = ar(\Delta ABX) + ar(\Delta AXY)$$

Hence, $ar(\Delta CBX) = ar(\Delta AXY)$

[Cancelling ar (ΔABX) from both sides]

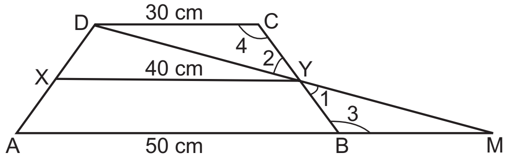
5. ABCD is a trapezium in which AB||DC, DC = 30 cm and AB = 50 cm. If X and Y are respectively the mid-points of AD and BC, prove that

$$ar(DCYX) = \frac{7}{9}\,ar(XYBA).$$

Sol. In ΔMBY and ΔDCY, we have

$\angle 1 = \angle 2$ [Vertically opposite $\angle s$]

$\angle 3 = \angle 4$ [$\because$ AB || DC and alt. $\angle s$ are equal]



$BY = CY$ [∵ Y is the mid-point of BC]

∴ $\Delta MBY \cong \Delta DCY$ [By ASA Cong. rule]

So, $MB = DC = 30$ cm [CPCT]

Now, $AM = AB + BM = 50 \text{ cm} + 30 \text{ cm} = 80 \text{ cm}$

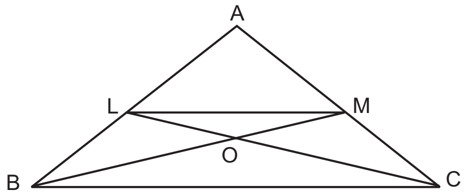
In ΔADM, we have $XY = \frac{1}{2} AM = \frac{1}{2} \times 80 \text{ cm} = 40 \text{ cm}$

As AB || XY || DC and X and Y are the mid-points of AD and BC, so height of trapezium DCYX and XYBA are equal. Let the equal height be h cm.

$$\frac{ar(DCYX)}{ar(XYBA)} = \frac{\frac{1}{2}(30+40)\times h}{\frac{1}{2}\times(40+50)\times h} = \frac{70}{90} = \frac{7}{9}$$

Hence, $ar(DCYX) = \frac{7}{9} ar(XYBA)$

6. In ΔABC, if L and M are the points on AB and AC, respectively such that LM || BC. Prove that ar (ΔLOB) = ar (ΔMOC).



Sol. Since triangles on the same base and between the same parallels are equal in area, so we have

∴ $ar(\Delta LBM) = ar(\Delta LCM)$

[ΔLBM and ΔLCM are on the same base LM and between the same parallels LM and BC]

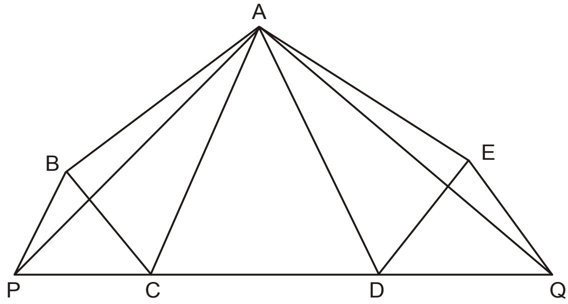
∴ $ar(\Delta LBM) = ar(\Delta LCM)$

⇒ $ar(\Delta LOM) + ar(\Delta LOB) = ar(\Delta LOM) + ar(\Delta MOC)$

Hence, $ar(\Delta LOB) = ar(\Delta MOC)$

[Cancelling ar (ΔLOM) from both sides]

7. In the given figure, ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that ar (ABCDE) = ar (Δ APQ).



Sol. BP || AC and AD || EQ,

Since triangles on the same base and between the same parallels are equal in area

$$\text{ar}(\Delta ABC) = \text{ar}(\Delta APC) \quad ...(1)$$

and $$\text{ar}(\Delta ADE) = \text{ar}(\Delta ADQ) \quad ...(2)$$

Adding (1) and (2), we get

$$\text{ar}(\Delta ABC) + \text{ar}(\Delta ADE) = \text{ar}(\Delta APC) + \text{ar}(\Delta ADQ)$$

Adding ar (ΔACD) to both sides, we get

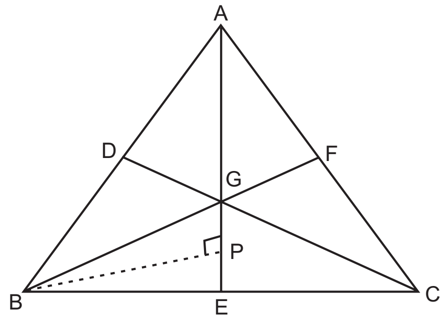
$$\text{ar}(\Delta ABC) + \text{ar}(\Delta ADE) + \text{ar}(\Delta ACD) = \text{ar}(\Delta APC) + \text{ar}(\Delta ADQ) + \text{ar}(\Delta ACD)$$

Hence, $$\text{ar}(ABCDE) = \text{ar}(\Delta APQ)$$

8. If the medians of a ΔABC intersect at G, show that

$$\text{ar}(AGB) = \text{ar}(AGC) = \text{ar}(BGC) = \frac{1}{3}\text{ar}(ABC).$$

Sol. Given : Medians AE, BF and CD of ΔABC intersect at G.



To prove:

$$\text{ar}(\Delta AGB) = \text{ar}(\Delta AGC) = \text{ar}(\Delta BGC) = \frac{1}{3}\text{ar}(\Delta ABC)$$

Construction: Draw BP ⊥ EG.

Proof : $AG = \frac{2}{3}AE$

[∵ Centroid divides the median in the ratio 2 : 1]

Now, $\text{ar}(\Delta AGB) = \frac{1}{2} \times AG \times BP$

$$= \frac{1}{2} \times \frac{2}{3} AE \times BP$$

$$= \frac{2}{3} \times \frac{1}{2} \times AE \times BP$$

$$= \frac{2}{3} \text{ar}(\Delta ABE)$$

$$= \frac{2}{3} \times \frac{1}{2} \text{ar}(\Delta ABC)$$ [∵ Median divides a triangle into two triangles equal in area]

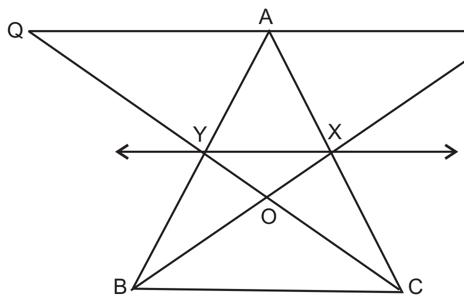
$$= \frac{1}{3} \text{ar}(\Delta ABC)$$

Similarly, ar (ΔAGC) = ar (ΔBGC) = $\frac{1}{3}$ ar (ΔABC)

∴ (ΔAGB) = ar (ΔAGC) = ar (ΔBGC) = $\frac{1}{3}$ ar (ΔABC)

Hence, proved.

9. In given figure, X and Y are the mid-points of AC and AB respectively, OP || BC and CYQ and BXP are straight lines. Prove that ar (ΔABP) = ar (ΔACQ).



Sol. In triangle ABC, X and Y are the mid-points of AB and AC.

∴ XY || BC [By mid-point theorem]

Since triangles on the same base (BC) and between the same parallels (XY|| BC) are equal in area

∴ ar (ΔBYC) = ar (ΔBXC) ...(1)

Subtracting ar (ΔBOC) from both sides, we get

ar (ΔBYC) – ar (ΔBOC) = ar (ΔBXC) – ar (ΔBOC)

⇒ ar (ΔBOY) = ar (ΔCOX) ...(2)

Adding ar (ΔXOY) to both sides of (2), we get

ar (ΔBOY) + ar (ΔXOY) = ar (ΔCOX) + ar (ΔXOY)

⇒ ar (ΔBXY) = ar (ΔCXY) ...(3)

Now, quad. XYAP and XYQA are on the same base XY and between the same parallels XY and PQ.

$\therefore$ ar (XYAP) = ar (XYQA) ...(4)

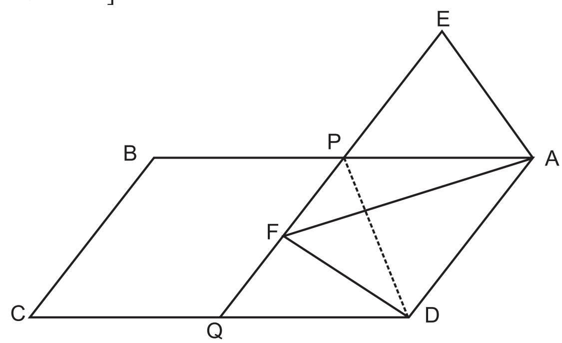
Adding (3) and (4), we get

$$\text{ar}(\Delta BXY) + \text{ar}(XYAP) = \text{ar}(\Delta CXY) + \text{ar}(XYQA)$$

Hence, ar (ΔABP) = ar (ΔACQ)

10. In the given figure, ABCD and AEFD are two parallelograms. Prove that ar (ΔPEA) = ar (ΔQFD).

[**Hint:** Join PD]



Sol. ABCD and AEFD are two parallelograms.

We have to prove that ar (ΔPEA) = ar (ΔQFD). Join PD.

In ΔPEA and ΔQFD, we have

$\angle$APE = $\angle$DQF [$\because$ Corresp. $\angle s$ are equal as AB || CD]

$\angle$AEP = $\angle$DFQ [$\because$ Corresp. $\angle s$ are equal as AE || DF]

AE = DF [$\because$ Opposite sides of a ||gm are equal]

$\therefore$ $\Delta PEA \cong \Delta QFD$ [By AAS Cong. rule]

Hence, ar (ΔPEA) = ar (ΔQFD).