JEE Main 2023 (Memory based)

29 January 2023 - Shift 1

Answer & Solutions

PHYSICS

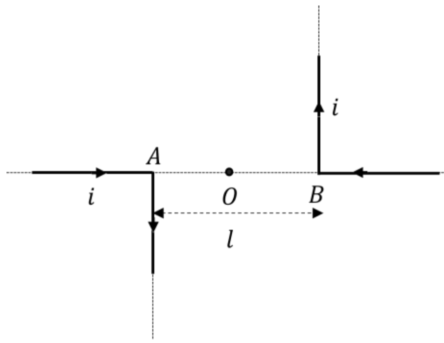
1. Point O and two long wires are kept in same plane such that point O lies at middle of the line. Then magnetic field at point O due to the current i flowing in both the wires is equal to

A. $\frac{\mu_o i}{2\pi l}$

B. $\frac{\mu_o i}{\pi l}$

C. $\frac{2\pi\mu_o i}{l}$

D. $\frac{\mu_o i}{2l}$

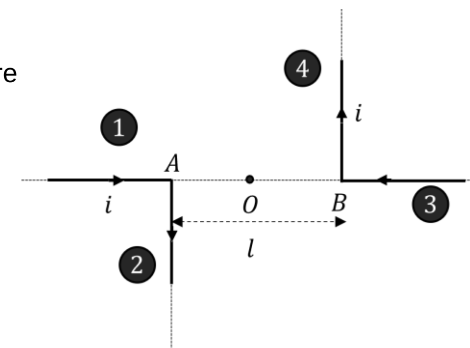


Answer (B)

Solution:

Magnetic field due to Wire section (1) and (3) shown in figure will not generate a field at O.
Magnetic field due to Wire section (2) and (4) will be equal to;

$$B = 2 \times \frac{\mu_0 i}{4\pi\left(\frac{l}{2}\right)}(\hat{k}) = \frac{\mu_0 i}{\pi l}(\hat{k})$$



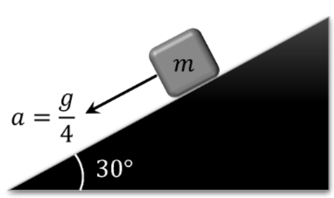
2. A block is sliding down an inclined plane of inclination $30°$, with an acceleration of $g/4$.
Find the co-efficient of friction between the block and incline.

A. $\frac{1}{\sqrt{3}}$

B. $\frac{1}{2\sqrt{3}}$

C. $\frac{1}{3}$

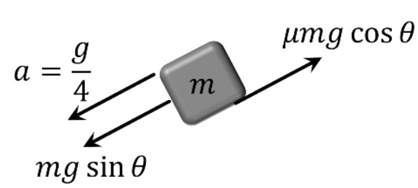
D. $\frac{1}{2}$



Answer (B)

Solution:

FBD for the given situation will be:



Balancing force gives:

$$mg\sin\theta - \mu mg\sin\theta = ma$$
$$\frac{g}{4} = g\sin\theta - \mu g\cos\theta$$
$$\mu = \frac{1}{2\sqrt{3}}$$

3. A car is moving on a circular track of radius $50\ cm$ with coefficient of friction being 0.34. On this horizontal track the maximum safe speed for turning is equal to $(g = 10\ \frac{m}{s^2})$

A. 1.03
B. 1.7
C. 1.3
D. 1.8

Answer (C)

Solution:

Friction will provide required centripetal acceleration to move in the circle:
So,

$$\frac{mv^2}{r} = \mu mg$$
$$v = \sqrt{\mu gR}$$
$$v = \sqrt{0.34 \times 10 \times \frac{1}{2}}$$
$$v = 1.3\ m/s$$

4. Find the ratio of maximum wavelength of $Lyman$ series of $Hydrogen$ atom to minimum wavelength of $Balmer$ series of $Helium$ ion.

A. $4/3$
B. 1
C. $3/2$
D. $3/4$

Answer (A)

Solution:

λ_{max} for Lyman series (for hydrogen atom)
ΔE will be for $n = 2$ to $n = 1$

$$\Delta E = 13.6 \times \left(1 - \frac{1}{4}\right)\ eV = \frac{3}{4} \times 13.6\ eV$$
$$\lambda_{max} = \frac{12400}{\frac{3}{4} \times 13.6}\ \text{Å}$$

λ_{min} for Balmer series (for He ion)
ΔE will be for $n = \infty$ to $n = 2$

$$\Delta E = 13.6 \times 4 \times \left(\frac{1}{4}\right)\ eV = 13.6\ eV$$
$$\lambda_{min} = \frac{12400}{13.6}\ \text{Å}$$
$$\frac{\lambda_{max}}{\lambda_{min}} = \frac{13.6}{\frac{3}{4} \times 13.6} = \frac{4}{3}$$

5. Find the excess pressure inside a soap bubble of radius 'R' and surface tension 'T'.

A. T/R
B. $2T/R$
C. $3T/R$
D. $4T/R$

Answer (D)

Solution:

We know that excess pressure inside soap bubble is:

$$(\Delta P)_{soap\ bubble} = \left(\frac{4T}{R}\right)$$

6. Two point masses (mass m each) are moving in a circle of radius R under mutual gravitational attraction. Find the speed of each mass.

A. $\sqrt{\frac{GM}{4R}}$
B. $\sqrt{\frac{GM}{2R}}$
C. $\sqrt{\frac{GM}{8R}}$
D. $\sqrt{\frac{GM}{R}}$

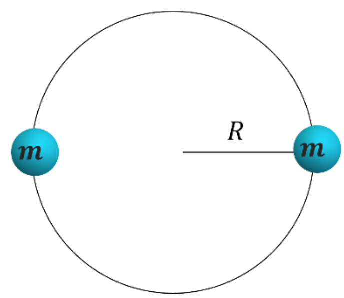
Answer (A)

Solution:

Masses will revolve around their center of mass,
Mutual gravitational force provides the centripetal force:

$$\frac{G \times m \times m}{(2R)^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{Gm}{4R}}$$

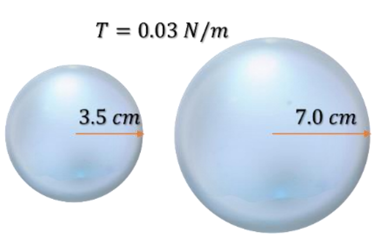


7. Find the work done in expanding the soap bubble from radius $r_1 = 3.5\ cm$ to $r_2 = 7.0\ cm$. (Given surface tension of soap solution, $T = 0.03\ N/m$)

A. $0.14\ mJ$
B. $1.4\ mJ$
C. $0.7\ mJ$
D. $2.8\ mJ$

Answer (D)

Solution:



Work done = Change in surface energy of soap bubble
Work done $= 2T(4\pi r_2^2 - 4\pi r_1^2)$
$W = 2 \times 4\pi T(r_2^2 - r_1^2)$
$W = 2 \times 4\pi \times 0.03(7^2 - 3.5^2) \times 10^{-4}\, J$
$W = 2.8\, mJ$

8. In an $isochoric$ process on an ideal gas initial temperature is equal to $27°\, C$ with an initial pressure being equal to $270\, kPa$. Now if final temperature is made equal to $36°\, C$ then final pressure is equal to approximately.

A. $298\, kPa$
B. $270\, kPa$
C. $360\, kPa$
D. $278\, kPa$

Answer (D)

Solution:

For an iso-choric process (Volume is constant):

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$
$$\frac{270}{300} = \frac{P_f}{309}$$
$$P_f = 278.1\, kPa$$

9. If half-life of a sample is $30\, minutes$. Find the fraction of undecayed sample after $90\, minutes$.

A. $1/4$
B. $3/4$
C. $1/8$
D. $7/8$

Answer (C)

Solution:

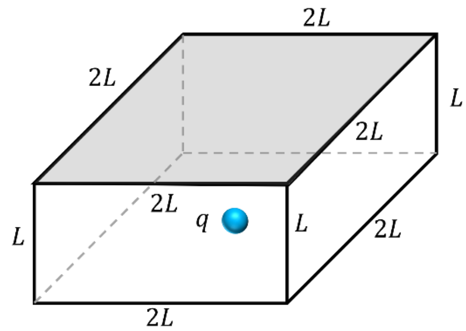
For first order decay:
$$N = N_0 e^{-\lambda t}$$

Half-life of a sample can be given as:
$$\lambda = \frac{ln2}{t_{\frac{1}{2}}}$$
$N =$ number of undecayed nuclei

$$N = N_0 e^{-\lambda \times 90}$$
$$N = N_0 e^{-\left(\frac{ln2}{30} \times 90\right)}$$
$$N = N_0 e^{-(ln8)}$$
$$N = \frac{N_0}{8}$$
$$\text{Fraction undecayed} = \frac{N}{N_0} = \left(\frac{1}{8}\right)$$

10. A charge q is placed at the centre of bottom face as shown:
Find the flux through the shaded surface.

A. $\frac{2q}{7\varepsilon_o}$
B. $\frac{q}{12\varepsilon_o}$
C. $\frac{q}{4\varepsilon_o}$
D. $\frac{q}{6\varepsilon_o}$



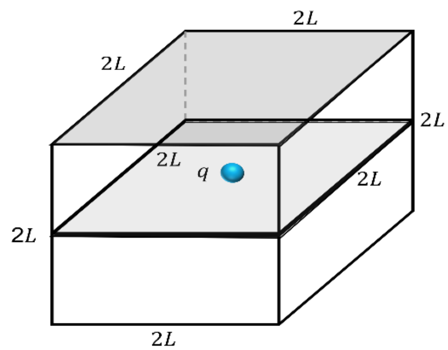
Answer (D)

Solution:

Place a similar box at the bottom as shown in figure. We get a cube of side $2L$. Charge q is then at the center of new cube.

From Gauss's Law:
$\phi = \left(\frac{q_{en}}{\epsilon_0}\right)$

From one surface of the cube, flux will be :
$\phi = \frac{1}{6}\left(\frac{q_{en}}{\epsilon_0}\right)$



11. Two coherent waves of amplitude $8\ cm$ each are superimposed on one another. If the amplitude of resultant wave is $8\ cm$ then the phase difference between two waves is equal to:

A. $2\pi/3$
B. $\pi/3$
C. $\pi/4$
D. $3\pi/4$

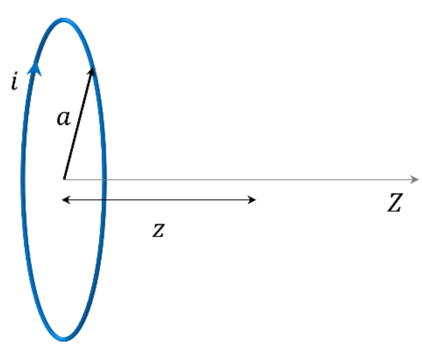
Answer (A)

Solution:

Given: $A_1 = A_2 = 8\ cm$
$A_R = 8\ cm$
Resultant Amplitude can be given as:
$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$
$8 = \sqrt{64 + 64 + 128 \cos\phi}$
$\cos\phi = -1/2$
$\phi = 2\pi/3$

12. A current carrying loop of radius a is placed in $X - Y$ Plane with its center at origin. Find magnetic field on the point $(0,0,a)$?

A. $\frac{\mu_0 i}{2\sqrt{2}\,a}$
B. $\frac{\mu_0 i}{4\sqrt{2}\,a}$
C. $\frac{2\mu_0 i}{a}$
D. $\frac{\mu_0 i}{4a}$



Answer (B)

Solution:

For a current carrying loop magnetic field can be given as:

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{2\vec{\mu}}{(a^2 + z^2)^{3/2}}$$

Magnetic moment:

$$|\vec{\mu}| = (\pi a^2)i$$

$$\vec{B} = \frac{\mu_0}{2} \cdot \frac{a^2 i}{(a^2 + z^2)^{3/2}}$$

For $z = a$;

$$\vec{B} = \frac{\mu_0}{2} \cdot \frac{a^2 i}{(2a^2)^{3/2}} = \frac{\mu_0 i}{4\sqrt{2}a}.$$

13. A ball of mass $2\ kg$ is dropped from a height of $9.8\ m$ and rebounds to a height of $4.9\ m$. If it remains in contact with ground for $0.2\ seconds$, the average force on the ball by the ground is $x(\sqrt{2} + 1)\ Newtons$. Find x (Take $g = 9.8\ m/s^2$)

Answer (98)

Solution:

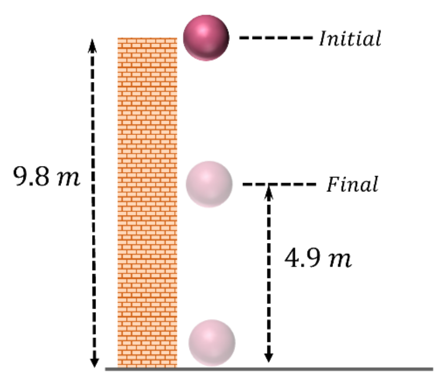
When ball hits the ground, velocity of ball is:

$v_{initial} = \sqrt{2g(9.8)}$

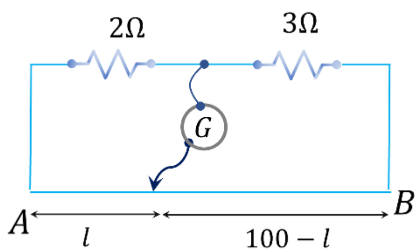
Velocity of ball after hitting the ground is:

$v_{final} = \sqrt{2g(4.9)}$

$$F\Delta t = m(v_{final} - v_{initial})$$
$$F(0.2) = 2 \times (\sqrt{2 \times 9.8 \times 9.8} - (-\sqrt{2 \times 9.8 \times 4.9}))$$
$$F = 98\ (\sqrt{2} + 1)$$



14. Consider the meter bridge setup shown:



If a shunt resistance $x\ \Omega$ is added to 3Ω resistor, balance point shift by $22.5\ cm$. Find x

Answer (2)

Solution:

When a resistance of $x\ \Omega$ connected in parallel with $3\ \Omega$ resistance. Effective resistance becomes less than $3\ \Omega$. So, balance point shifts to right

For balanced bridge:

$$\frac{2}{3} = \frac{l}{100 - l}$$
$$l = 40\ cm$$

$$\frac{2}{R} = \frac{62.5}{37.5}, where\ R = \frac{3x}{3+x}$$

$$\frac{3x}{3+x} = \frac{75}{62.5} \Rightarrow x = 2\ \Omega$$

15. If dimensional formula of pressure gradient is X, Electric field has Y, Energy density has W and Latent heat has Z. Find dimensional formula of $\frac{[X][Y]}{[Z][W]}$

A. $[ML^{-2}T^{-1}A^{1}]$
B. $[ML^{-2}T^{-1}A^{-1}]$
C. $[M^{-1}L^{2}T^{-1}A^{1}]$
D. $[ML^{2}T^{-1}A^{-1}]$

Answer (B)

Solution:

$$[X] = \left[\frac{\Delta P}{\Delta X}\right] = \left[\frac{MLT^{-2}}{L^3}\right] = [ML^{-2}T^{-2}]$$

$$[Y] = [E] = [MLT^{-3}A^{-1}]$$

$$[W] = \left[\frac{MLT^{-2}}{L^2}\right] = [ML^{-1}T^{-2}]$$

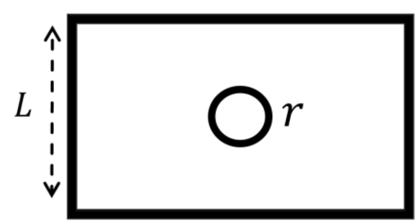
$$[Z] = \left[\frac{ML^2T^{-2}}{M}\right] = [L^2T^{-2}]$$

$$\frac{[X][Y]}{[Z][W]} = \left[\frac{[ML^{-2}T^{-2}][MLT^{-3}A^{-1}]}{[L^2T^{-2}][ML^{-1}T^{-2}]}\right]$$

$$= [ML^{-2}T^{-1}A^{-1}]$$

16. A small circular loop of radius r is placed in the plane of a square loop of side length $L(r \ll L)$. Circular loop is at the center of square as shown in the figure. Find mutual inductance.

A. $\frac{\mu_0 r^2}{\sqrt{2}L}$
B. $\frac{\pi\mu_0 r^2}{2L}$
C. $\frac{2\sqrt{2}\mu_0 r^2}{L}$
D. $\frac{4\mu_0 r^2}{L}$



Answer (C)

Solution:

B_{center} of rectangular loop

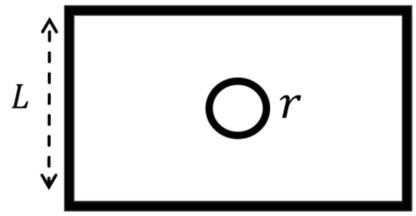
$$= \frac{\mu_0 i}{4\pi(\frac{L}{2})}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right] \times 4$$

$$= \frac{4\mu_0 i}{\sqrt{2}\pi L}$$

$$= \frac{2\sqrt{2}\mu_0 i}{\pi L}$$

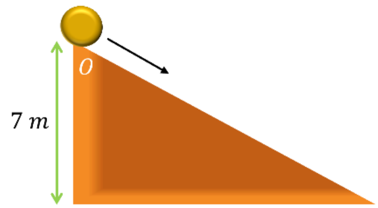
Flux in circular loop, $(\phi) = \pi r^2 \times B$

Self-inductance $= \frac{\phi}{i} = \frac{2\sqrt{2}\mu_0 r^2}{L}$



17. A solid sphere is released from point O at the top of an incline as shown. Find the value of velocity of centre of mass of sphere at the bottom most point of the incline after it reaches there doing pure rolling $(g = 10\ m/s^2)$

A. $3\ m/s$
B. $7\ m/s$
C. $10\ m/s$
D. $0.7\ m/s$



Answer (C)

Solution:

Using energy conservation

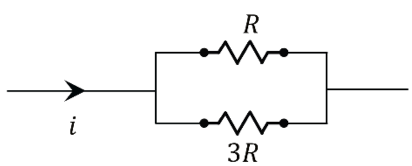
$$\frac{1}{2}mv_{cm}^2 + \frac{1}{2}\frac{2}{5}mR^2\left(\frac{v_{cm}}{R}\right)^2 = mgh$$

$$\frac{7}{10}mv_{cm}^2 = mgh$$

$$v_{cm} = \sqrt{\frac{10}{7}gh} = 10\ m/s$$

18. In the part of a circuit shown, find the ratio of rate of heat produced in R to that in $3R$.

A. $1:9$
B. $1:3$
C. $3:1$
D. $9:1$



Answer (C)

Solution:

$$Heat\ Produced(H) = \frac{V^2}{R}$$

Since V is same,

$$\frac{H_R}{H_{3R}} = \frac{3R}{R}$$

$$H_R : H_{3R} = 3:1$$

19. A disk of radius R is given by ω_o angular speed and placed gently on a rough horizontal surface. Find the velocity of center of disk when pure rolling starts.

A. $R\omega_0/3$

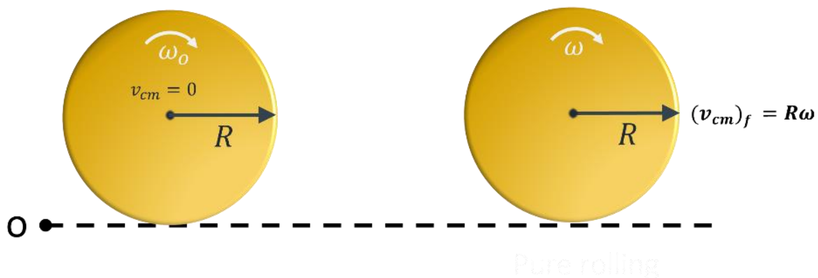
B. $R\omega_o$

C. $R\omega_o/4$

D. $2R\omega_o$

Answer (A)

Solution:



Applying angular momentum conservation about point 'O'

$I_{CM}\omega_0 = (I_{CM} + MR^2)\omega$

$\frac{1}{2}MR^2\omega_0 = \frac{3}{2}MR^2\omega$

$\omega_0 = 3\omega$

$\Rightarrow (v_{CM})_f = R\omega = \frac{R\omega_o}{3}$

20. In a standard YDSE first minima is obtained in front of the slit for $\lambda = 800\ nm$. If the distance between the slit and screen is $5\ m$ then separation between the slits is equal to

A. $5 \times 10^{-2}\ m$
B. $5\ mm$
C. $3\ mm$
D. $2\ mm$

Answer (D)

Solution:

$\frac{d}{2} = \frac{\lambda D}{2d}$

$d = \sqrt{\lambda D} = \sqrt{800 \times 10^{-9} \times 5} = 2\ mm$

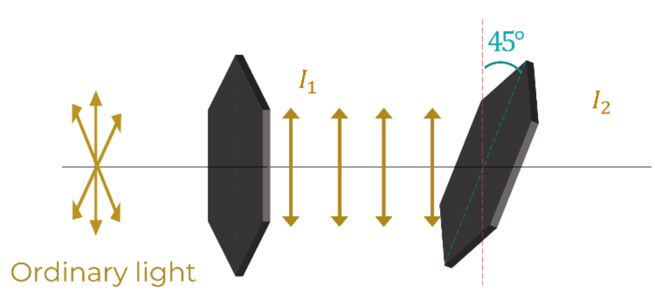
21. Two polarizers P_1 and P_2 are placed such that their transmission axis are at $45°$ from each other. Ordinary light is passed through P_1, I_1 intensity is observed and when this light is passed through P_2, I_2 intensity is observed. Find I_1/I_2 ?

Answer (2)

Solution:

$I_2 = I_1 \cos^2\theta = I_1 \cos^2 45°$

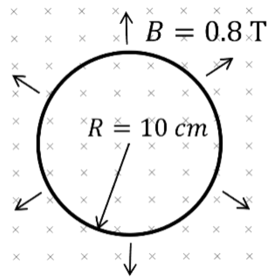
$\frac{I_1}{I_2} = \frac{1}{\cos^2 45°} = 2$



22. Magnetic field through a circular loop is $0.8\ T$. The radius of loop is expanding at $2\ cm/s$. The induced emf in the loop, when radius of the loop is $10\ cm$, is $x\pi \times 10^{-4}\ volts$. Find x.

Answer (32)

Solution:



$\phi = B(\pi r^2)$

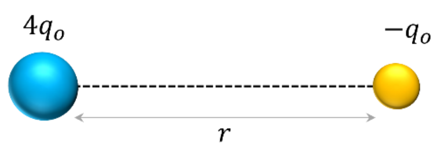
$$\varepsilon_{ind} = \left|\frac{d\phi}{dt}\right| = \frac{d(B\pi r^2)}{dt} = B\pi \times 2r\frac{dr}{dt}$$

$$= \pi \times 0.8 \times 2 \times \frac{10}{100} \times \frac{2}{100}$$

$$= 32 \times 10^{-4}\ Volts$$

23. Two point charges are arranged as shown:

Find the distance from $4q_o$ where net electric field is zero.



A. $4r$

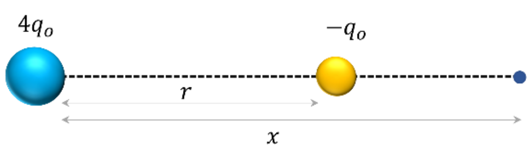
B. $3r$

C. $r/2$

D. $2r$

Answer (D)

Solution:



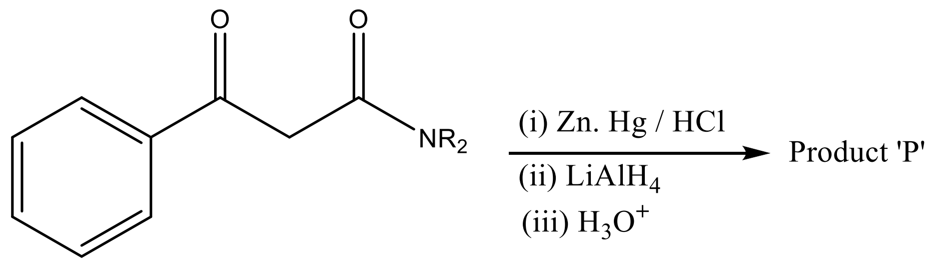
$$\frac{1}{4\pi\varepsilon_o}\frac{4q_o}{x^2} = \frac{1}{4\pi\varepsilon_o}\frac{q_o}{(x-r)^2}$$

$x^2 = 4(x-r)^2$

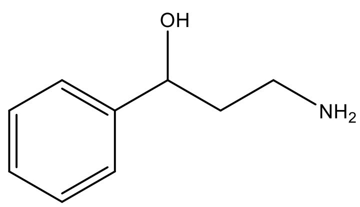
By solving,

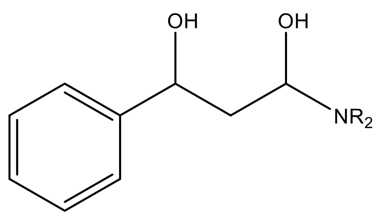
$\Rightarrow x = 2r$ and $x = 2r/3$ (at this point net electric field is not zero)

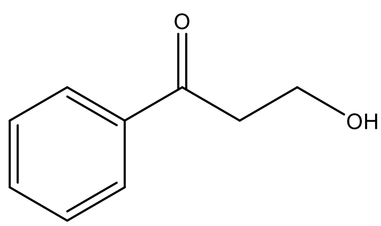
1. Consider the following sequence of reactions: -

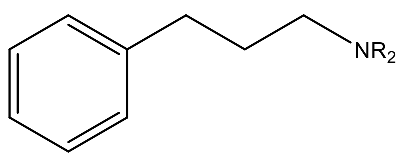


The Product ‘P’ is?

A. 

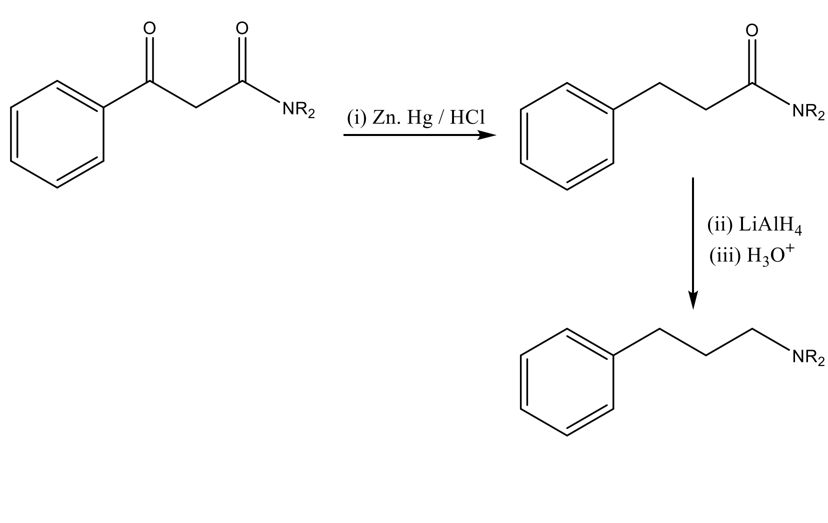
B. 

C. 

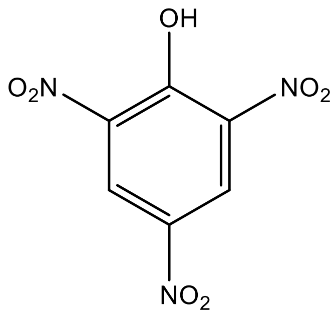
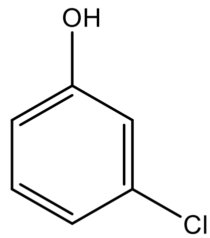
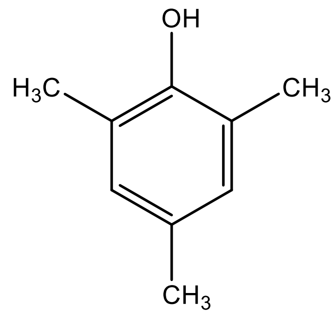
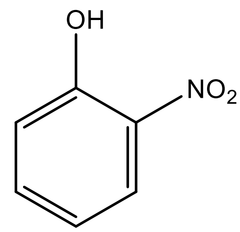
D. 

Answer (D)

Solution:



2. Following compounds are given

I) 
II) 
III) 
IV) 

Compare pK_a values

A. I > IV > II > III
B. I > IV > III > II
C. III > II > IV > I
D. IV > I > II > III

Answer (C)

Solution:

Acidic strength $\propto -I, -M$ Groups

Acidic strength $\propto \frac{1}{+I,+M}$ Groups

Acidic strength order: I > IV > II > III

pK_a order: III > II > IV > I

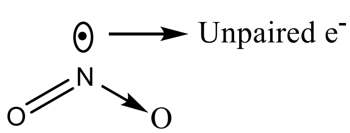
3. Which of the following compound(s) is/are paramagnetic

a) NO_2
b) NO
c) K_2O
d) Na_2O_2

A. a & b only
B. a, b & c only
C. a, b, c & d
D. a, b & d only

Answer (A)

Solution:

$\boldsymbol{NO_2}$: 

NO: (N_e = 15)

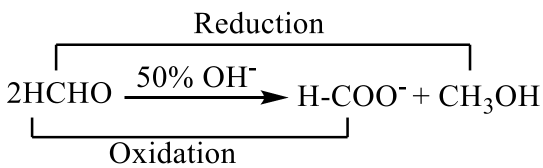
1 Unpaired electron as per MOT.

4. Cannizzaro reaction is an example of disproportionation reaction. What is the catalyst used in Cannizzaro reaction?

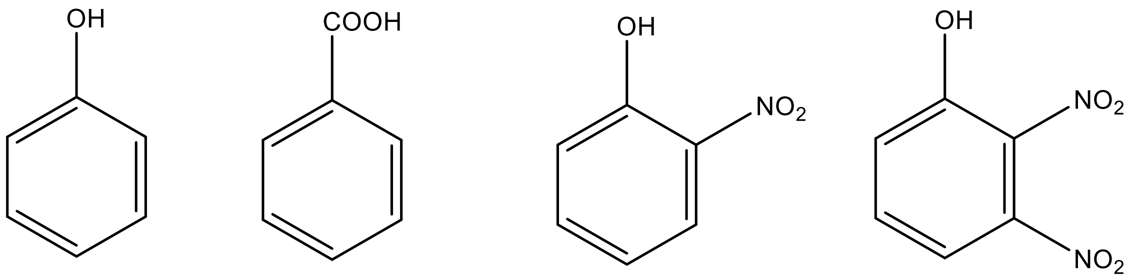
A. $FeCl_3$
B. $NaOH/H_2O$
C. $ZnCl_2/H^+$
D. $H_2/Pd/BaSO_4$

Answer (B)

Solution:



5. Arrange the following in decreasing pK_a values



(I) (II) (III) (IV)

A. IV > III > II > I
B. I > III > IV > II
C. IV > III > I > II
D. IV > II > III > I

Answer (B)

Solution:

$Acidity \propto \frac{1}{pK_a}$

The order of acidity is: II > IV > III > I

Therefore, their value of pK_a will be: I > III > IV > II

6. Which of the following reaction corresponds to Mond's process

A. $ZrI_4 \xrightarrow{1800\,K} Zr + 2I_2$
B. $Ni(CO)_4 \xrightarrow{450-470\,K} Ni + 4CO$
C. $2[Au(CN)_2]^-(aq) + Zn(s) \rightarrow 2Au(s) + [Zn(CN)_4]^{2-}(aq)$
D. $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

Answer (B)

Solution:

Mond process for refining Nickel

$Ni + 4CO \xrightarrow{330-350\ K} Ni(CO)_4$

$Ni(CO)_4 \xrightarrow{450\ -470\ K} Ni + 4CO$

7. Which of the following option contains the correct match

List – I	List - II
P. Clemmenson reduction	i. $Con. KOH$
Q. Reimer Tiemann reaction	ii. $Br_2/NaOH$
R. Cannizzaro reaction	iii. $CHCl_3/KOH$
S. Hoffmann bromamide degradation reaction	iv. $Zn - Hg/HCl$

A. P–(i), Q–(ii), R–(iii), S–(iv)
B. P–(iv), Q–(iii), R–(i), S–(ii)
C. P–(ii), Q–(iii), R–(iv), S–(i)
D. P–(iii), Q–(iv), R–(i), S–(ii)

Answer (B)

Solution:

Clemmenson reduction - $Zn - Hg/HCl$

Reimer - Tiemann reaction - $CHCl_3/KOH$

Cannizzaro reaction - $Con. KOH$

Hoffmann bromamide degradation reaction - . $Br_2/NaOH$

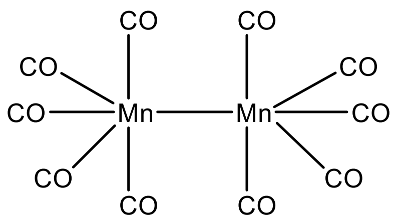
8. X: No of Bridge Bonds present in compound $Mn_2(CO)_{10}$
Y: No of Bridge Bonds present in compound $W(CO)_6$

Find out (X+Y)

Answer (00.00)

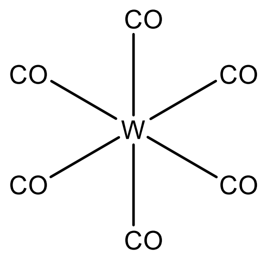
Solution:

$Mn_2(CO)_{10}$:



X = 0

$W(CO)_6$:



Y = 0

X + Y = 0

9. An element ${}^{239}_{92}X \to {}^{231}_{Z}Y + 2\alpha + 1\beta$

Then find the value of Z in the above reaction?

Answer (89)

Solution:

${}^{239}_{92}X \to {}^{231}_{89}Y + 2\,{}^{4}_{2}He^{2+}\,{}^{0}_{-1}e$

Therefore, the value of Z = 89

10. The shortest wavelength in Lyman series of H-atom is λ. of the longest wavelength in Balmer series He^+ is $\frac{x\lambda}{5}$. Find the value of x.

Answer (9)

Solution:

The shortest wavelength in Lyman series of H-atom is given by

$$\frac{1}{\lambda} = R_H\left[\frac{1}{(1)^2} - \frac{1}{(\infty)^2}\right] = R_H$$

$$\Rightarrow \lambda = 1/R_H$$

The longest wavelength in Balmer series He^+ ion is given by

$$\frac{1}{\lambda'} = (2)^2 R_H\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right] = \frac{5R_H}{9}$$

$$\lambda' = \frac{9}{5R_H} = \frac{9\lambda}{5}$$

∴ x = 9

11. Assertion: First law of thermodynamics has equation: $\Delta U = q + W$
Reason: First law of thermodynamics is based on the law of conservation of energy

A. Assertion and reason are correct and reason is the correct explanation of assertion
B. Assertion and reason are correct and reason is not the correct explanation of assertion
C. Assertion is correct but reason is incorrect
D. Assertion is incorrect but reason is correct.

Answer (A)

Solution:

First law of thermodynamics is based on the law of conservation of energy and its equation is, $\Delta U = q + W$

12. Match the following.

a. Siderite	i. $ZnCO_3$
b. Galena	ii. $FeCO_3$
c. Calamine	iii. PbS

A. (a) – (i) ; (b) – (ii) ; (c) – (iii)
B. (a) – (ii) ; (b) – (iii) ; (c) – (i)
C. (a) – (iii) ; (b) – (ii) ; (c) – (i)
D. (a) – (ii) ; (b) – (i) ; (c) – (iii)

Answer (B)

Solution:

The correct match is given as :

Siderite – $FeCO_3$; Galena – PbS ; Calamine – $ZnCO_3$

13. The number of cyclic tripeptides are formed with two amino acids A and B are :

A. 2
B. 3
C. 4
D. 5

Answer (C)

Solution:

Cyclic tripeptides contains three amino acids and the combinations possible using amino acids A and B are :

AAA, ABB, AAB, BBB

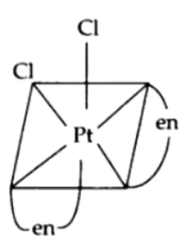
14. Which of the following complex is optically active?

A. Cis – $[Pt(NH_3)_2Cl_2]$
B. Trans – $[Pt(NH_3)_2Cl_2]$
C. Cis – $[Pt(en)_2Cl_2]$
D. Trans – $[Pt(en)_2Cl_2]$

Answer (C)

Solution:

cis – $[Pt(en)_2Cl_2]$ does not have POS and COS and hence is optically active.



15. Which of the following will give positive Lassaigne's test

A. NH_4OH

B. NH_4Cl
C. N_2H_4
D. $CH_3 - NH_2$

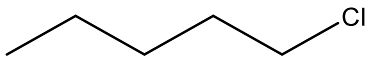
Answer (D)

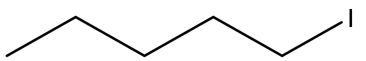
Solution:

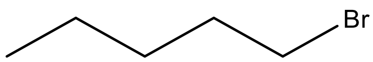
A compound with C-N bond will give positive Lassaigne's test. Hence, $CH_3 - NH_2$ will give positive Lassaigne's test.

16. The decreasing order of the boiling points for the following compounds is given as :

I. 

II. 

III. 

IV. 

A. I > II > III > IV
B. I > II > IV > III
C. III > IV > II > I
D. III > IV > I > II

Answer (C)

Solution:

Boiling point is directly proportional to molar mass.
Order of molar mass of the given compounds = III > IV > II > I
Hence, order of B.P. = III > IV > II > I

17. Which of the following molecule has the highest bond dissociation energy?

A. I_2
B. F_2
C. Cl_2
D. Br_2

Answer (C)

Solution:

Cl_2 has the highest bond dissociation energy among the halogens.

18. Select the correct statement among the following.

A. Photochemical smog has the high concentration of oxidising agent
B. Classical smog has the high concentration of oxidising agent

C. Classical smog contains NO_2
D. None of these

Answer (A)

Solution:

Photochemical smog has the high concentration of oxidising agent

19. Find out the magnetic character of Li_2O, KO_2 and MgO in that order.

A. Diamagnetic, Paramagnetic and Diamagnetic
B. Paramagnetic, Paramagnetic and Diamagnetic
C. Diamagnetic, Paramagnetic and Paramagnetic
D. Diamagnetic, Diamagnetic and Diamagnetic

Answer (A)

Solution:

Li_2O has Li^+ and O^{2-}. Both the cation and anion have all their electrons paired. So, it is diamagnetic.
KO_2 has K^+ and O_2^-. It is paramagnetic as O_2^- has one unpaired electron.
O_2^-: $\sigma_{1s^2}\ \sigma *_{1s^2}\ \sigma_{2s^2}\ \sigma *_{2s^2}\ \sigma_{2p_z^2}\ \pi_{2p_x^2} = \pi_{2p_y^2}\ \pi *_{2p_x^2} = \pi *_{2p_y^1}$
MgO has Mg^{2+} and O^{2-}. It is diamagnetic as Mg^{2+} and O^{2-} have all their electrons paired.

20. Which of the following option contains the correct decreasing order of hydration energy of the following ions?
K^+, Mg^{2+}, Cs^+, Ca^{2+}, Rb^+

A. $Mg^{2+} > Ca^{2+} > K^+ > Rb^+ > Cs^+$
B. $Ca^{2+} > Mg^{2+} > Cs^+ > Rb^+ > K^+$
C. $Mg^{2+} > Ca^{2+} > Cs^+ > Rb^+ > K^+$
D. $Cs^+ > Rb^+ > K^+ > Ca^{2+} > Mg^{2+}$

Answer (A)

Solution:

Hydration energy α Charge density
Therefore, the correct order is: - $Mg^{2+} > Ca^{2+} > K^+ > Rb^+ > Cs^+$

21. How many of the following compounds are odd electron species?
NO_2, NO_2^+, ICl_4^-, BrF_3, NO

Answer (2)

Solution:

NO and NO_2 are the odd electron species.

22. For a hypothetical reaction
$$A \leftrightharpoons B\ ;\ K_{eq} = 10^2$$
(Use $T = 27\ ℃$, $R = 8.3\ JK^{-1}mol^{-1}$, $ln10 = 2.3$)
If the value of $\Delta G°$ for the above reaction is $-x\ kJ$, the value of $2x$ will be
(Round off to the nearest integer)

Answer (23)

Solution:

$\Delta G° = -RTlnK_{eq}$

$= -8.3 \times 300 \times 2.3\log(10^2)$

$\Delta G° = -11454\,J$

$\Delta G° = -11.454\,kJ$

$x = 11.454$

$2x = 22.908$

23. A radioactive substance decays into products with half life of 30 min. The fraction left after 90 min is given by $(\frac{1}{2t})$. Find out "t".

Answer (4)

Solution:

$N_0 \xrightarrow{30\,min} \frac{N_0}{2} \xrightarrow{30\,min} \frac{N_0}{4} \xrightarrow{30\,min} \frac{N_0}{8}$

$\Rightarrow \frac{1}{8} = \frac{1}{2t}$

$t = 4$

24. How many elements can liberate H_2 from dilute acids?

V, Cr, Mn, Fe, Co, Ni, Cu

Answer (6)

Solution:

Except Cu, all other elements have negative $E^0_{M^{2+}/M}$. Hence, they can liberate H_2 from dilute acids.

No. of elements = 6

25. Consider the following reaction.

$H_2O(g) \leftrightharpoons H_2(g) + \frac{1}{2}\,O_2(g)$

If $K_{eq} = 2 \times 10^{-3}$ at 2300 K and initial pressure of $H_2O(g)$ is 1 atm, then degree of dissociation of above reaction will be $x \times 10^{-2}$, the value of x is:

Answer (2)

Solution:

$$\mathrm{K_{eq}} = \frac{(\mathrm{P_{H_2}})\,(\mathrm{P_{O_2}})^{1/2}}{\mathrm{P_{H_2O}}} = \frac{(\propto)\left(\frac{\propto}{2}\right)^{1/2}}{1-\propto} = 2 \times 10^{-3}$$

$$\Rightarrow \frac{\propto^{\frac{3}{2}}}{2^{1/2}} = 2 \times 10^{-3} \ \ (1\text{-}\propto \approx 1)$$

$$\Rightarrow \propto^{\frac{3}{2}} = 2^{\frac{3}{2}} \times (10^{-2})^{\frac{3}{2}}$$

$$\propto = 2 \times 10^{-2}$$

$$\mathrm{x} = 2$$

MATHEMATICS

1. Consider a function $f(x) = \frac{2x^2+x+1}{x^2+1}$, which of the following options is correct?

A. $f(x)$ is one-one for $x \in (0,\ 2)$
B. $f(x)$ is many-one for $x \in (0,\ 2)$
C. $f(x)$ is one-one for $x \in (0,\ \infty)$
D. $f(x)$ is one-one for $x \in (1,\ \infty)$

Answer (A)

Solution:

$f(x) = 2 + \frac{x-1}{x^2+1}$

$f'(x) = -\frac{-x^2+2x+1}{(x^2+1)^2}$

$Q(x) = -x^2 + 2x + 1$ is having $+$ ve sign in interval $(0,\ 2)$ so function is one-one

2. If real part of the product of z_1 and z_2 is zero, i.e., $Re(z_1z_2) = 0$ and $Re(z_1 + z_2) = 0$, then $Im(z_1)$ and $Im(z_2)$ is:

A. $> 0,\ > 0$
B. $< 0,\ < 0$
C. $= 0,\ = 0$
D. $> 0,\ < 0$

Answer (D)

Solution:

Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$

$Re(z_1z_2) = 0$ $\cdots$ (given)

$\Rightarrow a_1a_2 = b_1b_2 \ \cdots (1)$

$Re(z_1 + z_2) = 0 \Rightarrow a_2 = -a_1 \ \cdots (2)$

From (1) and (2), $b_1b_2 = -a_1^2 < 0$

$\Rightarrow Im(z_1)$ and $Im(z_2)$ are of opposite sign.

3. Consider $y = f(x)$ passing through $(1,\ 1)$ satisfying the following differential equation $y(x+1)dx + x^2dy = 0$, then $y = f(x)$ is given by:

A. $\ln xy = \frac{1}{x} - 1$
B. $\ln xy = \frac{1}{x}$
C. $\ln xy = \frac{1}{x} + 1$
D. $\ln xy = \frac{1}{x^2}$

Answer (A)

Solution:

$y(x+1)dx + x^2dy = 0$ ⋯ (given)

$y(x+1)dx = -x^2dy$

$\frac{(x+1)}{x^2}dx = -\frac{dy}{y}$

Integrating both sides, we get

$\int \frac{x+1}{x^2}dx = \int -\frac{1}{y}dy \Rightarrow \ln x - \frac{1}{x} = -\ln y + c$

As $y = f(x)$ passes through $(1,\ 1)$

$\Rightarrow \ln 1 - 1 = -\ln 1 + c \Rightarrow c = -1$

$\Rightarrow \ln x - \frac{1}{x} = -\ln y - 1 \Rightarrow \ln x + \ln y = \frac{1}{x} - 1$

$\Rightarrow \ln xy = \frac{1}{x} - 1$

4. If $[A]$ is 3×3 matrix and $A^2 = 3A + aI,\ A^4 = 21A + bI$, then $a + b$ is:

A. -9

B. -10

C. 9

D. 10

Answer (A)

Solution:

$A^4 = A^2 \cdot A^2$

$= (3A + aI)(3A + aI)$

$\Rightarrow A^4 = 9A^2 + 6aA + a^2I = 21A + bI$

Again using $A^2 = 3A + aI$ in LHS

$\Rightarrow 9(3A + aI) + 6aA + a^2I = 21A + bI$

$\Rightarrow (27 + 6a)A + (9a + a^2)I = 21A + bI$

$\therefore 27 + 6a = 21\ \&\ 9a + a^2 = b$

$\therefore a = -1,\ b = -8$

$\Rightarrow a + b = -9$

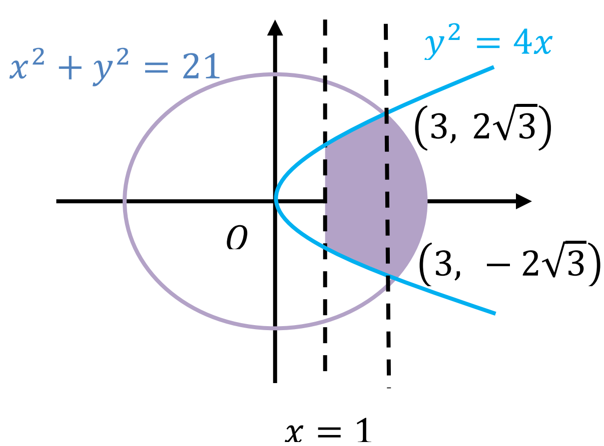
5. Find the area common to following region $x^2 + y^2 \le 21,\ x \ge 1\ \&\ y^2 \le 4x$.

A. $8\sqrt{3} - \frac{8}{3} + \frac{21}{2} - \frac{21}{2} sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$

B. $2\sqrt{3} + \frac{21\pi}{2} - \frac{8}{3} - 21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$

C. $8\sqrt{3} + \frac{21\pi}{2} - \frac{8}{3}$

D. $8\sqrt{3} + \frac{21\pi}{2} - \frac{8}{3} - 21\sin^{-1}\left(\sqrt{\frac{3}{11}}\right)$



Answer (B)

Solution:

Area of required region

$= 2\left(\int_1^3 2\sqrt{x}dx + \int_3^{\sqrt{21}} \sqrt{21 - x^2}dx\right)$

$$= 2\left(\left(2 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_1^3 + \left[\frac{21 \sin^{-1}\left(\frac{x}{\sqrt{21}}\right) + x\sqrt{21 - x^2}}{2}\right]_3^{\sqrt{21}}\right)$$

$$= 2\left(4\sqrt{3} - \frac{4}{3}\right) + (21 \sin^{-1} 1 + 0) - \left(21 \sin^{-1} \frac{3}{\sqrt{21}} + 3\sqrt{12}\right)$$

$$= 8\sqrt{3} - \frac{8}{3} + \frac{21\pi}{2} - 6\sqrt{3} - 21 \sin^{-1} \sqrt{\frac{3}{7}}$$

$$= 2\sqrt{3} + \frac{21\pi}{2} - \frac{8}{3} - 21 \sin^{-1} \sqrt{\frac{3}{7}}$$

6. Domain of $f(x) = \frac{\log_x(x-1)}{\log_{x-1}(x-4)}$ is:

A. $(0,\ 1)$
B. $(4,\ \infty)$
C. $[1,\ 4]$
D. $(4,\ \infty) - \{5\}$

Answer (D)

Solution:

For domain

$x > 0,\ x - 1 > 0, x \neq 1$

$\&\ x - 1 > 0, x - 1 \neq 1, x - 4 > 0$

$\log_{x-1}(x - 4) \neq 0$

$\Rightarrow x - 4 \neq 1$

$\Rightarrow x \neq 5$

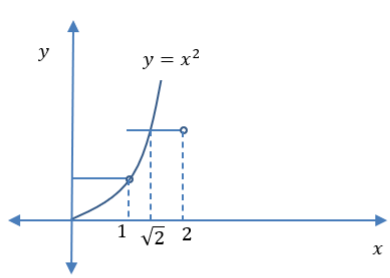
$\therefore x \in (4,\ \infty) - \{5\}$

7. Consider $f(x) = \max\{x^2,\ 1 + [x]\}$, where $[x]$ is greatest integer function. Then the value of $\int_0^2 f(x)dx$ is:

A. $\frac{4\sqrt{2}+5}{3}$
B. $\frac{6\sqrt{2}+5}{3}$
C. $\frac{8\sqrt{2}+5}{3}$
D. $\frac{8\sqrt{2}+3}{5}$

Answer (A)

Solution:



$$f(x) = \begin{cases} 1 + [x],\ 0 \leq x \leq \sqrt{2} \\ x^2,\ \ \sqrt{2} < x \leq 2 \end{cases}$$

$$\int_0^2 f(x)dx = \int_0^{\sqrt{2}} (1 + [x])dx + \int_{\sqrt{2}}^2 x^2 dx$$

$= \int_0^1 1dx + \int_1^{\sqrt{2}} 2dx + \frac{x^3}{3}\Big|_{\sqrt{2}}^{2}$

$= 1 + 2(\sqrt{2} - 1) + \frac{1}{3}(8 - 2\sqrt{2})$

$= \frac{4\sqrt{2}+5}{3}$

8. In a football club there are 15 players, each player has a T-shirt of their own name. Find the number of ways such that at least thirteen players pick the correct T-shirt of their own name.

A. 107
B. 106
C. 108
D. 109

Answer (B)

Solution:

At least 13 players pick correct T-shirt $=$ exactly 13 players pick correct T-shirt $+14$ players pick correct T-shirt $+$ exactly 15 players pick correct T-shirt

$= {}^{15}C_{13} \times 1 + 0 + 1$

$= 105 + 0 + 1$

$= 106$

9. If 3 bad and 7 good apples are mixed, then find probability of finding 4 good apples if 4 apples are drawn simultaneously.

A. $\frac{5}{12}$
B. $\frac{1}{6}$
C. $\frac{7}{13}$
D. $\frac{6}{7}$

Answer (B)

Solution:

$P(E) = \frac{{}^7C_4}{{}^{10}C_4} = \frac{7!}{4!3!} \cdot \frac{4!6!}{10!}$

$= \frac{4 \cdot 5 \cdot 6}{8 \cdot 9 \cdot 10}$

$= \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

10. If $x = 2$ is a root of $x^2 + px + q = 0$ and $f(x) = \begin{cases} \frac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^2}, & x \neq 0 \\ 0, & x = 2p \end{cases}$. Then $\lim\limits_{x \to 2p^+} f(x)$ is:

A. $\frac{1}{2}$
B. $\frac{1}{4}$
C. 0
D. $-\frac{1}{2}$

Answer (C)

Solution:

Since $x = 2$ is a root of $x^2 + px + q = 0$

$\Rightarrow 4 + 2p + q = 0$

$\Rightarrow 2p = -4 - q$

$\Rightarrow (q+4)^2 = 4p^2$

$\lim_{x\to 2p^+} f(x) = \lim_{x\to 2p^+} \frac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^2}$

$= \lim_{x\to 2p^+} \frac{1-\cos(x^2-4px+(q+4)^2)}{(x-2p)^2}$

$= \lim_{x\to 2p^+} \frac{1-\cos(x^2-4px+4p^2)}{(x-2p)^2}$

$= \lim_{x\to 2p^+} \frac{1-\cos(x-2p)^2}{(x-2p)^2}$

$= \lim_{x\to 2p^+} \frac{2\sin^2\frac{(x-2p)^2}{2}}{(x-2p)^2} \times \frac{(x-2p)^2\times 4}{4\times(x-2p)^2}$ $\left(\because\ 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right)$

$= \lim_{x\to 2p^+} 2\left(\frac{\sin\frac{(x-2p)^2}{2}}{\frac{(x-2p)^2}{2}}\right) \times \frac{(x-2p)^2}{4}$

$= 0$

11. If Incident ray $y = \frac{x}{\sqrt{3}}$ is incident on a reflecting surface $x + y = 1$. Then point of intersection of reflecting ray with x-axis is:

A. $\left(1 - \frac{1}{\sqrt{3}}, 0\right)$

B. $\left(1 + \frac{1}{\sqrt{3}}, 0\right)$

C. $\left(\frac{1}{\sqrt{3}}, 0\right)$

D. $\left(\frac{2}{\sqrt{3}}, 0\right)$

Answer (A)

Solution:

Solving,

$x + y = 1$ &

$y = \frac{x}{\sqrt{3}}$

$x + \frac{x}{\sqrt{3}} = 1$

$x = \frac{\sqrt{3}}{\sqrt{3}+1}$

$y = \frac{1}{\sqrt{3}+1}$

$\Rightarrow A = \left(\frac{\sqrt{3}}{\sqrt{3}+1}, \frac{1}{\sqrt{3}+1}\right)$

For line AP, $y - \frac{1}{\sqrt{3}+1} = \sqrt{3}\left(x - \frac{\sqrt{3}}{\sqrt{3}+1}\right)$

At P, $-\frac{1}{\sqrt{3}+1} = \sqrt{3}\left(h - \frac{\sqrt{3}}{\sqrt{3}+1}\right)$

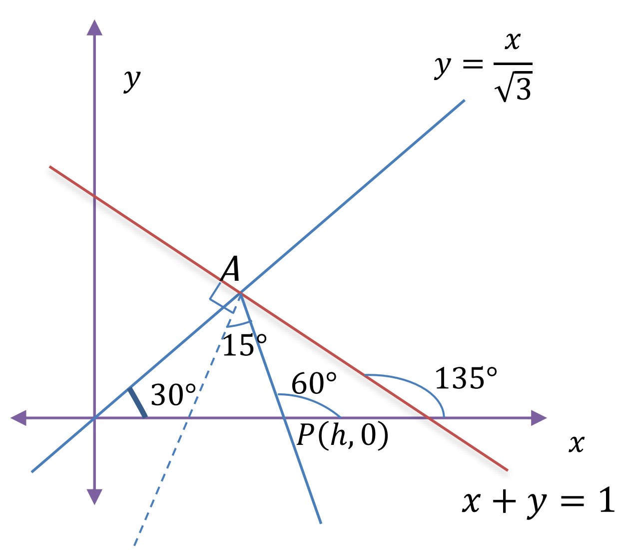
$\Rightarrow h = -\frac{1}{\sqrt{3}(\sqrt{3}+1)} + \frac{\sqrt{3}}{\sqrt{3}+1}$

$= \frac{1}{\sqrt{3}+1}\left(-\frac{1}{\sqrt{3}} + \sqrt{3}\right)$

$= \frac{2}{\sqrt{3}(\sqrt{3}+1)}$

$= \frac{\sqrt{3}-1}{\sqrt{3}}$

$\Rightarrow h = 1 - \frac{1}{\sqrt{3}}$

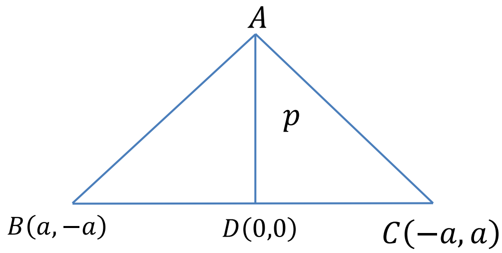


12. In an equilateral triangle ABC, point A lies on the line $y - 2x = 2$ and points B and C are lying on the line $y + x = 0$. If Points B and C are symmetric with respect to origin then area of the ΔABC (in sq. units) is :

A. $4\sqrt{3}$
B. 8
C. $\frac{8}{\sqrt{3}}$
D. $8\sqrt{3}$

Answer (A)

Solution:



Let coordinates of points B & C be $(a, -a)$& $(-a, a)$ respectively.

A lies on perpendicular bisector of BC

$i.e.,\ \Rightarrow y = x$

A is point of intersection of $y = x$ and $y - 2x = 2$

$A = (-2, -2)$

$p = AD = 2\sqrt{2}$

Area of $\Delta ABC = \frac{1}{2} . \frac{2p}{\sqrt{3}} . p$

$= \frac{p^2}{\sqrt{3}} = \frac{8}{\sqrt{3}}$(sq. units)

13. It is given that $\big((p \wedge q) \vee r\big) \vee (p \wedge r) \rightarrow (\sim q) \vee r$ is fallacy. Then truth values of p, q and r are given by:

A. p: true, q: true, r: false
B. p: false, q: false, r: false
C. p: true, q: true, r: true
D. None of these

Answer (A)

Solution:

$s \rightarrow t$ is always false if s is true and t is false.

$\therefore (\sim q) \vee r$ is false $\Rightarrow \sim q$ is false and r is false

$\Rightarrow q$ is true and r is false.

Also, if p is true, then $\big((p \wedge q) \vee r\big) \vee (p \wedge r)$ is true.

$\therefore$ option A is correct.

14. Let A & B be area of regions for $x \in [0,\ 1]$ given by

$A: 2x \le y \le \sqrt{4(x-1)^2}$ with y-axis

$B: y = \min\left\{2x,\ \sqrt{4(x-1)^2}\right\}$ with x-axis, then $\frac{A}{B}$ equals:

A. 1
B. 2
C. 3
D. 4

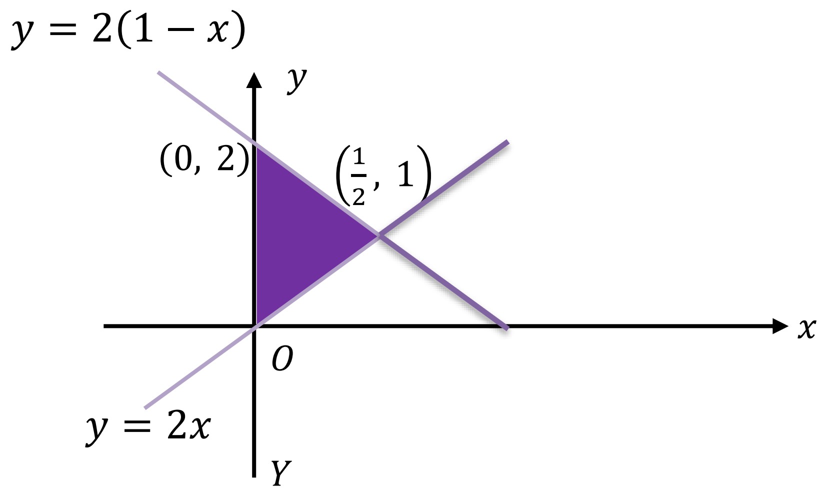
Answer (A)

Solution:

$A: 2x \le y \le \sqrt{4(x-1)^2}$ with y-axis

$y \le \sqrt{4(x-1)^2} \Rightarrow y \le 2|x-1|$

$\Rightarrow y \le 2(1-x) \because x \in [0, 1]$

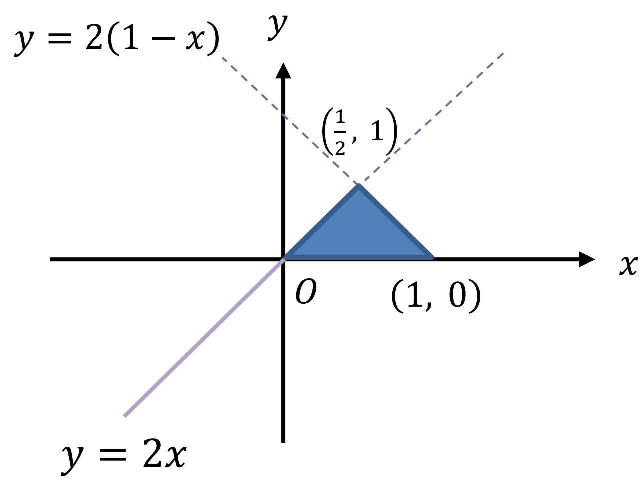


From the figure A is equal to area of highlighted triangle $A = \frac{1}{2} \times 2 \times \frac{1}{2} = \frac{1}{2}$

$B: y = \min\left\{2x, \sqrt{4(x-1)^2}\right\}$ with x-axis:

From this figure we get B as:

$B = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$



$\therefore \frac{A}{B} = 1$

15. A function $f(x)$ is such that $f(x+y) = f(x) + f(y) - 1 \; \forall x, y \in \mathbb{R}$, Also $f'(0) = 2$, then $|f(-2)|$ is:

Answer (3)

Solution:

$f(x+y) = f(x) + f(y) - 1$

Put $x = y = 0$

$\Rightarrow f(0) = f(0) + f(0) - 1$

$\Rightarrow f(0) = 1$

Now $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(x)+f(h)-1-f(x)}{h}$ $\quad [\because f(x+y) = f(x) + f(y) - 1]$

$\Rightarrow f'(x) = \lim_{h\to 0} \frac{f(h)-1}{h}$

$\Rightarrow f'(x) = f'(0) = 2 \quad \cdots (\because f'(0) = 2)$

Integrating both sides

$\Rightarrow f(x) = 2x + c$

$\because f(0) = 1,$

$\Rightarrow c = 1$
$\therefore f(x) = 2x + 1$
$f(-2) = (2 \times -2) + 1 = -3$
$|f(-2)| = 3$

16. If $a_1, a_2, \ldots$ are positive numbers in GP such that $a_5 + a_7 = 12$ and $a_4.a_6 = 9$ then $a_7 + a_9$ equals ______.

Answer (36)

Solution:

Let first term of GP be a with common ratio r
$\Rightarrow a, r > 0$
$a_5 + a_7 = 12 \cdots (1)$
$a_4.a_6 = 9$
$ar^3 \cdot ar^5 = 9$
$\Rightarrow a^2r^8 = 9$
$\Rightarrow ar^4 = 3 \Rightarrow a_5 = 3$
Substitute $a_5 = 3$ in eq. (1)
$3 + a_7 = 12$
$\Rightarrow a_7 = 9$
$\Rightarrow ar^6 = 9$
By taking the ratio of $\frac{a_7}{a_5}$, we get
$\frac{a_7}{a_5} = \frac{ar^6}{ar^4} = \frac{9}{3}$
$\Rightarrow r^2 = 3$
$\Rightarrow r = \sqrt{3}$
$\Rightarrow a = \frac{1}{3}$
$\therefore a_9 = ar^8$
$\Rightarrow a_9 = \frac{1}{3} \times \left(\sqrt{3}\right)^8 = 27$
$\therefore a_7 + a_9 = 9 + 27 = 36$

17. If $f(x + y) = f(x) + f(y)$, $f(1) = \frac{1}{5}$ and $\sum_{n=1}^{N} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$, then the value of N is ______.

Answer (10)

Solution:

$f(x + y) = f(x) + f(y) \qquad \cdots$ (given)
$\Rightarrow f(x) = kx$
$f(1) = \frac{1}{5} \Rightarrow k = \frac{1}{5}$
$\therefore f(x) = \frac{1}{5}x$
$\sum_{n=1}^{N} \frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12} \quad \cdots$ (given)
$\Rightarrow \sum_{n=1}^{N} \frac{\frac{1}{5}n}{n(n+1)(n+2)} = \frac{1}{12}$
$\Rightarrow \frac{1}{5}\sum_{n=1}^{N} \left(\frac{1}{n+1} - \frac{1}{n+2}\right) = \frac{1}{12}$
$\Rightarrow \frac{1}{5}\left(\frac{1}{2} - \frac{1}{N+2}\right) = \frac{1}{12}$
$\Rightarrow \frac{N}{10(N+2)} = \frac{1}{12}$
$\Rightarrow 12N = 10N + 20$
$\Rightarrow 2N = 20$
$\Rightarrow N = 10$

18. Let $S = \{1, 2, 3, 5, 7\}$. The rank of 35773, if all 5 digit numbers formed by the set S are arranged in a dictionary in ascending order & repetition of digits is allowed is __________.

Answer (1748)

Solution:

All five digit numbers starting from 1 and 2 will come first

i.e., $1 - \ - \ - - \rightarrow 5^4$

$2 - \ - \ - \ - \rightarrow 5^4$

If first digit is 3 (number of numbers that comes before 35773)

$3\ 1\ - \ - - \rightarrow 5^3$

$3\ 2 - - - \rightarrow 5^3$

$3\ 3 - - - \rightarrow 5^3$

$3\ 5\ 1 - - \rightarrow 5^2$

$3\ 5\ 2 - - \rightarrow 5^2$

$3\ 5\ 3 - - \rightarrow 5^2$

$3\ 5\ 5 - - \rightarrow 5^2$

$3\ 5\ 7\ 1 - \rightarrow 5$

$3\ 5\ 7\ 2 - \rightarrow 5$

$3\ 5\ 7\ 3 - \rightarrow 5$

$3\ 5\ 7\ 5 - \rightarrow 5$

$3\ 5\ 7\ 7\ 1 \rightarrow 1$

$3\ 5\ 7\ 7\ 2 \rightarrow 1$

$3\ 5\ 7\ 7\ 3 \rightarrow 1$

$\therefore$ rank $= 2(5^4) + 3(5^3) + 4(5^2) + 4(5) + 3$

$= 1250 + 375 + 100 + 20 + 3$

rank $= 1748$

19. If the ratio of coefficients of 3 consecutive terms in expansion of $(1+2x)^n$ is $10:35:84$. Then n is equal to __________.

Answer (10)

Solution:

$\frac{{}^nC_r 2^r}{{}^nC_{r+1} 2^{r+1}} = \frac{2}{7}$

$\Rightarrow \frac{r+1}{n-r} \cdot \frac{1}{2} = \frac{2}{7}$

$\Rightarrow n - r = \frac{7}{4}(r+1)$ ... (1)

$\frac{{}^nC_{r+1} 2^{r+1}}{{}^nC_{r+2} 2^{r+2}} = \frac{5}{12}$

$\Rightarrow \frac{r+2}{n-r-1} \cdot \frac{1}{2} = \frac{5}{12}$

$\Rightarrow n - r - 1 = \frac{6}{5}(r+2) \quad \cdots (2)$

Solving eq.(1) and eq.(2)

$r = 3$ & $n = 10$

20. Consider 3 coplanar vectors $\vec{a} = 3\hat{\imath} - 4\hat{\jmath} + \lambda\hat{k}$, $\vec{\mathrm{b}} = 4\hat{\imath} + 3\hat{\jmath} - \hat{\mathrm{k}}$ and $\vec{c} = \hat{\imath} + 3\hat{\jmath} - 4\hat{\mathrm{k}}$. Then 9λ is ________.

Answer (87)

Solution:

For coplanar vectors

$\begin{vmatrix} 3 & -4 & \lambda \\ 4 & 3 & -1 \\ 1 & 3 & -4 \end{vmatrix} = 0$

$\Rightarrow 3(-12+3) + 4(-16+1) + \lambda(12-3) = 0$

$\Rightarrow -27 - 60 + 9\lambda = 0$

$\Rightarrow 9\lambda = 87$