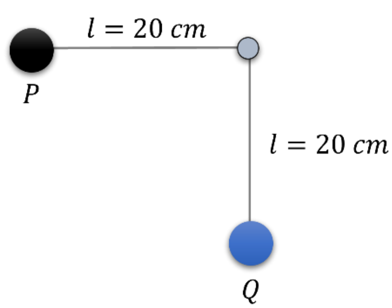
JEE Main 2023 (Memory based)

30 January 2023 - Shift 1

Answer & Solutions

PHYSICS

1. Bob P is released from the position of rest at the moment shown. If it collides elastically with an identical bob Q hanging freely then velocity of Q just after collision is $(g = 10\ m/s^2)$



Answer (C)

Solution:

Velocity of P just before collision is $\sqrt{2gl} = 2\ m/s$
As collision is elastic and the mass of P and Q is equal therefore just after collision,
velocity of P is 0 and that of Q is $2\ m/s$.

2. Choose the option showing the correct relation between Poisson's ratio(σ), Bulk modulus(B) and Modulus of rigidity(G).

A. $\sigma = \frac{3B-2G}{2G+6B}$

B. $\sigma = \frac{6B+2G}{3B-2G}$

C. $\sigma = \frac{9BG}{3B+G}$

D. $B = \frac{3\sigma-3G}{6\sigma+2G}$

Answer (A)

Solution:

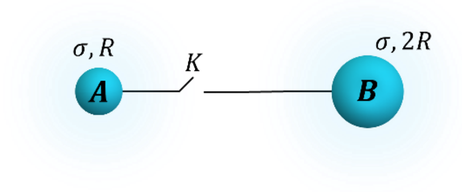
$E = 2G(1+\sigma) \dots\dots (1)$

$E = 3B(1-2\sigma) \dots\dots (2)$

$$1 = \frac{2G}{3B}\left(\frac{1+\sigma}{1-2\sigma}\right)$$

$3B - 6B\sigma = 2G + 2G\sigma$
$3B - 2G = \sigma(2G + 6B)$
$\sigma = \dfrac{3B - 2G}{2G + 6B}$

3. Two conducting solid spheres A and B are placed at a very large distance with charge densities and radii as shown. When the key K is closed, find the ratio of final charge densities.



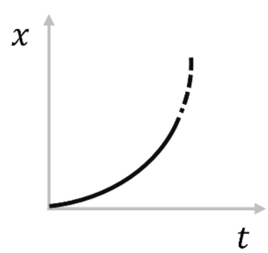
A. $4:1$
B. $1:2$
C. $2:1$
D. $1:4$

Answer (C)

Solution:

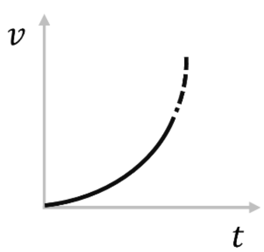
Final potential is same:
$\Rightarrow \dfrac{1}{4\pi\epsilon_0} \times \dfrac{Q_1}{R} = \dfrac{1}{4\pi\epsilon_0} \times \dfrac{Q_2}{2R} \ldots\ldots..(1)$
Also, $Q_1 + Q_2 = \sigma.4\pi R^2 + \sigma.4\pi(2R)^2 \ldots\ldots(2)$
From (1) and (2)
$\dfrac{\sigma_1}{\sigma_2} = 2$

4. Position-time graph for a particle is parabolic and is as shown:

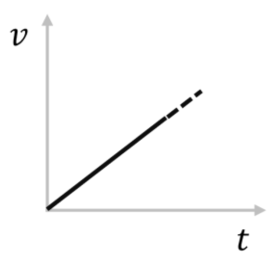


Choose the corresponding v-t graph.

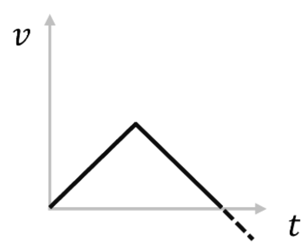
A.



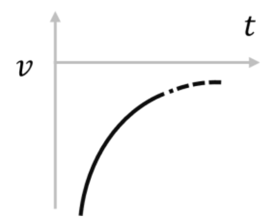
B.



C.



D.



Answer (B)

Solution:

Since, $x \propto t^2$
$\Rightarrow v = \dfrac{dx}{dt} \propto t$
So, it will be a linear plot and option (B) is correct.

5. Electromagnetic wave beam of power $20\ mW$ is incident on a perfectly absorbing body for $300\ ns$. The total momentum transferred by the beam to the body is equal to

A. $2\ \times 10^{-17}\ Ns$
B. $1\ \times 10^{-17}\ Ns$
C. $3\ \times 10^{-17}\ Ns$
D. $5\ \times 10^{-17}\ Ns$

Answer (A)

Solution:

Total energy incident $= Pt$

So,

Total initial momentum $= Pt/c$

Total final momentum $= 0$

Total momentum transferred:

$$\frac{Pt}{c} = \frac{20 \times 10^{-3} \times 300 \times 10^{-9}}{3 \times 10^{8}} = 2 \times 10^{-17}\ Ns$$

6. The velocity of an electron in the seventh orbit of hydrogen like atom is $3.6 \times 10^{6}\ m/s$. Find the velocity of the electron in the $3rd$ orbit.

A. $4.2 \times 10^{6}\ m/s$
B. $8.4 \times 10^{6}\ m/s$
C. $2.1 \times 10^{6}\ m/s$
D. $3.6 \times 10^{6}\ m/s$

Answer (B)

Solution:

For hydrogen like atom,

$$v \propto \frac{1}{n}$$

$$\frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$\frac{3.6 \times 10^{6}}{v_2} = \frac{3}{7}$$

$$v_2 = 8.4 \times 10^{6}\ m/s$$

7. Electric field in a region is given by $\vec{E} = \frac{a}{x^2}\ \hat{\imath} + \frac{b}{y^3}\hat{\jmath}$, where $x\ \&\ y$ are coordinates. Find SI units of $a\ \&\ b$.

A. $a - Nm^2C^{-1}, b - Nm^3C^{-1}$
B. $a - Nm^3C^{-1},, b - Nm^2C^{-1}$
C. $a - Nm\ C^{-1},, b - Nm^2C^{-1}$
D. $a - Nm^2C^{-1},, b - Nm^2C^{-1}$

Answer (A)

Solution:

Unit of Electric Field: $E \Rightarrow N/C$

$x^2 - m^2$

$y^3 - m^3$

$a - Nm^2C^{-1},, b - Nm^3C^{-1}$

8. Coil A of radius $10\ cm$ has N_A number of turns and I_A current is flowing through it. Coil B of radius $20\ cm$ has N_B number of turns and I_B current is flowing through it. If magnetic dipole moment of both the coils is same then

A. $I_A N_A = 4 I_B N_B$
B. $I_A N_A = \frac{1}{4} I_B N_B$
C. $I_A N_A = 2 I_B N_B$
D. $I_A N_A = \frac{1}{2} I_B N_B$

Answer (A)

Solution:

Magnetic dipole moment, $\mu = NiA = Ni\pi R^2$

So,

$$\frac{\mu_A}{\mu_B} = \frac{N_A i_A R_A^2}{N_B i_B R_B^2} = 1$$

$$\frac{N_A i_A 10^2}{N_B i_B 20^2} = 1$$

$$N_A i_A = 4 N_B i_B$$

9. An ideal gas undergoes a thermodynamic process following the relation $PT^2 = constant$. Assuming symbols have their usual meaning then volume expansion coefficient of the gas is equal to:

A. $2/T$
B. $3/T$
C. $1/2T$
D. $1/T$

Answer (C)

Solution:

Volume expansion coefficient= $V \times \left(\frac{dV}{dT}\right)$

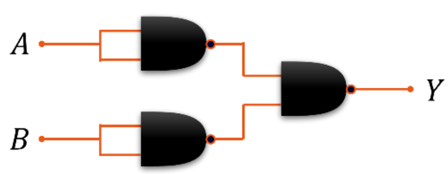
So,

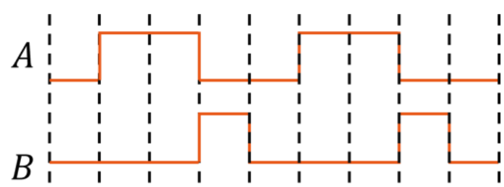
$PT^2 = constant$

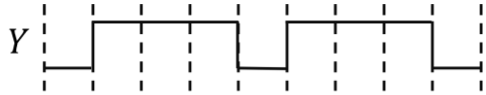
Or,

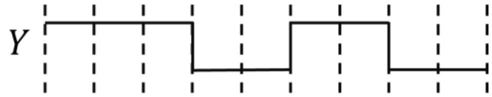
$$\frac{T^3}{V} = constant$$
$$\frac{dV}{dT} = C \times 3T^2$$
$$\frac{1}{V} \times \frac{dV}{dT} = \frac{3T^2}{T^3} = \frac{3}{T}$$

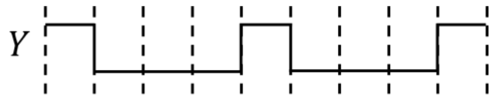
10. Consider a combination of gates as shown:





A. 

B. 

C. 

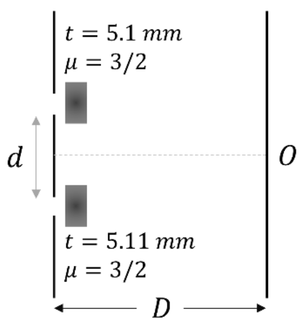
D. 

Answer (A)

Solution:

$Y = (A'B')' = A + B$
$\Rightarrow$OR gate

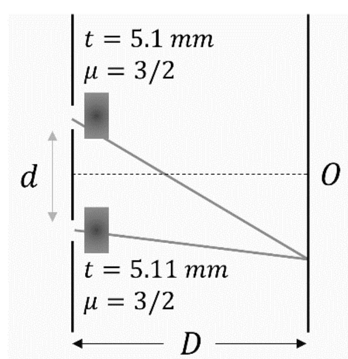
11. For the given $YDSE$ setup, find the number of fringes by which the central maxima gets shifted from point O.
(Given $d = 1\ mm,\ D = 1\ m,\ \lambda = 5000\dot{A}$)



A. 10
B. 15
C. 8
D. 12

Answer (A)

Solution:



At central position, path difference is,
$\Delta x = (\mu - 1)(t_1 - t_2)$
$$\Delta x = \left(\frac{3}{2} - 1\right)(5.11 - 5.1)\ mm = \frac{1}{2} \times 0.01 = 0.005\ mm$$

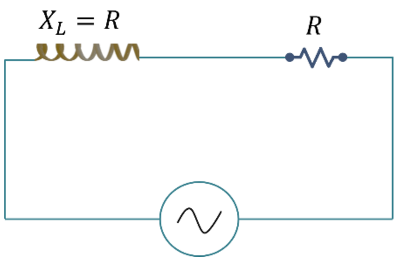
Number of fringes shifted= $\Delta x / \lambda$
$$\frac{\Delta x}{\lambda} = \frac{5 \times 10^{-6}}{5 \times 10^{-7}} = 10$$

12. If an insulator with inductive reactance $X_L = R$ is connected in series with resistance R across an $A.C.$ Voltage, power factor comes out to be P_1. Now, if a capacitor with capacitive reactance $X_C = R$ is also connected in series with the inductor and resistor in the same circuit, power factor becomes P_2. Find P_1/P_2.

A. $\sqrt{2}:1$
B. $1:\sqrt{2}$
C. $1:1$
D. $1:2$

Answer (B)

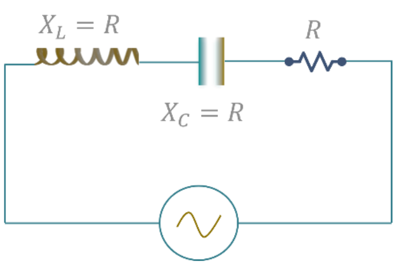
Solution:



Impedance of circuit:

$Z = \sqrt{R^2 + R^2} = \sqrt{2}R$

$P_1 = \cos\phi = \text{Power Factor} = \frac{R}{Z} = \frac{1}{\sqrt{2}}$



When capacitor is also connected in series:

The LCR circuit is in resonance stage
So,

$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$

$P_2 = \cos\phi = Power\ Factor = \frac{R}{Z} = \frac{R}{R} = 1$

So,

$\frac{P_1}{P_2} = \frac{\left(\frac{1}{\sqrt{2}}\right)}{1} = \frac{1}{\sqrt{2}}$

13. For a system undergoing isothermal process, heat energy is supplied to the system. Choose the option showing correct statements.
1) Internal energy will increase.
2) Internal energy will decrease.
3) Work done by system is positive.
4) Work done by system is negative.
5) Internal energy remains constant.

A. (1), (3), (5)
B. (2), (4)
C. (3), (5)
D. (1), (4), (5)

Answer (C)

Solution:

From First Law of thermodynamics, $\Delta Q = \Delta U + \Delta W$
For Isothermal process, $\Delta T = 0 \Rightarrow U = Constant \Rightarrow \Delta U = 0$
So,
$\Delta Q = \Delta W$
As heat is supplied, ΔW is positive. Hence, work is done by the system.

14. The heat passing through the cross-section of a conductor, varies with time 't' as $Q(t) = \alpha t - \beta t^2 + \gamma t^3 (\alpha, \beta$ and γ are positive constants). The minimum heat current through the conductor is

A. $\alpha - \frac{\beta^2}{2\gamma}$
B. $\alpha - \frac{\beta^2}{3\gamma}$
C. $\alpha - \frac{\beta^2}{\gamma}$
D. $\alpha - \frac{3\beta^2}{\gamma}$

Answer (B)

Solution:

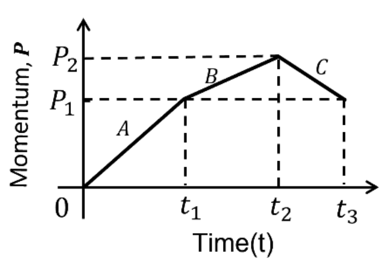
Heat through cross section of rod, $Q = \alpha t - \beta t^2 + \gamma t^3$
Heat current, $i = \frac{dQ}{dt} = \alpha - 2\beta t + 3\gamma t^2$

For heat current to be minimum, $\frac{di}{dt} = 0$
$\Rightarrow -2\beta + 6\gamma t = 0$
$\Rightarrow t = \dfrac{\beta}{3\gamma}$

So minimum heat current, $i_{min} = \alpha - 2\beta \times \frac{\beta}{3\gamma} + 3\gamma \left(\frac{\beta}{3\gamma}\right)^2$

$$= \alpha - \frac{\beta^2}{3\gamma}$$

15. Momentum-time graph of an object moving along a straight line is as shown in figure. If $(P_2 - P_1) < P_1$ and $(t_2 - t_1) = t_1 < (t_3 - t_2)$ then at which points among A, B and C the magnitude of force experienced by the object is maximum and minimum respectively



A. A, B
B. A, C
C. B, C
D. B, A

Answer (B)

Solution:

$$F_A = \frac{P_1 - 0}{t_1 - 0}$$
$$F_B = \frac{P_2 - P_1}{t_2 - t_1}$$

$$|F_C| = \frac{P_2 - P_1}{t_3 - t_2}$$

As given, $(P_2 - P_1) < P_1$ and $(t_2 - t_1) = t_1$ So, $F_B < F_A$

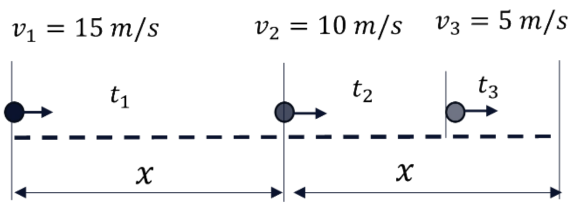
As given, $(t_2 - t_1) < (t_3 - t_2)$ So, $F_B > |F_C|$

16. A particle moving in unidirectional motion travels half of the total distance with a constant speed of $15\ m/s$. Now first half of the remaining journey time, it travels at $10\ m/s$ and second half of the remaining journey time, it travels at $5\ m/s$. Average speed of the particle is:

A. $12\ m/s$
B. $10\ m/s$
C. $7\ m/s$
D. $9\ m/s$

Answer (B)

Solution:



$$v_{av} = \frac{2x}{t_1 + t_2 + t_3}$$

As $t_2 = t_3 = t$

So, $x = t(v_2 + v_3) \Rightarrow t = x/(v_2 + v_3)$

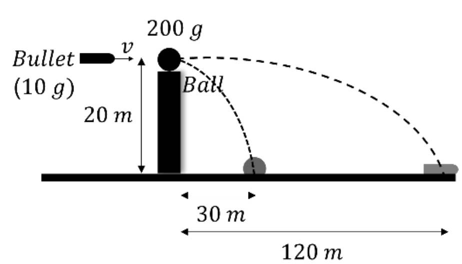
$$v_{av} = \frac{2x}{\frac{x}{15} + 2t}$$

$$v_{av} = \frac{2x}{\frac{x}{15} + \frac{2x}{10+5}}$$

$$v_{av} = 10\ m/s$$

17. A bullet strikes a stationary ball kept at a height as shown. After collision, range of bullet is $120\ m$ and that of ball is $30\ m$. Find initial speed of bullet. Collision is along horizontal direction.$(Take\ g = 10\ m/s^2)$

A. $150\ m/s$
B. $90\ m/s$
C. $240\ m/s$
D. $360\ m/s$



Answer (D)

Solution:

Applying conservation of momentum,
$m_1 v + m_2(0) = m_1 v_1' + m_2 v_2'$

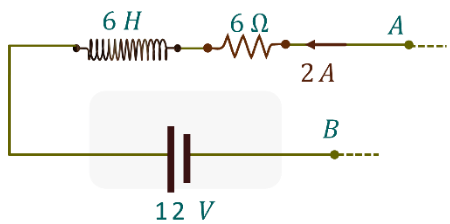
$$\Delta t = \sqrt{\frac{2h}{g}} = 2\ s$$

$$v_1' = \frac{120}{2} = 60\ m/s$$

$$v_2' = \frac{30}{2} = 15\, m/s$$
$$v = 360\, m/s$$

18. In a part of circuit as shown, find $V_A - V_B$ in $Volts$. It is given that current is decreasing at a rate of $1\ Ampere/s$

Answer (18)



Solution:

$$V_A - iR - \frac{Ldi}{dt} - 12 = V_b$$
$$V_A - 2 \times 6 - 6(-1)\ - 12 = V_B$$
$$V_A - V_B = 18$$

19. A particle undergoing SHM, follows the position-time equation given as $x = A\sin(\omega t + \pi/3)$. If the SHM motion has a time period of T, then velocity will be maximum at time $t = T/\beta$ for first time after $t = 0$. value of β is equal to ______.

Answer (3)

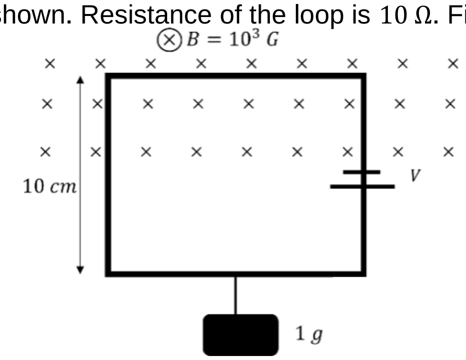
Solution:

$$x = A\ Sin\ (\omega t + \frac{\pi}{3})$$
$$v = A\omega Cos\left(\omega t + \frac{\pi}{3}\right)$$

For maximum value of v,

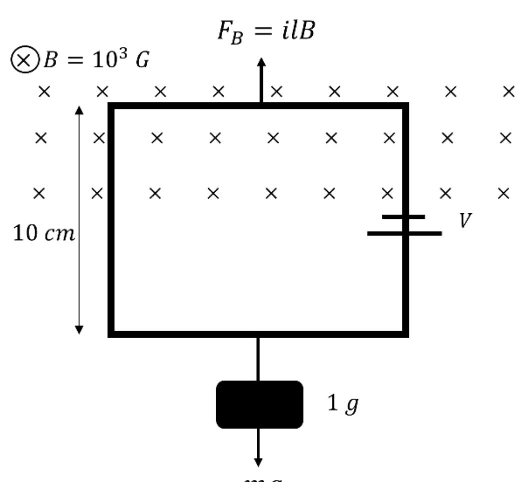
$$Cos\ \left(\omega t + \frac{\pi}{3}\right) = \pm 1$$
$$\Rightarrow \omega t + \frac{\pi}{3} = \pi$$
$$\Rightarrow \omega t = \frac{2\pi}{3}$$
$$\Rightarrow t = \frac{T}{3}$$

20. A block of mass $1\ kg$ is in equilibrium with the help of current carrying square loop which is partially laying in constant magnetic field (B) as shown. Resistance of the loop is $10\ \Omega$. Find the voltage V of the battery in the loop.



Answer (10)

Solution:



In equilibrium, $ilB = mg$

$$\Rightarrow i = \frac{mg}{lB} = \frac{1 \times 10^{-3}\ kg \times 10 \frac{m}{s^2}}{0.1\ m \times 0.1\ T} = 1\ A$$

As resistance of loop =1 Ω

$$V = iR = 10\ V$$

21. Initial volume of $1\ mole$ of a $monoatomic$ gas is $2\ litres$. It is expanded isothermally to a volume of $6\ litres$. Change in internal energy is xR. Find x.

Answer (0)

Solution:

As internal energy is a function of temperature only.
$\Delta U = nC_v\Delta T$
So, $\Delta U = 0$

22. An object is placed at a distance of $40\ cm$ from the pole of a converging mirror. The image is formed at a distance of $120\ cm$ from the mirror on the same side. The focal length is measured with a scale where each $1\ cm$ has 20 equal divisions. If the fractional error in the measurement of focal length is $1/10K$, then find K.

Answer (60)

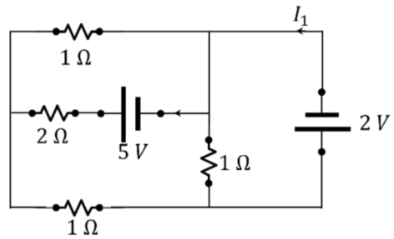
Solution:

$u = -40\ cm \ \ v = -120\ cm$

Mirror formula,
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$
$$\Rightarrow -\frac{1}{120} - \frac{1}{40} = \frac{1}{f}$$
$f = -30\ cm$

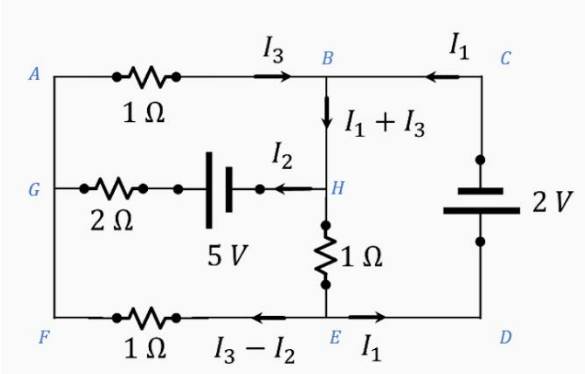
$$Least\ count\ of\ scale = \frac{1}{20}\ cm$$
$$Fractional\ error\ = \frac{\left(\frac{1}{20}\right)}{30} = \frac{1}{600}$$
So, $K = 60$

23. In the circuit shown below, the value of $current\ I_1$(in $amperes$) is equal to $-\frac{y}{5}\ Amp$. The value of y is equal to:



Answer (11)

Solution:



Using Kirchhoff's voltage law,

For loop $BCDEHB$, $(I_1 + I_3 - I_2) \times 1 = -2 \ldots\ldots(1)$

For loop $ABHGA$, $(I_3 + 2I_2) = 5 \ldots\ldots\ldots\ldots\ldots\ldots(2)$

For loop $GHEFG$, $2I_2 - (I_3 - I_2) - (I_1 + I_3 - I_2) = 5 \ldots\ldots(3)$

Solving (1), (2) and (3)

$$I_1 = -\frac{11}{5}$$

$y = 11$

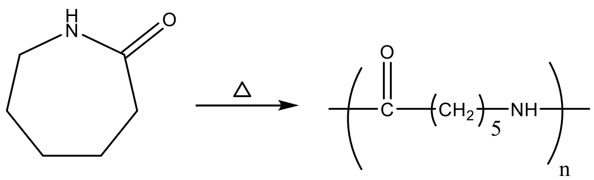
1. Caprolactam when heated at high temperature gives

A. Nylon 6,6
B. Dacron
C. Teflon
D. Nylon 6

Answer (D)

Solution:

Caprolactam on heated at high temperature gives Nylon 6 polymer



Caprolactam Nylon 6

2. Molarity of CO_2 in soft drink is 0.01 M. The volume of soft drink is 300 mL. Mass of CO_2 in soft drink is

A. 0.132 g
B. 0.481 g
C. 0.312 g
D. 0.190 g

Answer (A)

Solution:

Molarity = $\frac{moles\ of\ solute}{Volume\ (L)} = \frac{millimoles}{Volume(mL)}$

millimoles = MV (mL)

millimoles of CO_2 = 0.01 × 300 = 3 or moles of CO_2 = 3×10^{-3}

Mass of CO_2 = moles × Mol.wt

$= 3 \times 10^{-3} \times 44$

= 132 $\times 10^{-3}\ g$

Mass of CO_2 = 0.132 g

3. During the qualitative analysis of SO_3^{2-} using acidified H_2SO_4, SO_2 gas evolved which turns $K_2Cr_2O_7$ solution

A. Green
B. Black
C. Blue
D. Red

Answer (A)

Solution:

Orange of dichromate solution $K_2Cr_2O_7$ converts to green Cr^{3+}

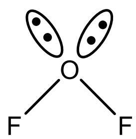
$Cr_2O_7^{2-} + 2SO_3^{2-} + 8H^+ \rightarrow 2Cr^{3+} + 2SO_4^{2-} + 4H_2O$

4. Shape of OF_2 molecule is

A. Bent
B. Linear
C. Tetrahedral
D. T- Shaped

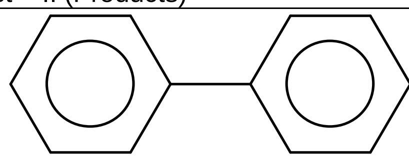
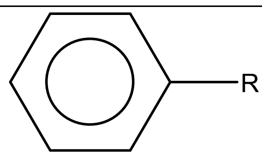
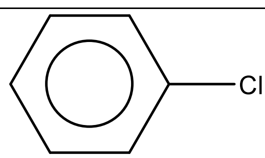
Answer (A)

Solution:



It is sp^3 hybridized therefore it's shape will be Bent or V - Shaped

5. Which of the following option contains correct match:

List – I (Reactions)	List – II (Products)
A. Wurtz	P. 
B. Fittig	Q. R—R
C. Wurtz Fittig	R. 
D. Sandmeyer	S. 

A. A – Q, B – P, C – R, D - S
B. A – P, B – Q, C – R, D - S
C. A – S, B – R, C – Q, D - P
D. A – R, B – S, C – P, D - Q

Answer (A)

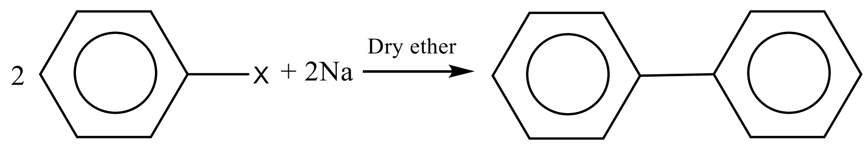
Solution:

The correct matches are

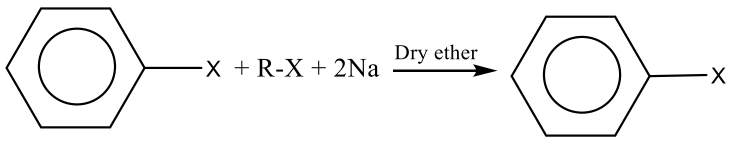
A. Wurtz reaction

$$2\ R\text{-}X + 2Na \xrightarrow{\text{Dry ether}} R\text{-}R + 2NaX$$

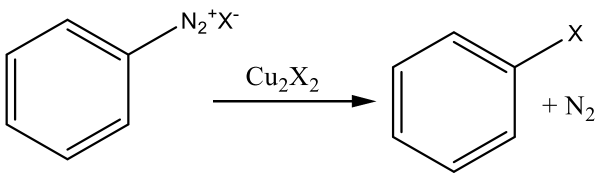
B. Fittig reaction



C. Wurtz-Fittig reaction



D. Sandmeyer reaction



6. For a given cell at T K,
Pt / H_2 (g)/ H^+ // Fe^{3+}/ Fe^{2+}/ Pt
(1 bar) (1 M)
E cell = 0.712 V
E^0 cell = 0.770 V
If $\frac{[Fe^{2+}]}{[Fe^{3+}]}$ is t ($\frac{2.303\,RT}{F} = 0.058$)
Find ($\frac{t}{5}$)

Answer (2)

Solution:

$$0.712 = 0.770 - \frac{0.058}{2}\log\left(\frac{Fe^{2+}}{Fe^{3+}}\right)^2$$

$$-0.058 = -0.058\log\frac{[Fe^{2+}]}{[Fe^{3+}]}$$

$$\frac{Fe^{2+}}{Fe^{3+}} = 10 = t$$

$$\frac{t}{5} = 2$$

7. How many moles of electrons are required to reduce 1 mole of permanganate ions into manganese dioxide

Answer (3)

$$\underset{+7}{MnO_4^-} \longrightarrow \underset{+4}{MnO_2}$$

n-factor = 3

Therefore, 3 moles of electrons are required.

8. 600 mL of 0.04M HCl is mixed with 400mL of 0.02M H_2SO_4. Find the pH of the resulting solution.

Answer (1.40)

Solution:

moles of H^+ from HCl = 0.04 x 600

= 24 mol

moles of H^+ from H_2SO_4 = 0.02 x 2 x 400

= 16 mol

Total moles of H^+ = 24+16 = 40 mol

Final volume of solution = 1000 mL

$[H^+] = \frac{40}{1000} = 0.04$ M

pH = -log (0.04) = 1.4

9. A solution of 2g of a solute and 20g water has boiling point 373.52 K. Then find the molecular mass of solute? [Given: K_b = 0.52 K kg/mole and solute is non-electrolyte]

Answer (100)

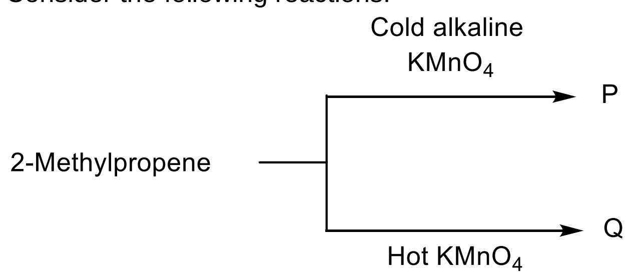
Solution:

$\Delta T_b = K_b . m$

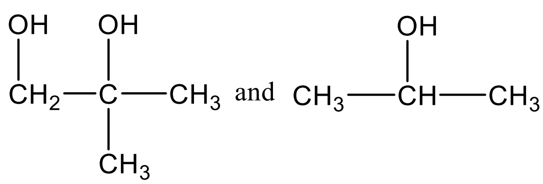
$0.52 = 0.52 \times \frac{2/M}{0.02}$ (M indicates molecular mass of solute)

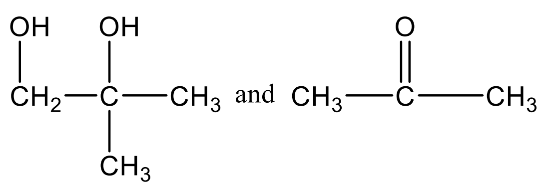
M = 100 g

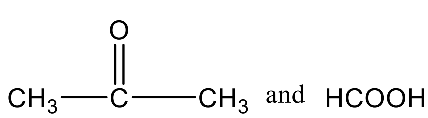
10. Consider the following reactions:

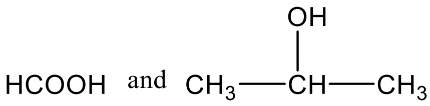


The Product P and Q respectively are?

A. 

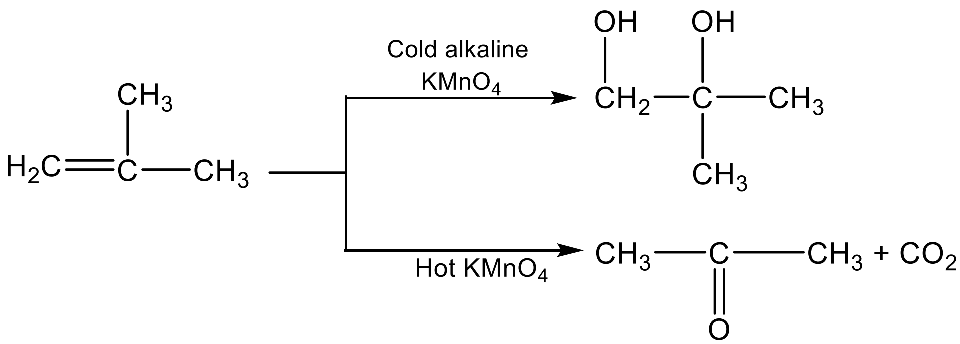
B. 

C. 

D. 

Answer (B)

Solution:



11. Assertion: ketos gives seliwanoff test
Reason: ketos undergoes β- elimination to form furfural

A. Assertion and reason both are correct and reason is the correct explanation of assertion
B. Assertion and reason both are correct but reason is not the correct explanation of assertion
C. Assertion is correct and reason is incorrect
D. Assertion is incorrect but reason is correct

Answer (A)

Solution:

Seliwanoff's reagent is a mixture of resorcinol and concentrated hydrochloric acid. This test distinguishes ketoses like fructose from other sugars, because in this test, only ketose sugars can produce the furfurals which form colored complexes with resorcinol.

12. The role of SiO_2 in Cu extraction is:

A. Converts FeO to $FeSiO_3$
B. Converts $\mathrm{Ca}O$ to $\mathrm{Ca}SiO_3$
C. Reduces $\mathrm{C}u_2S$ to Cu
D. None of these

Answer (A)

Solution:

SiO_2 behaves as flux and reacts with impurity (FeO) to form slag $(FeSiO_3)$
$FeO + SiO_2 \rightarrow FeSiO_3$.

13. Which of the following compound is used as antacid?

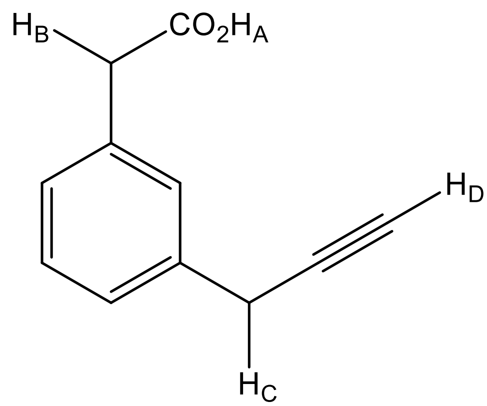
A. Ranitidine
B. Prontosil
C. Norethindrone
D. Codeine

Answer (A)

Solution:

Ranitidine is used as an antacid

14. Consider the following molecule



Select the correct order of the acidic strength

A. $H_A > H_D > H_B > H_C$
B. $H_B > H_A > H_D > H_C$
C. $H_A > H_B > H_C > H_D$
D. $H_C > H_B > H_D > H_A$

Answer (A)

Solution:

Acidic strength α Stability of conjugate base

Therefore,
The correct order of acidic strength $H_A > H_D > H_B > H_C$

15. Arrange the following ligands according to their increasing order of field strength
$S^{2-}, C_2O_4^{2-}, NH_3, en, CO$

A. $S^{2-} < CO < NH_3 < en < C_2O_4^{2-}$
B. $S^{2-} < NH_3 < en < CO < C_2O_4^{2-}$
C. $S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$
D. $CO < C_2O_4^{2-} < NH_3 < en < S^{2-}$

Answer (C)

Solution:

The correct order of field strength as per the spectrochemical series is
$S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$

16. If volume of ideal gas is increased isothermally than its internal energy

A. Increases
B. Remains constant
C. Decreases
D. Can be increased or decreased

Answer (B)

Solution:

$\Delta U = nC_v\Delta T$

And for an isothermal process $\Delta T = 0$

Therefore,

For isothermal expansion of ideal gas $\Delta U = 0$

17. For first order kinetic rate constant $2.011\text{x } 10^{-3}\ sec^{-1}$. The time taken for the decomposition of substance from 7g to 2g will be:
(Use log7 = 0.845 and log2 = 0.301)

Answer (623)

Solution:

A → products

Initial moles of A = $\frac{7}{M}$ (M is molar mass of A)

Final moles of A = $\frac{2}{M}$

Rate constant k = $2.011 \times 10^{-3} s^{-1}$

For a first order reaction

$t = \frac{2.303}{k}\log\frac{7}{2}$

$= \frac{2.303}{2.011} \times 10^{-3}[0.845 - 0.301]$

= 623 sec

18. Consider the following reactions

$NO_2 \xrightarrow{UV} A + B$

$A + O_2 \rightarrow C$

$B + C \rightarrow NO_2 + O_2$

A, B and C respectively are

A. O, NO, O_3
B. NO, O, O_3
C. NO, O_3, O
D. O_3, O, NO

Answer (A)

Solution:

$NO_2 \xrightarrow{UV} NO + O$

(B) (A)

$O + O_2 \rightarrow O_3$

(C)

$NO + O_3 \rightarrow NO_2 + O_2$

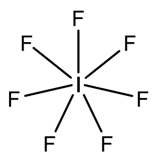
19. No. of lone pairs of central atoms are given.
Match the following.

Column 1	Column 2
A. IF_7	P. 0
B. ICl_4^-	Q. 1
C. XeF_2	R. 2
D. XeF_6	S. 3

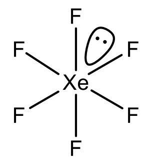
A – P, B – Q, C – R, D – S
E. A – P, B – R, C – S, D - Q
F. A – R, B – S, C – P, D - Q
G. A – S, B – R, C – Q, D - P

Answer (B)

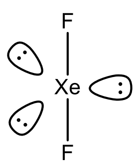
Solution:



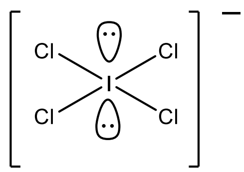
L.P = 0



L.P = 1



L.P = 3



L.P = 2

20. Which one of the following is water soluble?

a. $BeSO_4$
b. $MgSO_4$
c. $CaSO_4$
d. $SrSO_4$
e. $BaSO_4$

A. Only a & b
B. Only a, b, c
C. Only d & e
D. Only a & e

Answer (A)

Solution:

Solubility of sulphates of group-2 elements decreases down the group. $BeSO_4$ and $MgSO_4$ are appreciably soluble in water. $CaSO_4$, $SrSO_4$ and $BaSO_4$ are practically insoluble in water.

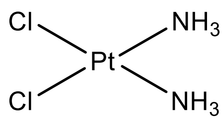
21. Inhibitor of cancer growth

A. Cis-platin
B. EDTA
C. Cobalt
D. Ethanol 1, 2-diamine

Answer (A)

Solution:

Cis-platin acts as an anticancer agent.



22. Speed of e⁻ in 7th orbit is 3.6 x 10^6 m/s, then find the speed in 3rd orbit.

A. 3.6 x 10^6 m/s
B. 8.4 x 10^6 m/s
C. 7.5 x 10^6 m/s
D. 1.8 x 10^6 m/s

Answer (B)

Solution:

Speed of electron in nth orbit of a Bohr atom is given by

$v_n = (v_1)_H \times \frac{z}{n}$

If n = 7

$v_7 = (v_1)_H \times \frac{z}{n} = 3.6 \times 10^6 \ m/s$

$\Rightarrow (v_1)_H \times Z = 3.6 \times 10^6 \times 7$ → Eq -1

If n = 3

$v_3 = (v_1)_H \times \frac{z}{3}$

Putting value of $(v_1)_H \times Z$ from Eq - 1

$= \frac{7 \times 3.6 \times 10^6}{3}$

$= 8.4 \times 10^6 \ m/s$

23. Match the following.

Atomic no	Group
i. 52	P. s
ii. 37	Q. p
iii. 65	R. f
iv. 74	S. d

A. (i) - Q, (ii) - P, (iii) - R, (iv) - S
B. (i) - Q, (ii) - P, (iii) - S, (iv) - R
C. (i) - S, (ii) - R, (iii) – P, (iv) - Q
D. (i) - R, (ii) - P, (iii) - Q, (iv) – S

Answer (B)

Solution:

f - block elements → Lanthanoids = 57 - 71
f - block elements → Actinoids = 89 - 103

65: f - block

37: $[Kr]5s^1 \rightarrow s - block$

52: $[Kr]5s^2 4d^{10} 5p^4 \rightarrow p - block$

74: $[Xe]6s^2 4f^{14} 5d^4 \rightarrow d - block$

MATHEMATICS

1. Coefficient of x^{301} in $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \cdots + x^{500}$ is equal to

A. ${}^{506}C_{306}$
B. ${}^{501}C_{300}$
C. ${}^{501}C_{301}$
D. ${}^{500}C_{300}$

Answer (C)

Solution:

Coefficient of $x^{301} = {}^{500}C_{301} + {}^{499}C_{300} + {}^{498}C_{299} + \cdots + {}^{199}C_0$

$= {}^{500}C_{199} + {}^{499}C_{199} + {}^{498}C_{199} + \cdots + {}^{199}C_{199}$

$= {}^{501}C_{200}$

$= {}^{501}C_{301}$ $\cdots$ (since ${}^{n}C_r = {}^{n}C_{n-r}$)

2. $\tan 15° + \frac{1}{\tan 165°} + \frac{1}{\tan 105°} + \tan 195° = 2a$. Then the value of $\left(a + \frac{1}{a}\right)$ is __________.

A. $4 - 2\sqrt{3}$

B. $-\frac{4}{\sqrt{3}}$

C. 2

D. $5 - \frac{3}{2}\sqrt{3}$

Answer (B)

Solution:

$\tan 15° + \cot 165° + \cot 105° + \tan 195°$

$= \tan 15° - \cot 15° - \tan 15° + \tan 15°$

$= \tan 15° - \cot 15°$

$= -2\sqrt{3}$

$\Rightarrow a = -\sqrt{3}$

$\Rightarrow a + \frac{1}{a} = -\sqrt{3} - \frac{1}{\sqrt{3}} = -\frac{4}{\sqrt{3}}$

3. If set $A = \{a, b, c\}$, $R: A \to A$, $R = \{(a, b), (b, c)\}$. How many elements should be added for making it symmetric and transitive?

A. 2
B. 3
C. 4
D. 7

Answer (D)

Solution:

For symmetric

$(a, b), (b, c) \in R$

$\Rightarrow (b, a), (c, b) \in R$

For Transitive

$(a, b), (b, c) \in R$

$\Rightarrow (a, c) \in R$

Now,

$(a, c) \in R$

$(c, a) \in R$ (for symmetric)

$(a, b), (b, a) \in R$

$\Rightarrow (a, a) \in R$

$(b, c), (c, b) \in R$

$\Rightarrow (b, b) \in R$

$(c, b), (b, c) \in R$

$\Rightarrow (c, c) \in R$

$\therefore$ elements to be added

$\{(b, a), (c, b), (b, b), (a, a), (a, c), (c, a), (c, c)\}$

Total 7 elements

4. Let $P(h, k)$ be any point on $x^2 = 4y$ which is at shortest distance from $Q(0, 33)$, then difference of distances of $P(h, k)$ from directrix of $y^2 = 4(x + y)$ is:

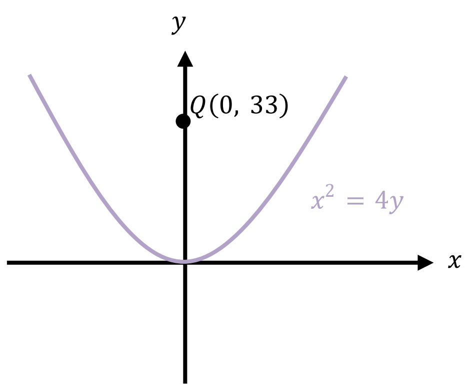
A. 2

B. 4

C. 6

D. 8

Answer (B)



Solution:

For normal through $Q(0, 33)$

Normal at point $(2t, t^2)$

$x = -ty + 2at + at^3$

$\Rightarrow 0 = -t \cdot 33 + 2a + t^3$

$\Rightarrow t = 0$ or $\pm\sqrt{31}$

Points at which normal are drawn are

$A(0, 0), B\left(2\sqrt{31}, 31\right), C\left(-2\sqrt{31}, 31\right)$

S.D $= QB = QC = \sqrt{124 + 4} = 8\sqrt{2}$ units

Given parabola $(y - 2)^2 = 4(x + 1)$

Directrix is $x = -2$, that is line L

$B_l - C_l = \left|\left(-2 + 2\sqrt{31}\right) - \left(2 + 2\sqrt{31}\right)\right| = 4$

5. Area bounded by larger part in I^{st} quadrant by $x = 4y^2$, $x = 2$ and $y = x$ is A, then $3A$ equals :

A. $6 + \frac{1}{32} - 2\sqrt{2}$

B. $2 + \frac{1}{96} - \frac{2\sqrt{2}}{3}$

C. $\frac{2\sqrt{2}}{3}$

D. 96

Answer (A)

Solution:

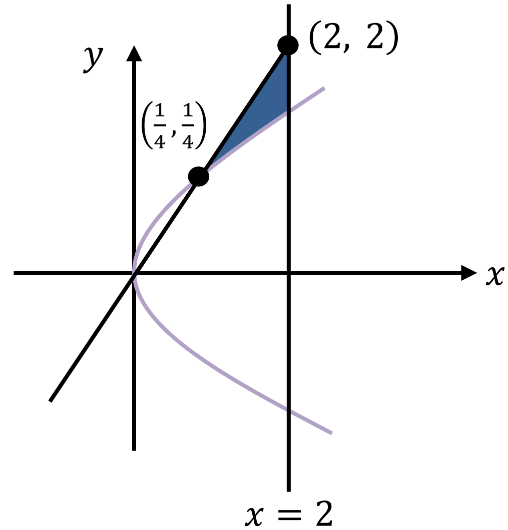
$A = \int_{\frac{1}{4}}^{2} \left(x - \frac{\sqrt{x}}{2}\right) dx$

$= \frac{x^2}{2} - \frac{x^{\frac{3}{2}}}{3}\Big|_{\frac{1}{4}}^{2}$

$= \left(2 - \frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{32} - \frac{1}{24}\right)$

$= 2 + \frac{1}{96} - \frac{2\sqrt{2}}{3}$

$\Rightarrow 3A = 6 + \frac{1}{32} - 2\sqrt{2}$ sq. units



6. A die with points $(2,\ 1,\ 0,\ -1,\ -2,\ 3)$ is thrown 5 times. The probability that the product of outcomes on all throws is positive is _______.

Answer ($\frac{521}{2592}$)

Solution:

$(2,\ 1,\ 0,\ -1,\ -2,\ 3)$

P(positive number) $= \frac{3}{6} = \frac{1}{2}$

P(negative number) $= \frac{2}{6} = \frac{1}{3}$

$E =$ product is positive

$E =$ (5 positive (or) 3 positive 2 negative (or) 1 positive 4 negative)

$P(E) = {}^5C_5 \left(\frac{1}{2}\right)^5 + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{3}\right)^2 + {}^5C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{3}\right)^4$

$= \frac{1}{32} + \frac{5}{36} + \frac{5}{81 \cdot 2} = \frac{521}{2592}$

7. Let $S\{1,\ 2,\ 3,\ 4,\ 5\}$. If $f: S \to P(S)$, where $P(S)$ is power set of S. Then number of one-one function f can be made is:

A. $(32)^5$

B. $\frac{32!}{27!}$

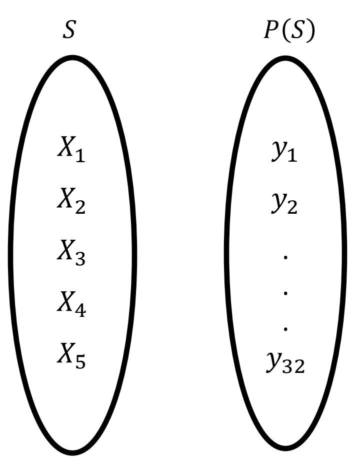
C. ${}^{32}C_{27}$

D. ${}^{32}P_{27}$

Answer (B)
Solution:

$n(S) = 5$

$n\big(P(S)\big) = 2^5 = 32$



$\therefore$ No. of one-one function

$= 32 \times 31 \times 30 \times 29 \times 28$

$= \frac{32!}{27!}$

8. A line is cutting x-axis and y-axis at two points A and B respectively, where $OA = a,\ OB = b$. A perpendicular is drawn is drawn from O (origin) to AB at an angle of $\frac{\pi}{6}$ from positive x-axis. If area of triangle $OAB = \frac{98\sqrt{3}}{3}$ sq. units, then $\sqrt{3}a + b$ is equal to:

A. 28

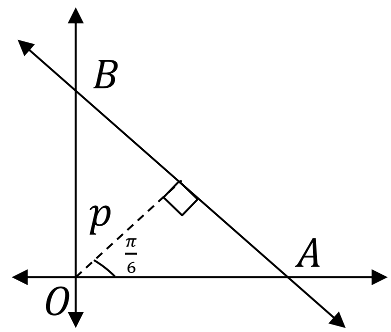
B. 14

C. 12

D. 7

Answer (A)

Solution:



Let the perpendicular distance of line from origin is p.

$\Rightarrow$ Equation of AB: $x\cos\frac{\pi}{6} + y\sin\frac{\pi}{6} = p$

$\Rightarrow \frac{x\sqrt{3}}{2} + \frac{y}{2} = p$

$\Rightarrow \frac{x}{\frac{2p}{\sqrt{3}}} + \frac{y}{2p} = 1$

Compare above equation to intercept form of line $\frac{x}{a} + \frac{y}{b} = 1$

$OA = a = \frac{2p}{\sqrt{3}},\ OB = b = 2p$

$\therefore$ Area of triangle $OAB = \frac{1}{2} \cdot \frac{2p}{\sqrt{3}} \cdot 2p = \frac{98\sqrt{3}}{3}$

$\Rightarrow p = 7$

$OA = a = \frac{14}{\sqrt{3}}$

$OB = b = 14$

$\Rightarrow \sqrt{3}a + b = 14 + 14 = 28$

9. $\frac{3(e-1)}{e}\int_1^2 x^2 e^{[x]+[x^3]}dx$ equals:

A. $e^9 - e$

B. $e^8 - 1$

C. $e^8 - e$

D. $e^9 - 1$

Answer (C)

Solution:

$I = \int_1^2 x^2 e^{[x]+[x^3]}dx = e\int_1^2 x^2 e^{[x^3]}dx$

Let $x^3 = t$

$I = e\int_1^8 \frac{dt}{3} \cdot e^{[t]} = \frac{e}{3}(e + e^2 + \cdots e^7)$

$= \frac{e^2}{3}\left(\frac{e^7-1}{e-1}\right)$

So $\frac{3(e-1)}{e} \cdot \frac{e^2}{3} \cdot \frac{e^7-1}{e-1} = e^8 - e$

10. $\vec{n}$ is a vector, $\vec{a} \neq 0$, $\vec{b} \neq 0$. If $\vec{n} \perp \vec{c}$, $\vec{a} = \alpha\vec{b} - \hat{n}$ and $\vec{b} \cdot \vec{c} = 12$, then the value of $\left|\vec{c} \times \left(\vec{a} \times \vec{b}\right)\right|$ equals:
(where $\hat{n}$ represents unit vector in direction of $\vec{n}$)

A. 144
B. $\sqrt{12}$
C. 12
D. 24

Answer (C)

Solution:

$\vec{a} = \alpha\vec{b} - \hat{n}$
$\Rightarrow \vec{a} \times \vec{b} = -\hat{n} \times \vec{b}$
$\left|\vec{c} \times \left(\vec{a} \times \vec{b}\right)\right| = \left|\vec{c} \times \left(-\hat{n} \times \vec{b}\right)\right| = \left|-\hat{n}\left(\vec{c} \cdot \vec{b}\right) - \vec{b}\left(\vec{c} \cdot (-\hat{n})\right)\right|$
$\Rightarrow \left|\vec{c} \times \left(\vec{a} \times \vec{b}\right)\right| = \left|-\hat{n}(12) - \vec{b}(0)\right| = 12$

11. $\lim\limits_{x \to 0} \frac{48\int_0^x \frac{t^3}{1+t^6}dt}{x^4}$ equals ___________.

Answer (12)

Solution:

$\lim\limits_{x \to 0} \frac{48\int_0^x \frac{t^3}{1+t^6}dt}{x^4}$
As $\frac{0}{0}$ form, applying L'hospital rule we get,
$\lim\limits_{x \to 0} \frac{48\,x^3}{(x^6+1)\cdot 4x^3} = 48 \cdot \frac{1}{4} = 12$

12. If $a_n = \frac{-2}{4n^2-16n+15}$, and $a_1 + a_2 + \cdots + a_{25} = \frac{m}{n}$ where m and n are coprime, then the value of $m + n$ is ______.

Answer (191)

Solution:

We have,
$a_n = \frac{-2}{4n^2-16n+15}$
$= \frac{-2}{(2n-3)(2n-5)}$
$= \frac{1}{(2n-3)} - \frac{1}{(2n-5)}$
$\Rightarrow a_1 + a_2 + \cdots + a_{25} = \left(\frac{1}{-1} - \frac{1}{-3}\right) + \cdots + \left(\frac{1}{47} - \frac{1}{45}\right)$
$= \frac{1}{47} + \frac{1}{3}$
$= \frac{50}{141} = \frac{m}{n}$
$\Rightarrow m + n = 191$

13. If $z = 1 + i$ and $z_1 = \frac{i+\bar{z}(1-i)}{\bar{z}(1-z)}$, then the value of $\frac{12}{\pi}\arg(z_1)$ is ______.

Answer (3)

Solution:

$z_1 = \frac{i+\bar{z}(1-i)}{\bar{z}(1-z)}$
$= \frac{i+(1-i)(1-i)}{(1-i)(-i)}$

$= \frac{i-2i}{(1-i)(-i)}$
$= \frac{1}{1-i}$
$\arg(z_1) = \arg\left(\frac{1}{1-i}\right) = \arg\left(\frac{1+i}{(1-i)(1+i)}\right) = \arg\left(\frac{1}{2} + \frac{i}{2}\right) = \frac{\pi}{4}$
$\frac{12}{\pi}\arg(z_1) = \frac{12}{\pi} \times \frac{\pi}{4} = 3$

14. Mean and variance of 7 observations are 8 & 16 respectively. If number 14 is omitted, then a & b are new mean and variance. The value of $a + b$ is _______.

Answer (19)

Solution:

Let $x_1,\ x_2,\ \ldots,\ x_7$ be the 7 observations
New mean $(a)\ = \frac{8\times7-14}{6} = 7$
$\frac{\sum_{i=1}^{7} x_i^2}{7} - 64 = 16$
$\Rightarrow\ \sum x_i^2 = 560$
$\sum {x_i^2}_{new} = 560 - 14^2$
$\therefore b = \frac{364}{6} - 7^2 = \frac{70}{6} = \frac{35}{3}$
$a = 7; b = \frac{35}{3}$
$a + b = 7 + \frac{35}{3} = \frac{56}{3} = 18.67 \approx 19$ (Rounding off gives 19)

15. If coefficient of x^{15} in expansion of $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ is equal to coefficient of x^{-15} in expansion of $\left(ax^{\frac{1}{3}} + \frac{1}{bx^3}\right)^{15}$, then $|ab - 5|$ is equal to __________.

Answer (4)

Solution:

For expansion of $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$
$T_{r+1} = {}^{15}C_r \times a^{15-r} \times (x^3)^{15-r} \times b^{-r} \times x^{\frac{-r}{3}}$
Here we need coefficient of x^{15}
$\Rightarrow 45 - 3r - \frac{r}{3} = 15$
$\Rightarrow \frac{10r}{3} = 30$
$\Rightarrow r = 9$
$\therefore$ Coefficient of x^{15} in $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15} = {}^{15}C_9 \times a^6 \times b^{-9}$

For expansion of $\left(ax^{\frac{1}{3}} + \frac{1}{bx^3}\right)^{15}$
$T_{r+1} = {}^{15}C_r \times a^{15-r} \times (x)^{\frac{15-r}{3}} \times b^{-r} \times x^{-3r}$
Here we need coefficient of x^{-15}
$\Rightarrow \frac{15-r}{3} - 3r = -15$
$\Rightarrow 15 - r - 9r = -45$
$\Rightarrow r = 6$
$\therefore$ Coefficient of x^{-15} in $\left(ax^{\frac{1}{3}} + \frac{1}{bx^3}\right)^{15} = {}^{15}C_6 \times a^9 \times b^{-6}$
Coefficient of x^{15} in $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ = Coefficient of x^{-15} in $\left(ax^{\frac{1}{3}} + \frac{1}{bx^3}\right)^{15}$ $\cdots$ (given)
$\Rightarrow a^{-3}b^{-3} = 1$
$\Rightarrow ab = 1$
$\Rightarrow |ab - 5| = |1 - 5| = 4$

16. Using $1,2,3$ and 5, four digit numbers are formed, where repetition is allowed. The number of numbers divisible by 15 are _______.

Answer (21)

Solution:

Unit digit will be 5
$\underline{a}\ \ \underline{b}\ \ \underline{c}\ \ \underline{5}$
$a+b+c=(3\lambda+1)$ type
For (a,b,c) possibilities are
$(2,2,3)\ (1,1,5)\ (1,1,2)\ (3,3,1)\ (5,5,3)\ (2,3,5)$
For $(2,2,3) \Rightarrow \frac{3!}{2!}=3$
For $(1,1,5) \Rightarrow \frac{3!}{2!}=3$
For $(1,1,2) \Rightarrow \frac{3!}{2!}=3$
For $(3,3,1) \Rightarrow \frac{3!}{2!}=3$
For $(5,5,3) \Rightarrow \frac{3!}{2!}=3$
For $(2,3,5) \Rightarrow \frac{3!}{2!}=3$
Total $=21$

17. If $5f(x+y)=f(x)\cdot f(y)$ and $f(3)=320$, then the value of $f(1)$ is ______.

Answer (20)

Solution:

$5f(x+y)=f(x)\cdot f(y)\ \cdots(1)$
Put $x=1,\ y=2$ in (1)
$5f(3)=f(1)\cdot f(2)$
$\Rightarrow f(1)\cdot f(2)=5\times 320=1600\ \cdots(2)$
Put $x=y=1$ in (1)
$f(2)=\frac{(f(1))^2}{5}\ \cdots(3)$
$f(3)=320$
Using (2) and (3),
$f(1)\cdot\frac{(f(1))^2}{5}=1600$
$(f(1))^3=8000$
$f(1)=20$

18. If for $\log_{\cos x}(\cot x)-4\log_{\sin x}(\cot x)=1,\ x=\sin^{-1}\left(\frac{\alpha+\sqrt{\beta}}{2}\right)$. Then the value of $(\alpha+\beta)$ is __________.
(given $x\in\left(0,\frac{\pi}{2}\right)$)

Answer (4)

Solution:

$\log_{\cos x}(\cot x)-4\log_{\sin x}(\cot x)=1$
$\Rightarrow 1-\log_{\cos x}(\sin x)-4(\log_{\sin x}(\cos x)-1)=1$
Let $\log_{\cos x}(\sin x)=t$
$\Rightarrow -t-4\left(\frac{1}{t}-1\right)=0$
$\Rightarrow t+\frac{4}{t}=4$
$\Rightarrow t=2$
$\therefore t=\log_{\cos x}(\sin x)=2$
$\Rightarrow \cos^2 x=sin\,x$
$\Rightarrow 1-\sin^2 x=\sin x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$
$\Rightarrow \sin x = \frac{-1 \pm \sqrt{5}}{2}$
Comparing, $\alpha = -1,\ \beta = 5$
$\Rightarrow \alpha + \beta = 4$