JEE Main 2023 (Memory based)

1st February 2023 - Shift 1

Answer & Solutions

PHYSICS

1. Statement 1: Value of acceleration due to gravity is same at all the points inside earth assuming it to be made up of uniform density.
Statement 2: Value of gravitational field increases as we go towards centre in a uniform spherical shell.

A. Both statement 1 and statement 2 are true.
B. Statement 1 is true but statement 2 is false.
C. Statement 1 is false but statement 2 is true.
D. Both statement 1 and statement 2 are false.

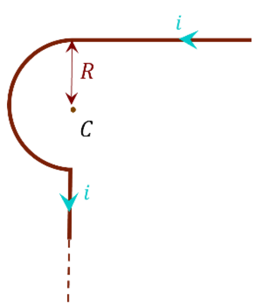
Answer (D)

Solution:

Value of acceleration due to gravity decreases as we go inside the earth.
Value of gravitational field does not change as we go towards centre in a uniform spherical shell.

2. An infinite wire is bent in the shape as shown. Find the magnetic field at point C.

A. $\frac{\mu_0 i}{4\pi r}(1+\pi)$
B. $\frac{\mu_0 i}{4\pi r}(2+\pi)$
C. $\frac{\mu_0 i}{2\pi r}(1+\pi)$
D. $\frac{\mu_0 i}{4r}$



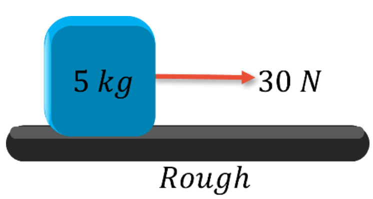
Answer (A)

Solution:

$$B_C = \frac{\mu_0 i}{4\pi R}[\sin 90° + \sin 0°] + \frac{\mu_0 i}{4R} + 0$$
$$= \frac{\mu_0 i}{4\pi R}[1+\pi]$$

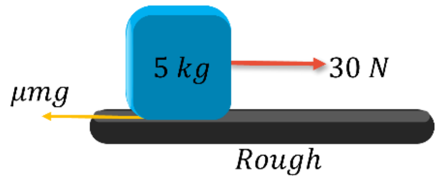
3. A force of $30\ N$ is applied on a block of mass $5\ kg$. the block travels a distance of $50\ m$ in $10\ sec$ starting from rest. Find the coefficient of friction.

A. 0.5
B. 0.7
C. 0.3
D. 0.8



Answer (A)

Solution:



Applying Newtons' second law,

$30 - \mu mg = ma$

$\Rightarrow a = \left(\frac{30 - 50\mu}{5}\right)$

As acceleration is uniform and block start from rest,

$S = \frac{1}{2}at^2$

$\Rightarrow 50 = \frac{1}{2}\left(\frac{30 - 50\mu}{5}\right)10^2$

$\Rightarrow 5 = 30 - 50\mu$

$\Rightarrow \mu = \frac{25}{50} = 0.5$

4. Which of the following is not the frequency of frequency modulated (FM) signal?

A. $90\ MHz$
B. $89\ MHz$
C. $106\ MHz$
D. $100\ kHz$

Answer (D)

Solution:

Frequency of FM signal is in MHz.

5. For a real gas the equation of gas is given by $\left(P + \frac{an^2}{V^2}\right)(V - bn) = nRT$. If symbols have their usual meaning, then the dimensions of $\frac{V^2}{an^2}$ is same as that of

A. Compressibility
B. Bulk modulus
C. Viscosity
D. Energy Density

Answer (A)

Solution:

$[P] = \left[\frac{an^2}{V^2}\right] = dimension\ of\ bulk\ modulus$

$So, \left[\frac{an^2}{V^2}\right] has\ dimension\ of\ compressibility.$

6. A stone is thrown vertically up with speed v_o from a cliff of height H. Find the average speed of the ball till the moment it reaches ground. Given that $H = 100\ m,\ v_o = 10\ m/s,\ g = 10\ m/s^2$.

A. $\frac{64}{1+\sqrt{21}}\ m/s$

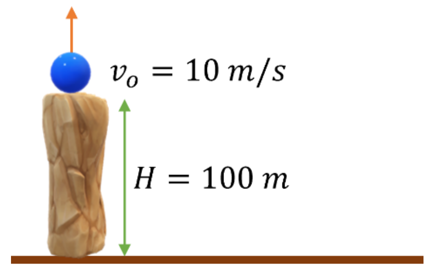
B. $55\ m/s$

C. $110(1 + \sqrt{21})\ m/s$

D. $\frac{110}{1+\sqrt{21}}\ m/s$

Answer (D)

Solution:



Total distance $= \frac{v_o^2}{2g} \times 2 + 100 = 110\ m$

Total time $= t_0$

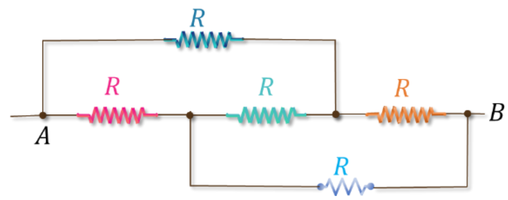
$S = ut_0 + \frac{1}{2}at_0^2$

$\Rightarrow -100 = 10\ t_o - \frac{1}{2} \times 10 \times t_o^2$

$\Rightarrow t_o = 1 + \sqrt{21}\ s$

$\Rightarrow$ Average speed $= \dfrac{110}{1+\sqrt{21}}\ m/s$

7. In the circuit shown find the equivalent resistance between terminals A and B.

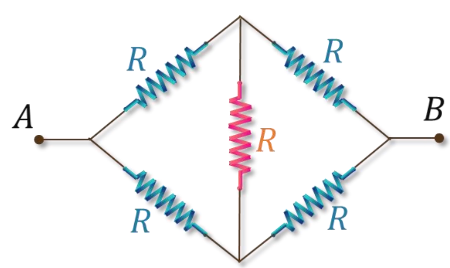


A. $3R/2$
B. $2R$
C. $4R$
D. R

Answer (D)

Solution:

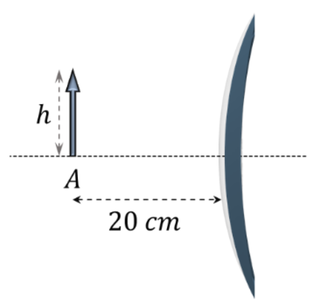
Redrawing the structure, we will get the circuit as shown here:



It is a balanced Wheatstone bridge.

The equivalent resistance of circuit: $R_{eq} = R$

8. An object of height h is placed in front of a convex mirror (radius of curvature $= 20\ cm$). Find the height of image.



A. $h/2$
B. $h/3$
C. $h/6$
D. $h/4$

Answer (B)

Solution:

From mirror formula:

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} + \frac{1}{-20} = \frac{1}{10}$

$$\Rightarrow \frac{1}{v} = \frac{3}{20} \Rightarrow v = \frac{20}{3}$$

Magnification of mirror:

$$m = -\frac{v}{u} = \frac{1}{3} = \frac{h_i}{h}$$

$$h_i = \frac{h}{3}$$

9. A uniform solid cylinder of radius R, is released from a $600\,m$ long ramp, inclined at $30°$ from the horizontal. Find the time taken to reach the bottom of the ramp. (Consider sufficient friction for pure rolling)

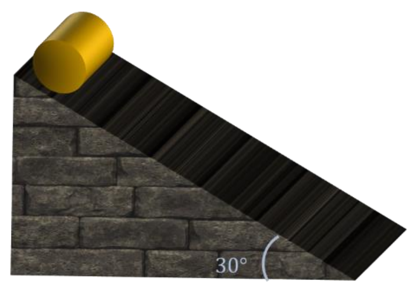
A. $60\,sec$
B. $6\sqrt{10}\,sec$
C. $3\sqrt{10}\,sec$
D. $20\,sec$

Answer (B)

Solution:

$$mg\sin\theta - f_r = ma$$

Also,



$$\frac{3}{2}mR^2\alpha = mg\sin\theta \times R$$

$$\Rightarrow \frac{3}{2}ma = mg\sin\theta$$

$$a = \frac{2}{3}g\sin 30° = \frac{g}{3} = \frac{10}{3}\ m/s^2$$

Ramp length, $s = 600\,m$

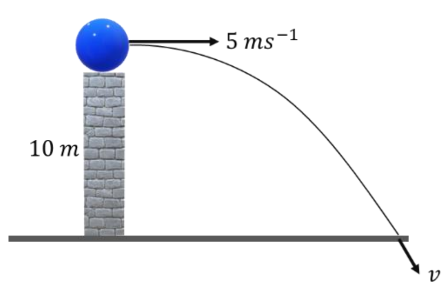
$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 600 \times 3}{10}} = 6\sqrt{10}\ seconds$$

10. A ball is thrown horizontally from height of $10\,m$ with a speed of $5\,ms^{-1}$ as shown. Find the speed with which it strikes the ground.

A. $15\,m/s$
B. $5\,m/s$
C. $10\,m/s$
D. $20\,m/s$

Answer (A)

Solution:



$$v^2 = u^2 + 2gh$$
$$v^2 = 25 + 2 \times 10 \times 10$$
$$v = 15\,m/s$$

11. An ideal gas ($adiabatic\ constant\ = 3/2$) undergoes an adiabatic expansion process where change in temperature is $-T$. If there are $2\,moles$ of the gas, find the work done by the gas.

A. $3RT$

B. $2RT$
C. $4RT$
D. $-RT$

Answer (C)

Solution:

Work done for adiabatic expansion can be given as:

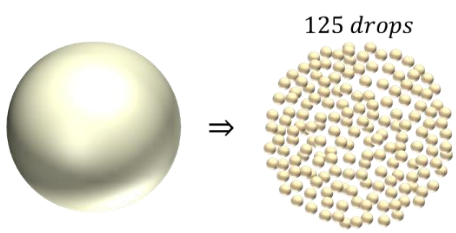
$$W = \frac{nR\Delta T}{1-\gamma} = \frac{2 \times R(-T)}{1-3/2} = 4RT$$

12. A drop of $Mercury$ is divided into 125 drops of equal radius $10^{-3}\ m$ each. If surface tension of $Mercury$ is equal to $0.45\ Nm^{-1}$. Magnitude of change in surface energy is equal to nearly:

A. $1.14 \times 10^{-4}\ J$
B. $7.06 \times 10^{-4}\ J$
C. $8.47 \times 10^{-4}\ J$
D. $5.65 \times 10^{-4}\ J$

Answer (D)

Solution:



Let radius of bigger drop was R So,

$$\frac{4}{3}\pi R^3 = 125 \times \frac{4}{3}\pi(10^{-3})^3$$
$$R = 5 \times 10^{-3}\ m$$
$$U_i = 4\pi R^2\sigma = 4\pi(5 \times 10^{-3})^2 \times 0.45 = 1.41 \times 10^{-4}\ J$$
$$U_f = 125 \times 4\pi r^2\sigma = 500 \times \pi(10^{-3})^2 \times 0.45 = 7.06 \times 10^{-4}\ J$$

So,

$$\Delta U = U_f - U_i = 5.65 \times 10^{-4}\ J$$

13. A charged particle with charge $2 \times 10^{-6}\ C$, at rest, is first accelerated through a potential difference of $100\ V$ and then it is subjected to a transverse magnetic field of $4mT$. In region of magnetic field it undergoes a circular path of radius $3\ cm$. Mass of the particle is equal to

A. $1.44 \times 10^{-16}\ kg$
B. $7.2 \times 10^{-16}\ kg$
C. $1.44 \times 10^{-10}\ kg$
D. $7.2 \times 10^{-10}\ kg$

Answer (A)

Solution:

Radius of circular path can be given as:

$$R = \frac{\sqrt{2mqV}}{qB}$$

$$3 \times 10^{-2} = \frac{\sqrt{2m \times 100}}{\sqrt{2 \times 10^{-6}} \times 4 \times 10^{-3}} \Rightarrow m = 1.44 \times 10^{-16}\, kg$$

14. A string of mass per unit length equal to $7 \times 10^{-3}\, kg/m$ is subjected to a tension equal to $70\, N$. The speed of transverse wave on this string is equal to

A. $10\, m/s$
B. $50\, m/s$
C. $100\, m/s$
D. $200\, m/s$

Answer (C)

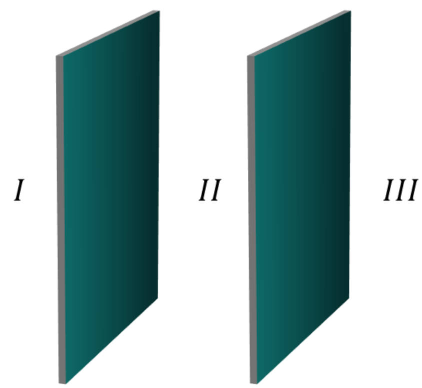
Solution:

Velocity of transverse wave can be given as:

$$v = \sqrt{\frac{T}{\mu}} == \sqrt{\frac{70}{7 \times 10^{-3}}} = 100\, m/s$$

15. Two thin insulating sheets (each having charge density $+\sigma$) are arranged as shown. Then find the net electric field magnitude in the 3 regions:

A. $E_1 = \frac{\sigma}{\epsilon_0}; E_2 = 0; E_3 = \frac{\sigma}{\epsilon_0}$
B. $E_1 = E_2 = E_3 = 0$
C. $E_1 = 0; E_2 = \frac{\sigma}{2\epsilon_0}; E_3 = \frac{\sigma}{\epsilon_0}$
D. $E_1 = \frac{\sigma}{\epsilon_0}; E_2 = 0; E_3 = \frac{\sigma}{2\epsilon_0}$



Answer (A)

Solution:

Electric field in different zones can be written as:

$$E_{I(1)} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$
$$E_{II(2)} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$
$$E_{III(3)} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

16. In a series LCR circuit connected across $220\, V$, $50\, Hz$ AC supply. If the inductive reactance of the circuit is $79.6\, \Omega$. If the power delivered in the circuit is maximum, the capacitance of the circuit is $x\, \mu F$. Find x.

Answer (40)

Solution:

For maximum power, LCR should be in resonance condition,
$X_L = X_C$

$$\Rightarrow 79.6 = \frac{1}{\omega c} = \frac{1}{2\pi f c} = \frac{1}{2\pi \times 50 \times c}$$
$$\Rightarrow c = \frac{1}{79.6 \times 100\pi} = 40 \times 10^{-6}\, F = 40\, \mu F$$

17. An $alpha$ particle and a $proton$ having same $de-Broglie$ wavelengths will have $kinetic\ energies$ in the ratio ____________.

Answer (0.25)

Solution:

$\text{charge on } \alpha \text{ particle} = 2\text{e}$
$\text{mass of proton} = \text{m}$
$\text{mass of} \alpha \text{ particle} = 4\text{m}$

$$\frac{\lambda_P}{\lambda_\alpha} = \frac{(P_\alpha)}{(P_P)} = \frac{\sqrt{2K_\alpha m_\alpha}}{\sqrt{2K_P m_P}} = 1$$
$$\frac{K_\alpha}{\text{K}_\text{P}} \times \left(\frac{m_\alpha}{m_P}\right) = 1$$
$$\frac{K_\alpha}{\text{K}_\text{P}} \times (4) = 1$$
$$\frac{K_\alpha}{\text{K}_\text{P}} = \frac{1}{4} = 0.25$$

18. If mass of a planet is 9 times that of the earth and radius is 2 times that of the earth, then escape speed from this planet is $\frac{xv_e}{\sqrt{2}}$. Find x.
(v_e is escape speed from the Earth.)

Answer (3)

Solution:

$$\text{Escape speed from earth, } \text{v}_\text{e} = \sqrt{\frac{2\text{GM}_\text{e}}{\text{R}_\text{e}}}$$
$$\text{Escape speed from planet, } \text{v}'_\text{e} = \sqrt{\frac{2\text{GM}'}{\text{R}'}} = \sqrt{\frac{2\text{G} \times 9\text{M}_\text{e}}{2\text{R}_\text{e}}} = \text{v}_\text{e} \times \frac{3}{\sqrt{2}}$$

19. There are n number of polarizers arranged one after the other. Each polarizer pass axis is inclined at 45^0 with respect to the previous polarizer. Unpolarized light of intensity I_0 is incident on this setup. Final transmitted light has intensity $\frac{I_0}{64}$. Find n

Answer (6)

Solution:

Intensity of light passing through 1^{st} polarizer will be $I_0/2$
Intensity of light passing through 2^{nd} polarizer will be $\frac{I_0}{2} \times \cos^2 45°$
Intensity of light passing through 3^{rd} polarizer will be $\frac{I_0}{2} \times (\cos^2 45°)^2$
Similarly, for n polarizers:
$$I = \frac{I_0}{2} \times \cos^2 45° \times \cos^2 45° \times \ldots\ldots\ldots\ldots \quad (\text{upto } n-1 \text{ times})$$

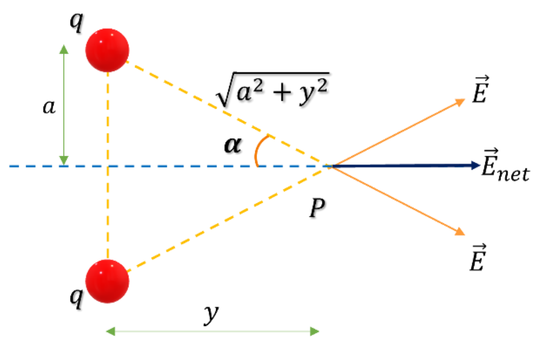
$$\Rightarrow \frac{I_0}{64} = \frac{I_0}{2} \times \left(\frac{1}{2}\right)^{n-1}$$
$$\Rightarrow n - 1 = 5 \; or \; n = 6$$

20. Two-point charges each of magnitude q is kept at a separation of $2a$. The distance from mid point on perpendicular bisector where a point charge will experience maximum force is $\frac{a}{\sqrt{x}}$. Find the value of x.

Answer (2)

Solution:

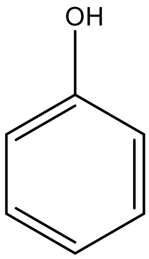
$$E \text{ due to one charge} = \frac{kq}{a^2 + y^2}$$
$$E_{net} \text{ at point } P = 2E\cos\alpha$$
$$= \frac{2Kq}{a^2 + y^2} \times \frac{y}{(a^2 + y^2)^{\frac{1}{2}}}$$
$$= \frac{2Kqy}{(a^2 + y^2)^{\frac{3}{2}}}$$

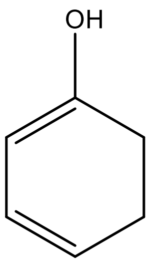


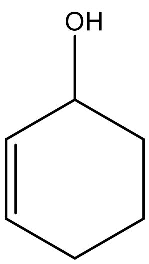
$$Force = qE_{net}$$

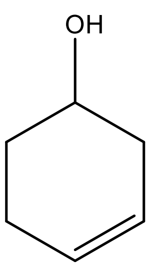
$$\frac{dF}{dy} = 0, \; for \; maximum \; force$$
$$\text{On solving,} \frac{dF}{dy} = 0$$
$$\Rightarrow y = \left(\frac{a}{\sqrt{2}}\right)$$
$$\text{So,} x = 2$$

1. Which one of the following compounds shows fastest rate of dehydration?

A. 

B. 

C. 

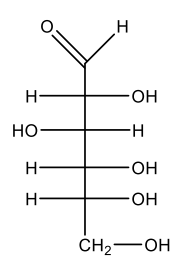
D. 

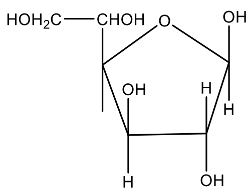
Answer (C)

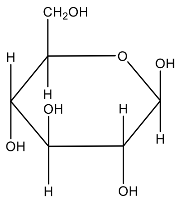
Solution:

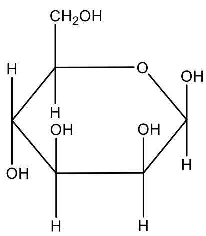
The alcohol which produces stable carbocation shows faster rate of dehydration.

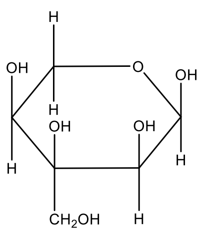
2. Pyranose form of the given compound is



A. 

B. 

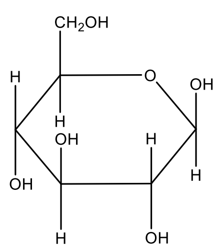
C. 

D. 

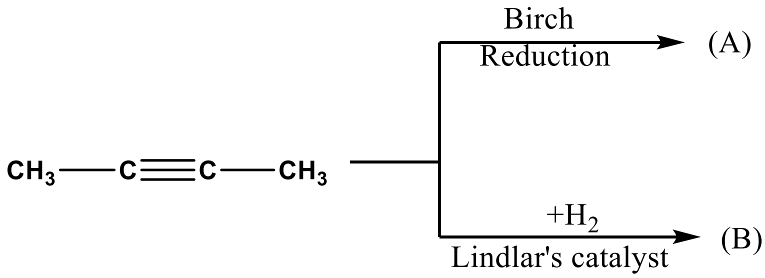
Answer (B)

Solution:

The correct pyranose form is



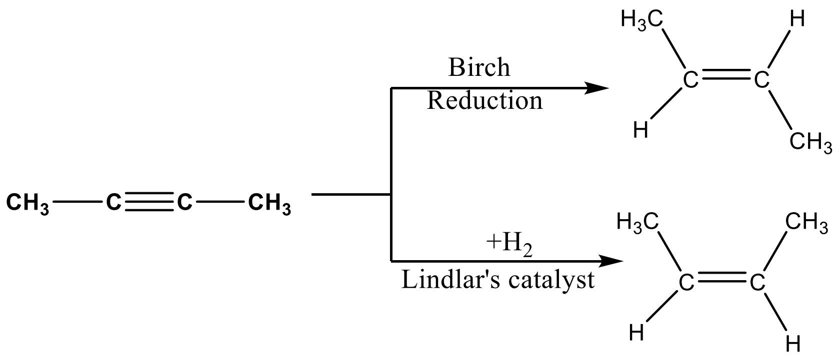
3. Identify the correct statement on physical properties of (A) and (B)



A. Melting Point: A >B; Boiling Point: A>B
B. Melting Point: A <B; Boiling Point: A>B
C. Melting Point: A >B; Boiling Point: A<B
D. Melting Point: A <B; Boiling Point: A<B

Answer (C)

Solution:



Compound (A) is non polar, whereas compound (B) is polar. Compound (A) will have lower boiling point than (B) due to dipole – dipole interaction in (B)

Compound (A) is symmetrical with respect to compound (B), thus the packing will be better in (A). Hence, (A) will have higher melting point than B.

4. Choose correct statement.

Consider the following statements.

A: Beryllium oxide is an acidic oxide

B: Beryllium sulphate is soluble in aqueous medium

C: Beryllium carbonate is thermally stable

D: Beryllium shows anomalous behavior in comparison to another Group 2 elements

A. A&B
B. B&C
C. B&D
D. C&D

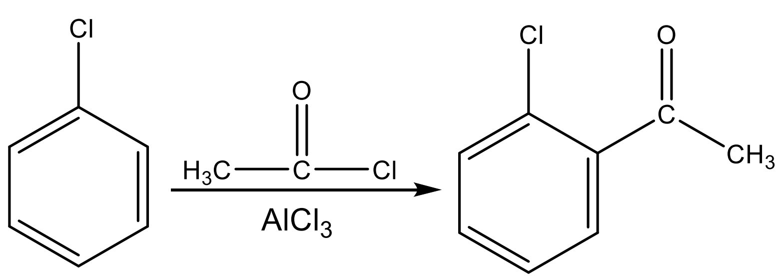
Answer (C)

Solution:

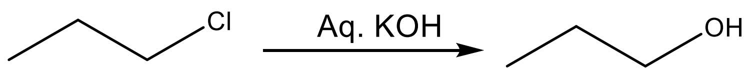
BeO is amphoteric oxide and $BeSO_4$ is soluble in water. $BeCO_3$ is thermally unstable.

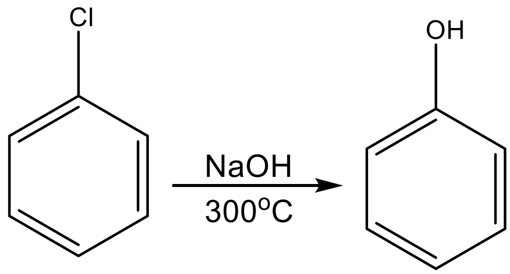
$BeCO_3 \xrightarrow{\Delta} BeO + CO_2$

5. In which of the following option the reaction does not matches with their correct product?



A.

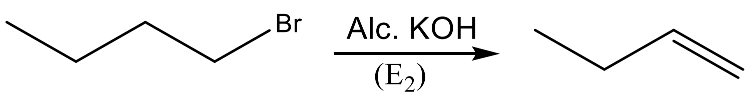
B. 

C. 

D. 

Answer (D)

Solution:



With alc. KOH alkyl halides shows elimination reaction

6. Statement 1: Chlorine easily forms oxides and the compounds are explosive.

Statement 2: The higher oxidation states of chlorine, bromine and iodine are released when halogens form oxides and fluorides.

A. Statement 1 is correct Statement 2 is incorrect
B. Statement 1 is incorrect Statement 2 is correct
C. Statement 1 is correct Statement 2 is correct
D. Statement 1 is incorrect Statement 2 is incorrect

Answer (C)

Solution:

The oxides of chlorine are easily formed, and the compounds are explosive. The higher oxidation states of chlorine, bromine and iodine are stable in their oxides and fluorides. Therefore, both the statement are correct.

7. For given reaction in acidic medium,

$5e^- + 8H^+ + MnO_4^- \rightarrow Mn^{2+} + 4H_2O$

$0.001\ M \quad 0.1\ M\ ?$

Given: $E^o_{cell} = 1.54\ V, E_{cell} = 1.2832\ V$

Find out p^H of the solution

A. 2.46
B. 3.52
C. 6.38
D. 1.02

Answer (A)

Solution:

$$1.2832 = 1.54 - \frac{0.0591}{5}\log\frac{10^{-1}}{(10^{-3})(H^+)^8}$$

$$-0.2568 = -\frac{0.0591}{5}(\log 10^2 - 8\log H^+)$$

$$21.72 = 2 + 8\, p^H$$

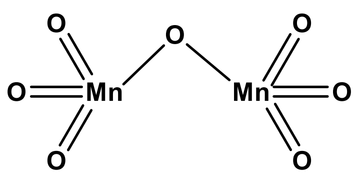
$$19.72 = 8p^H$$

$$p^H = \frac{19.72}{8} = 2.46$$

8. Consider the structure of Mn_2O_7.
X: No.of Mn-Mn bonds
Y: No. of Mn- O- Mn bonds
Find out (X+Y)

Answer (1)

Solution:



X= 0 and Y = 1

X+Y = 1

9. X: Oxidation number of Br in bromic acid
Y: Oxidation number of Br in perbromic acid

Find out (X+Y)

Answer (12)

Solution:

Bromic Acid ($HBrO_3$)

Oxidation number of Br = +5 = X

Perbromic acid ($HBrO_4$)

Oxidation number of Br = +7 = Y

X+Y = 12

10. Consider the following first order reaction

$A \rightarrow C; \frac{t_1}{2}$ = 15min, $B \rightarrow D; \frac{t_1}{2}$ = 5min

The initial concentrations of A and B are 1 molar and 8 molar respectively. The time when the concentration of A and B becomes equal is 'X' minutes. Find 2X (to the nearest integer)

Answer (45)

Solution:

$$\frac{1}{K_1} ln \frac{1}{[A]} = \frac{1}{K_2} ln \frac{8}{[A]}$$

$A = \frac{1}{\sqrt{8}}$

$\frac{t_1}{2} = \frac{15}{0.693}\ ln\sqrt{8}$

= 22.5 min

2x = 45

11. H- atom in ground state absorbs 12.75 eV of energy. The orbital angular momentum of the electron becomes $nh/2\pi$, the value of n is

Answer (4)

Solution:

$\Delta \boldsymbol{E} = \mathbf{13.6}\left(\mathbf{1} - \frac{\mathbf{1}}{\boldsymbol{n}^{\mathbf{2}}}\right) = \mathbf{12.75}$

$\Rightarrow\ -\frac{1}{n^2} = \frac{12.75}{13.60} - 1$

$n^2 = 16\ \Rightarrow n = 4$

12. Find out ΔT_f of a KCl solution (i = 2), if 25 mL of this KCl solution requires 20 mL of 1 M $AgNO_3$ solution for complete precipitation of KCl solution.

$K_f =\ 1.86\ K.kg\ mol^{-1}$. Assume molarity = molality) (Round off to nearest integer)

Answer (3)

Solution:

$\Delta T_f = i\ K_f m$

$\Delta T_f = 2 \times 1.86 \times m$ – (Eq – 1)

Let's find m

$KCl + AgNO_3 \rightarrow KNO_3 + AgCl \downarrow$

From law of chemical equivalance

millequivalents of KCl reacted = milliequivalents of $AgNO_3$ reacted

$(M \times nf \times V)_{KCl} = (M \times nf \times V)_{AgNO_3}$

$M \times 1 \times 25 = 1 \times 1 \times 20$

$M = \frac{20}{25} = \frac{4}{5}$

Given, molarity = molality

Therefore, molality = $\frac{4}{5}m$

Putting value of molality in (Eq – 1)

$\Delta T_f = 2 \times 1.86 \times \frac{4}{5} \sim 3$

13. Which of the following complexes have maximum splitting?

A. $[Fe(CN)_6]^{4-}$

B. $[Fe(NH_3)_6]^{2+}$

C. $[FeCl_6]^{4-}$

D. $[Fe(ox)_3]^{4-}$

Answer (A)

Solution:

All the above complexes shows octahedral crystal field splitting
And Octahedral crystal field splitting is dependant on the nature of the ligand
i.e, in case of strong field the splitting will be more when compared to a weak field ligand.
As per the electrochemical series CN^- is a strong field ligand and stronger than $NH_3, ox^{2-} and\ Cl^-$.
Hence maximum splitting is seen in case of $[Fe(CN)_6]^{4-}$

14. Average kinetic energy of an ideal gas depends on-

A. Nature of the gas
B. Pressure of the gas
C. Temperature of the gas
D. Volume of the gas

Answer (C)

Solution:

Average kinetic energy of an ideal gas per mole is given by

$$Average\ K.E = \frac{3}{2}RT$$

Hence, average K.E depends on temperature.

15. Assertion: Hydrogen is an environment/eco-friendly fuel.
Reason: Hydrogen is the lightest element.

A. Both Assertion and reason are true, and reason is the correct explanation of assertion
B. Both Assertion and reason are true, but reason is not the correct explanation of assertion
C. Assertion is true but reason is false
D. Assertion is false but reason is true

Answer (B)

Solution:

The correct option is option(B).

16. $X(g) \rightleftharpoons 2Y(g)$ K_{P1} ----(i)
$A(g) \rightleftharpoons B(g) + C(g)$ K_{P2} ----(ii)
If degree of dissociation is same for both the reactions. Find out the ratio of total pressure P_1 & P_2 respectively.

A. $\frac{K_{P_1}}{K_{P_2}}$
B. $\frac{4K_{P_1}}{K_{P_2}}$
C. $\frac{K_{P_1}}{4K_{P_2}}$
D. $\frac{K_{P_1}}{2K_{P_2}}$

Answer (C)

Solution:

X(g) ⇌ 2Y(g)

$$K_{P_1} = \frac{4\alpha^2}{(1+\alpha)} \times \frac{P_1}{1+\alpha}$$

A(g) ⇌ B(g) + C(g)

$$K_{P_2} = \frac{\alpha^2}{(1-\alpha)} \times \frac{P_2}{1-\alpha}$$

$$\frac{K_{P_1}}{K_{P_2}} = \frac{4\alpha^2 \times P1(1+\alpha)(1-\alpha)}{(1+\alpha)(1-\alpha)\times\alpha^2\times P_2}$$

$$\frac{P_1}{P_2} = \frac{K_{P_1}}{4KP_2}$$

17. Which of the following is not correctly matched.

List - I	List - II
A. Antibiotic	Penicillin
B. Antiseptic	Chloroxylenol
C. Tranquilizer	Erythromycin
D. Analgesic	Aspirin

A. A
B. B
C. C
D. D

Answer (C)

Solution:

Erythromycin is an antibiotic. Hence, the correct answer is option(C).

18. Which of the following option contains the correct match

List - I	List - II
A. Caustic soda	P. $CaSO_4$
B. Washing soda	Q. $Na_2CO_3.10H_2O$
C. Dead burnt plaster	R. $Ca(OH)_2$
D. Slaked lime	S. NaOH

A. A – S, B – Q, C – P, D – R
B. A – P, B – Q, C – R, D – S
C. A – S, B – P, C – Q, D – R
D. A – R, B – S, C – Q, D – P

Answer (A)

Solution:

Caustic soda – NaOH

Washing soda – $Na_2CO_3.10H_2O$

Dead burnt plaster – $CaSO_4$

Slaked lime – $Ca(OH)_2$

19. How photochemical smog can be controlled in automobiles?

A. Using catalytic convertors which will increase release of nitrogen oxide
B. Using catalytic convertors which will decrease the release of nitrogen oxide
C. By increasing sulphur in fuel content
D. By decreasing sulphur in fuel content

Answer (B)

Solution:

Catalytic convertors will prevent the release of nitrogen oxide and hydrocarbons to the atmosphere.

20. When $K_4[Fe(CN)_6]$ is added to $FeCl_3$, the Prussian blue complex compound formed is:

A. $Fe_3[Fe(CN)_6]_4$
B. $Fe_4[Fe(CN)_6]_3$
C. $K_2Fe[Fe(CN)_6]$
D. $K_2Fe_3[Fe(CN)_6]_2$

Answer (B)

Solution:

Prussian Blue is $Fe_4[Fe(CN)_6]_3$

21. Match the tests given in column-I with the compounds given in column-II

Column - I	Column - II
A. Schiff's test	1. Carbohydrate
B. Carbylamine test	2. Peptide
C. Molisch test	3. Aldehyde
D. Biuret test	4. 1° Amine

A. A – 1, B – 2, C – 4, D – 3
B. A – 2, B – 4, C – 3, D – 1
C. A – 4, B – 3, C – 2, D – 1
D. A – 3, B – 4, C – 1, D – 2

Answer (D)

Solution:

Schiff's test is given by aldehydes. Carbylamine test is given by primary amines only. Molisch test is given by carbohydrates. Biuret test is given by peptide.

22. Electrons are emitted in cathode ray tube with a velocity of 1000 m/s. Select the correct statement among the following.

A. The de Broglie wavelength of e^- is 666.67 nm
B. The cathode rays travel from cathode to anode
C. The characteristics of e^- depends on the metal used in cathode
D. The characteristics of e^- depends on the gas filled inside the cathode tube

Answer (B)

Solution:

$$\Lambda = \frac{h}{mv} = \frac{6.6\times10^{-34}\ J.s}{9.1\times10^{-31}kg\times10^{3}}$$
$$= 725 \times 10^{-9} m$$
$$= 725\ nm$$

Cathode rays travels from cathode to anode. Hence, the correct answer is option (B).

23. The density of a 3M NaCl solution is 1 g/mL. The molality of the solution is ‘x’. Then find 2x. (Round off to nearest integer)

Answer (7)

Solution:

$$\boldsymbol{m} = \frac{\boldsymbol{M\times1000}}{\boldsymbol{1000\times d_{solution} - M\times MM_{solute}}}$$

m = molality

M = Molarity

d = density

MM_{solute} = Molar mass of solute

$$Molality = X = \frac{3}{824.5}\times 1000$$
$$= 3.63\ molal$$

Hence, 2x ≈ 7

1. $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}\right)$ equals :

A. $\ln 2$
B. $\ln\frac{3}{2}$
C. $\ln\frac{2}{3}$
D. 0

Answer (A)

Solution:

$\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}\right)$

$=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{n+r}\right)$

$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{1}{1+\frac{r}{n}}\right)$

$0<\lim_{n\to\infty}\frac{r}{n}<1$

$=\int_0^1\frac{dx}{1+x}$

$=\ln(1+x)|_0^1=\ln\ 2$

2. For solution of $\frac{dy}{dx}+y\tan x=\sec x$, $y(0)=1$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

A. $\frac{\sqrt{3}}{2}$

B. $\frac{1+\sqrt{3}}{2}$

C. $\frac{1}{2}$

D. $-\frac{\sqrt{3}}{2}$

Answer (B)

Solution:

$\frac{dy}{dx}+y\tan x=\sec x$

I. F $=e^{\int\tan x dx}=\sec x$

Solution of equation is

$y\cdot\sec x=\int\sec x\cdot\sec x$

$\Rightarrow y\cdot\sec x=\tan x+C$

At $x=0,\ y=1$ (given)

$\Rightarrow C=1$

At $x=\frac{\pi}{6}$

$\Rightarrow y\cdot\frac{2}{\sqrt{3}}=\frac{1}{\sqrt{3}}+1$

$\Rightarrow y = \frac{1+\sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2}$

$\Rightarrow y = \frac{1+\sqrt{3}}{2}$

3. The sum of $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \cdots \infty$ terms equals to:

A. $\frac{1}{2}$
B. $\frac{1}{3}$
C. $\frac{1}{4}$
D. $\frac{1}{5}$

Answer (A)

Solution:

$$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \cdots \infty$$
$$= \sum_{r=1}^{\infty} \frac{r}{1+r^2+r^4}$$
$$= \frac{1}{2} \sum_{r=1}^{\infty} \frac{(r^2+r+1)-(r^2-r+1)}{(r^2+r+1)(r^2-r+1)}$$
$$= \frac{1}{2} \sum_{r=1}^{\infty} \frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}$$
$$= \frac{1}{2}\left(1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{7} + \frac{1}{7} - \frac{1}{13} + \cdots\right)$$
$$= \frac{1}{2} \times 1 = \frac{1}{2}$$

4. The number of ways by which letter of word $ASSASSINATION$ can be arranged such that all vowels come together is:

A. $\frac{8!3!}{6!}$
B. $\frac{8!}{4!3!}$
C. $\frac{8!6!}{4!(2!)^2 3!}$
D. $\frac{8!6!}{4!\,3!2!}$

Answer (C)

Solution:

$A \rightarrow 3$ times repeated

$S \rightarrow 4$ times repeated

$I \rightarrow 2$ times repeated

$N \rightarrow 2$ times repeated

$T \rightarrow 1$

$O \rightarrow 1$

A, I & O are vowels

$\therefore$ Number of ways $= \frac{8!}{4!2!} \cdot \frac{6!}{3!2!}$

5. $f(x) + f'(x) = \int_0^2 f(t)dt$ and $f(0) = e^{-2}$, then the value of $f(2) - 2f(0)$ is:

A. 0
B. -1
C. 1
D. 2

Answer (B)

Solution:

$f(x) + f'(x) = \int_0^2 f(t)dt$
Let $k = \int_0^2 f(t)dt$
$\Rightarrow \frac{dy}{dx} + y = k$
$\Rightarrow ye^x = ke^x + C$
$\because f(0) = e^{-2}$
$\Rightarrow e^{-2} = k + C$
$\Rightarrow C = e^{-2} - k$
$\Rightarrow ye^x = ke^x + e^{-2} - k$
$\Rightarrow y = k + (e^{-2} - k)e^{-x}$
Now, $\int_0^2 f(t)\,dt = k$
$\Rightarrow \int_0^2 (k + (e^{-2} - k)e^{-t})\,dt = k$
$\Rightarrow [kt]_0^2 - [e^{-t}(e^{-2} - k)]_0^2 = k$
$\Rightarrow 2k - (e^{-2} - k)(e^{-2} - 1) = k$
$\Rightarrow 2k - (e^{-4} - ke^{-2} - e^{-2} + k) = k$
$\Rightarrow 2k - e^{-4} + ke^{-2} + e^{-2} - k = k$
$\Rightarrow ke^{-2} = e^{-4} - e^{-2}$
$\Rightarrow k = e^{-2} - 1$
$\Rightarrow f(x) = e^{-2} - 1 + e^{-x}$
Now, $f(2) - 2f(0) = (e^{-2} - 1 + e^{-2}) - 2(e^{-2} - 1 + 1)$
$\Rightarrow f(2) - 2f(0) = 2e^{-2} - 1 - 2e^{-2}$
$\Rightarrow f(2) - 2f(0) = -1$

6. If set $S = \left\{\left(\sqrt{3} + \sqrt{2}\right)^{x^2-4} + \left(\sqrt{3} - \sqrt{2}\right)^{x^2-4} = 10\right\}$ then $n(S)$ equals:

A. 2
B. 3
C. 4
D. 6

Answer (C)

Solution:

$\left(\sqrt{3} + \sqrt{2}\right)^{x^2-4} + \left(\sqrt{3} - \sqrt{2}\right)^{x^2-4} = 10$

Let $(\sqrt{2} + \sqrt{3})^{x^2-4} = t$

$\therefore\ t + \frac{1}{t} = 10$

$\Rightarrow\ t^2 - 10t + 1 = 0$

$\Rightarrow\ (t - 5)^2 = 24$

$\Rightarrow t = 5 \pm 2\sqrt{6}$

$\Rightarrow\ \left(\sqrt{2} + \sqrt{3}\right)^{x^2-4} = 5 \pm 2\sqrt{6}$

If $\left(\sqrt{2}+\sqrt{3}\right)^{x^2-4}=5+2\sqrt{6}$

$\Rightarrow \left(\sqrt{2}+\sqrt{3}\right)^{x^2-4}=\left(\sqrt{2}+\sqrt{3}\right)^2$

$\Rightarrow x^2-4=2 \Rightarrow x=\pm\sqrt{6}$

if $\left(\sqrt{2}+\sqrt{3}\right)^{x^2-4}=5-2\sqrt{6}$

$\Rightarrow \left(\sqrt{2}+\sqrt{3}\right)^{x^2-4}=\left(\sqrt{2}+\sqrt{3}\right)^{-2}$

$\Rightarrow x^2-4=-2 \Rightarrow x^2=\pm\sqrt{2}$

$\therefore$ 4 solutions are possible in total.

7. $1,\ 3,\ 5,\ x,\ y$ are 5 observations. Mean of these observations is 5 and variance is 8. Sum of the cubes of the two missing number equals:

A. 1072
B. 513
C. 1079
D. 516

Answer (A)

Solution:

$\bar{x}=5$

$\Rightarrow 1+3+5+x+y=25$

$\Rightarrow x+y=16 \cdots (i)$

$\sigma^2=8=\frac{\sum x_i^2}{5}-(\bar{x})^2$

$\Rightarrow 8=\frac{1^2+3^2+5^2+x^2+y^2}{5}-25$

$\Rightarrow 165=35+x^2+y^2$

$\Rightarrow x^2+y^2=130$

$\Rightarrow (x+y)^2-2xy=130$

$\Rightarrow xy=63 \cdots (ii)$

From (*i*) & (ii),

$x=7, y=9$

Now, $x^3+y^3=7^3+9^3$

$x^3+y^3=343+729=1072$

8. Sum of the series $\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+\cdots+\frac{1}{5!0!}$ is:

A. $\frac{2^{51}}{50!}$
B. 2^{51}
C. $5!\cdot 2^{51}$
D. $\frac{2^{50}}{51!}$

Answer (D)

Solution:

$\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+\cdots+\frac{1}{5!0!}$

$=\frac{1}{51!}\left(\frac{51!}{1!50!}+\frac{51!}{3!48!}+\frac{51!}{5!46!}+\cdots+\frac{51!}{51!0!}\right)$

$=\frac{1}{51!}\left({}^{51}c_1+{}^{51}c_3+-+{}^{51}c_{51}\right)$

$=\frac{1}{51!}\left(\frac{2^{51}}{2}\right)=\frac{2^{50}}{51!}$

9. If $R=\{(a,b):3a-3b+\sqrt{7}$ is irrational$\}$.Then which among the following options are correct

A. R is an equivalence relation

B. R is symmetric but not reflexive

C. R is reflexive but not symmetric

D. R is reflexive and symmetric but not transitive

Answer (C)

Solution:

For reflexive

$3a-3a+\sqrt{7}=\sqrt{7}$ is irrational

$\therefore (a,a)\in R$, $\therefore$ reflexive

For symmetric

$\left(\frac{\sqrt{7}}{3},0\right)\in R\ \ but\ \left(0,\frac{\sqrt{7}}{3}\right)\notin R$

$\Rightarrow$ Relation is not symmetric

For transitive

$\left(\frac{\sqrt{7}}{3},0\right)\in R$, $\left(0,\frac{2\sqrt{7}}{3}\right)\in R$

But $\left(\frac{\sqrt{7}}{3},\frac{2\sqrt{7}}{3}\right)\notin R$

$\Rightarrow$ Relation is not transitive

10. Negation of the statement $p\vee(p\wedge\sim q)$ is:

A. p

B. $\sim p$

C. q

D. $\sim q$

Answer (B)

Solution:

$\because \big(p\vee(p\wedge\sim q)\big)\equiv p$

$\Rightarrow \sim\big(p\vee(p\wedge\sim q)\big)\equiv\sim p$

11. Let S be solution set for values of x satisfying $\cos^{-1}(2x)+\cos^{-1}\sqrt{1-x^2}=\pi$, then $\sum_{x\in S}2\sin^{-1}(x^2-1)$ is equal to:

A. 0

B. $-\sin^{-1}\left(\frac{24}{25}\right)$

C. $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

D. $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Answer (B)

Solution:

$\frac{\pi}{2} - \sin^{-1}(2x) + \frac{\pi}{2} - \sin^{-1}\sqrt{1-x^2} = \pi$
$\Rightarrow \sin^{-1}(2x) + \sin^{-1}\sqrt{1-x^2} = 0$
$\Rightarrow \sin^{-1}(-2x) = \sin^{-1}\sqrt{1-x^2}$
$\Rightarrow -2x = \sqrt{1-x^2}$
$4x^2 = 1 - x^2$
$\Rightarrow x = \pm\sqrt{\frac{1}{5}}$
$x = -\frac{1}{\sqrt{5}}$ is only possible solutions
$\sum_{x\in S} 2\sin^{-1}(x^2-1) = 2\sin^{-1}\left(-\frac{4}{5}\right)$
$= -2\sin^{-1}\frac{4}{5} \quad \cdots\left(2\sin^{-1}x = \sin^{-1}(2x\sqrt{(1-x^2)}\right)$
$= -\sin^{-1}\left(\frac{24}{25}\right)$

12. A triangle be such that $\cos 2A + \cos 2B + \cos 2C$ is minimum. If inradius of the triangle is 3, then which of the following is CORRECT?

A. Area of Δ *is* $\frac{6\sqrt{3}}{2}$ Sq. Units

B. Perimeter of Δ is $18\sqrt{3}$ Units

C. Area of Δ *is* $2\sqrt{3}$ Sq. Units

D. Perimeter of Δ is $9\sqrt{3}$ Units

Answer (B)

Solution:

If $K = \cos 2A + \cos 2\beta + \cos 2C$ is minimum then $k = \frac{-3}{2}$

& $A = B = C = \pi/3$

$\therefore r = \frac{\Delta}{s} = 3 = \frac{\sqrt{3}a^2}{4\times 3a} \times 2$

$\Rightarrow a = 6\sqrt{3}$

$\therefore$ Area $= \frac{\sqrt{3}}{4} \times 36 \times 3 = 27\sqrt{3}$ Sq. Units

$s = 3a = 18\sqrt{3}$ units

$\therefore$ Perimeter is $18\sqrt{3}$ units

13. Area bounded by $y = x|x-3|$ & x −axis between $x = -1$ & $x = 2$ is A then $12A$ equals _______.

Answer 62

Solution:

$$y = x|x-3| = \begin{cases} x(x-3)\ ; x \ge 3 \\ -x(x-3); x \le 3 \end{cases}$$

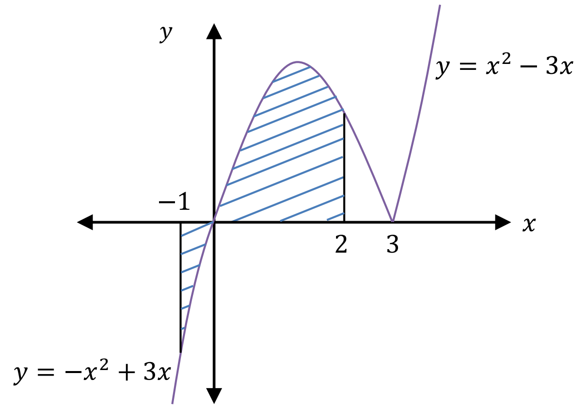
Area= $\int_{-1}^{0}(x^2-3x)dx + \int_{0}^{2}(-x^2+3x)dx$

$= \left[\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-1}^{0} + \left[-\frac{x^3}{3} + \frac{3x^2}{2}\right]_{0}^{2}$

$= \left[0 - \left(-\frac{11}{6}\right)\right] - \left[\frac{-10}{3} - 0\right]$

$= \frac{11}{6} + \frac{10}{3} = \frac{31}{6}$

$\Rightarrow 12A = 12 \times \frac{31}{6} = 62$



14. Remainder when $23^{200} + 19^{200}$ is divided by 49 equals _______.

Answer (2)

Solution:

$23^{200} + 19^{200} = (21+2)^{200} + (21-2)^{200}$
$= 2[\,{}^{200}C_0 21^{200} + {}^{200}C_2 21^{198} + {}^{200}C_4 21^{196} + \cdots + {}^{200}C_{198} 21^2 + {}^{200}C_{200}(21)^0]$
$= 2(49k+1)$
Remainder $= 2$

15. $8, a_1, a_2, \ldots, a_n$ are terms in A.P. Sum of first 4 terms of series is 50 and sum of last 4 terms of series is 170. Then the product of middle terms of series is ___________.

Answer (754)

Solution:

$\frac{4}{2}[16 + 3d] = 50$
$\Rightarrow d = 3$
$\frac{4}{2}[2a_n + 3(-d)] = 170$
$\Rightarrow 2a_n - 3d = 85$
$\Rightarrow 2a_n = 94$
$\Rightarrow a_n = 47$
$\Rightarrow 8 + (n-1)d = 47$
$\Rightarrow n = 14$
So 7^{th} & 8^{th} are middle Term

$T_7 = 8 + 6 \cdot 3 = 26$

$T_8 = 8 + 7 \cdot 3 = 29$

$\therefore T_7 \cdot T_8 = 754$

16. A circle is represented by $\frac{|z-2|}{|z-3|} = 2$. Its radius is γ units and centre is (α, β), then $3(\alpha + \beta + \gamma)$ is equal to _______.

Answer (12)

Solution:

Let $z = x + iy$
$\Rightarrow (x-2)^2 + y^2 = 4(x-3)^2 + 4y^2$
$\Rightarrow x^2 + y^2 - 4x + 4 = 4x^2 - 24x + 36 + 4y^2$

$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$
$\Rightarrow x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$
Centre$\equiv \left(\frac{10}{3}, 0\right)$
$\Rightarrow r = \sqrt{\left(\frac{10}{3}\right)^2 + 0^2 - \frac{32}{3}} = \frac{2}{3}$
$\Rightarrow 3(\alpha + \beta + \gamma) = 12$

17. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = 2x + f'(1)$ then $f(4) - g(4)$ equals _______.

Answer (12)

Solution:

$g(x) = 2x + f'(1)$
$\Rightarrow g'(x) = 2$
$\Rightarrow g'(1) = 2$ and $g''(x) = 0$
Now, $f(x) = x^2 + xg'(1) + g''(2)$
$f(x) = x^2 + 2x$
$\Rightarrow f'(x) = 2x + 2 \Rightarrow f'(1) = 4$
$\therefore g(x) = 2x + 4$
$f(4) - g(4) = (16 + 8) - (8 + 4)$
$= 12$

18. For some values of λ, system of equations
$\lambda x + y + z = 1,$
$x + \lambda y + z = 1,$
$x + y + \lambda z = 1$ has no solution, then $\sum(|\lambda|^2 + |\lambda|)$ equals __________.

Answer (6)

Solution:

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$
$\Rightarrow \lambda(\lambda^2 - 1) - 1(\lambda - 1) + 1(1 - \lambda) = 0$
$\Rightarrow (\lambda - 1)(\lambda^2 + \lambda - 1 - 1) = 0$
$\Rightarrow \lambda = 1, -2$
For $\lambda = 1$ There are infinite solution
For $\lambda = -2$ system has no solution
$\sum(|\lambda|^2 + |\lambda|) = 4 + 2 = 6$

19. If solution of $\frac{dy}{dx} + \frac{x+a}{y-2} = 0$ is a circle and $y(0) = 1$, area of circle is 2π. P and Q are point of intersection of circle with y-axis. Normal at P and Q intersect x −axis at R and S. The length of RS is:

Answer (4)

Solution:

$\frac{dy}{dx} + \frac{x+a}{y-2} = 0$
$\Rightarrow (y - 2)dy = -(x + a)dx$
$\Rightarrow \frac{(y-2)^2}{2} = -\frac{(x+a)^2}{2} + C$
$\Rightarrow (x + a)^2 + (y - 2)^2 = 2C$
$\because y(0) = 1$
$\Rightarrow a^2 + 1 = 2C$
Area $= 2\pi$

$\Rightarrow \pi(2C) = 2\pi \Rightarrow C = 1$
$\Rightarrow a^2 + 1 = 2 \Rightarrow a = \pm 1$
CASE I:
Equation of circle $(x+1)^2 + (y-2)^2 = 2$
$C \equiv (-1, 2)$
For P & Q, $x = 0$
$\Rightarrow y - 2 = \pm 1$
$\Rightarrow P \,\&\, Q \equiv (0, 3) \& (0, 1)$
Normal equation$\Rightarrow y - 3 = \frac{3-2}{(0+1)}(x - 0)$
$\Rightarrow x - y + 3 = 0$
$y - 1 = \frac{1-2}{0+1}(x - 0)$
$\Rightarrow y + x - 1 = 0$
$R \,\&\, S \equiv (-3, 0) \,\&\, (1, 0)$
$\Rightarrow RS = 4$
CASE II:
Equation of circle $(x-1)^2 + (y-2)^2 = 2$
$C \equiv (1, 2)$
For P & Q, $x = 0$
$\Rightarrow y - 2 = \pm 1$
$\Rightarrow P \,\&\, Q \equiv (0, 3) \& (0, 1)$
Normal equations at P & Q are
$y - 3 = \frac{3-2}{(0-1)}(x - 0)$
$\Rightarrow x + y - 3 = 0$ and
$y - 1 = \frac{1-2}{(0-1)}(x - 0)$
$\Rightarrow x - y + 1 = 0$
$R \,\&\, S \equiv (3, 0) \,\&\, (-1, 0)$
$\Rightarrow RS = 4$

20. Number of 3-digit numbers which are divisible by 2 or 3 but not divisible by 7 is ____________.

Answer (514)

Solution:

We know that
$T_n = a + (n-1)d$
So, numbers divisible by 2 is:
$998 = 100 + (n_2 - 1)2$
$\Rightarrow n_2 = 450$
Numbers divisible by 3 is:
$999 = 102 + (n_3 - 1)3$
$\Rightarrow n_3 = 300$
Numbers divisible by 2 & 3 is:
$996 = 102 + (n_{2\,\&\,3} - 1)6$
$\Rightarrow n_{2\,\&\,3} = 150$
Numbers divisible by 2 & 7 is:
$994 = 112 + (n_{2\,\&\,7} - 1)14$
$\Rightarrow n_{2\,\&\,7} = 64$
Numbers divisible by 3 & 7 is:
$987 = 105 + (n_{3\,\&\,7} - 1)21$
$\Rightarrow n_{3\,\&\,7} = 43$
Numbers divisible by 2, 3 & 7 is:
$966 = 126 + \left(n_{2,3\,\&\,7} - 1\right)42$
$\Rightarrow n_{2,3\,\&\,7} = 21$
Only $A = 450 - (43 + 150) = 257$
Only $B = 300 - (22 + 150) = 128$
Total numbers which are divisible by 2 or 3 but not divisible by 7 $= 257 + 129 + 128 = 514$